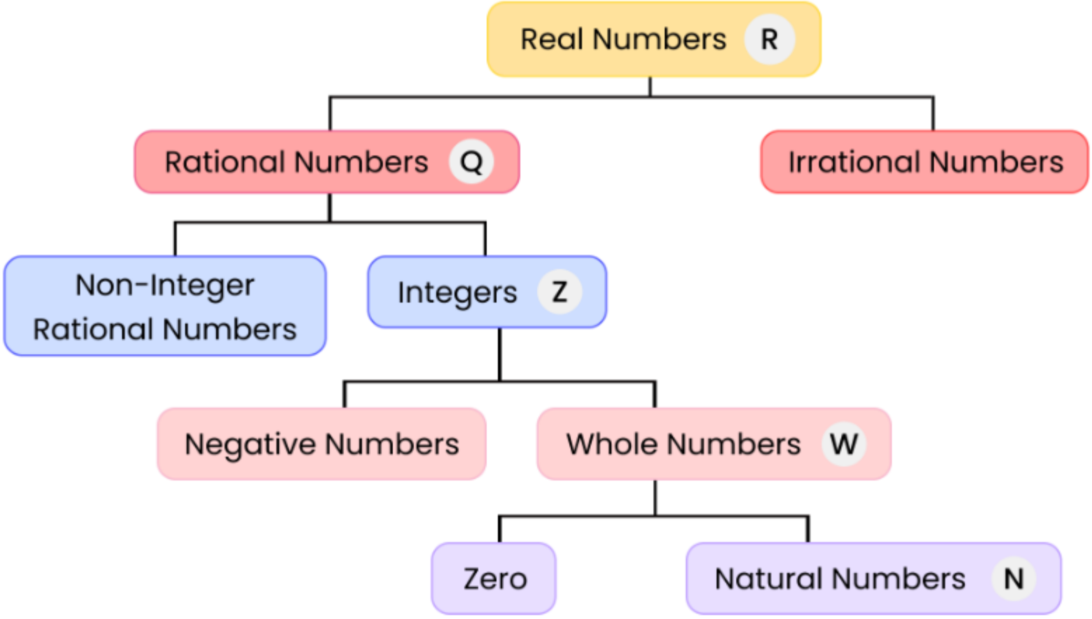
1. Real Numbers



- Set of Natural Number (N) = {1, 2, 3, 4, 5, . . . }
- Set of Whole Number (W) = {0, 1, 2, 3, . . . }
- Set of Integers (Z) = {. . ., -3, -2, -1, 0, 1, 2, 3, . . . }
- Rational Numbers:- The numbers which can be written in the form of $\frac{p}{q}$, where p, q are integers and q $\neq$ 0 are called rational numbers. Set of rational numbers is denoted by the letter Q.

  Example: 0, 2, -3, $\frac{2}{3}$, . . .

- Irrational Numbers:- The numbers which cannot be written in the form of $\frac{p}{q}$, where p, q are integers and q $\neq$ 0 are called irrational numbers. Set of irrational numbers is denoted by the letter Q^1 or S.

  Example: $\sqrt{2}$, $\sqrt{3}$, π, . . .
- Real Numbers (R) : Combination of rational and irrational numbers are called real numbers.
- $\sqrt{p}$ is an irrational when 'p' is a prime.
- If a prime number p divides a^2, then p divides a.
- The sum or difference of a rational and an irrational number is irrational.
- The product and quotient of a non-zero rational and irrational number are irrational.
- Prime numbers are whole numbers whose only factors are 1 and itself.

  Example : 2, 3, 5, 7, 11, 13, . . .
- Composite number are the positive integers which has factors other than 1 and itself.

  Example : 4, 6, 8, 9, 10, 12, . . .

The Fundamental Theorem of Arithmetic (Theorem 1.1): Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Example : $32760 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 13$

$= 2^3 \times 3^2 \times 5 \times 7 \times 13$

Example 1 : Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution : If any number ends with the digit 0 that means it should be divisible by both 2 and 5. That is, if 4^n ends with the digit 0, then the prime factorization of 4^n would contain the primes 2 and 5.

Prime factors of $4^n = (2\times 2)^n = (2^2)^n = 2^{2n}$

We can observe clearly, 5 is not in the prime factors of 4^n .

That means 4^n will not be divisible by 5.

∴ 4^n cannot end with the digit 0 for any natural number n.

Finding HCF and LCM by using Fundamental theorem of Arithmetic :

- HCF of the given pair of integers = Product of the smallest power of each common prime factor in the numbers.
- LCM of the given pair of integers = Product of the greatest power of each prime factor, involved in the number.

Example 2 : Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Solution : $6 = 2 \times 3 = 2^1 \times 3^1$.

$20 = 2 \times 2 \times 5 = 2^2 \times 5^1$.

HCF (6, 20) $= 2^1 = 2$

LCM (6, 20) $= 2^2 \times 3^1 \times 5^1 = 4 \times 3 \times 5 = 60$

Example 3: Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Solution : $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3^1$,

$404 = 2 \times 2 \times 101 = 2^2 \times 101^1$

∴ HCF of 96 and 404 $= 2^2 = 4$.

We know that, LCM x HCF = Product of two numbers

⇨ LCM x 4 = 96 x 404

$\Rightarrow$ LCM = $\frac{96 \times 404}{4}$ = 96 x 101 = 9696

Example 4 : Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

Solution : $6 = 2 \times 3 = 2^1 \times 3^1$,

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$,

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$

$\therefore$ HCF of 6, 72, 120 = $2^1 \times 3^1 = 2 \times 3 = 6$

and LCM of 6, 72, 120 = $2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

Note : *The product of three numbers is not equal to the product of their HCF and LCM.*

EXERCISE 1.1

1. Express each number as a product of its prime factors:

(i) 140

$$\begin{array}{r|l} 2 & 140 \\ 2 & 70 \\ 5 & 35 \\ & 7 \end{array}$$

Solution : Prime factors of 140 = 2 x 2 x 5 x 7

$= 2^2 \times 5 \times 7$

(iii) 3825

$$\begin{array}{r|l} 3 & 3825 \\ 3 & 1275 \\ 5 & 425 \\ 5 & 85 \\ & 17 \end{array}$$

Solution : Prime factors of 3825 = 3 x 3 x 5 x 5 x 17

$= 3^2 \times 5^2 \times 17^1$

(v) 7429

$$\begin{array}{r|l} 17 & 7429 \\ 19 & 437 \\ & 23 \end{array}$$

Solution : Prime factors of 7429 = 17 x 19 x 23

$= 17^1 \times 19^1 \times 23^1$

(ii) and (iv) Home Work

2. Find the LCM and HCF of the following pairs of integers and verify that

LCM × HCF = product of the two numbers.

(i) 26 and 91

Solution : Prime factors of 26 = 2 x 13 = $2^1 \times 13^1$

Prime factors of 91 = 7 x 13 = $7^1 \times 13^1$

HCF of 26 and 91 = 13^1 = 13

LCM of 26 and 91 = $2^1 \times 7^1 \times 13^1$ = 2 x 7 x 13 = 182

Product of two numbers = 26 x 91 = 2366

LCM x HCF = 182 x 13 = 2366

So, product of two numbers = LCM × HCF

(iii) 336 and 54

Solution : Prime factors of 336 = 2 x2 x 2 x 2 x 3 x 7 = $2^4 \times 3^1 \times 7^1$

Prime factors of 54 = 2 x 3 x 3 x 3 = $2^1 \times 3^3$

HCF of two numbers = $2^1 \times 3^1$ = 2 x 3 = 6

LCM of two numbers = $2^4 \times 3^3 \times 7^1$ = 16 x 27 x 7 = 3024

Product of two numbers 336 x 54 = 18144

LCM x HCF = 3024 x 6 = 18144

So, Product of two numbers = LCM × HCF

(ii) → Home work

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Solution : Prime factors of 12 = 2 x 2 x 3 = $2^2 \times 3^1$

Prime factors of 15 = 3 x 5 = $3^1 \times 5^1$

Prime factors of 21 = 3 x 7 = $3^1 \times 7^1$

HCF of 12 15 and 21 = 3^1 = 3

LCM of 12 15 and 21 = $2^2 \times 3^1 \times 5^1 \times 7^1$ = 4 x 3 x 5 x 7 = 420

(ii) 17, 23 and 29

Solution : Prime factors of 17 = 17^1

Prime factors of 23 = 23^1

Prime factors of 29 = 29^1

HCF of 17, 23 and 29 = 1 (Since, they have no common factors)

LCM of 17 23 and 29 = 17 x 23 x 29 = 11339

Note : HCF of co-primes is always 1 & LCM of co-primes is the product of the numbers.

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution : We know that LCM x HCF = Product of two numbers

⇨ LCM x 9 = 306 x 657

⇨ LCM = $\frac{306 \times 657}{9}$

⇨ LCM = 34 x 657

⇨ LCM = 22338

5. Check whether 6^n can end with the digit 0 for any natural number n. (IMP)

Solution : If any number ends with the digit 0 that means it should be divisible by both 2 and 5. That is, if 6^n ends with the digit 0, then the prime factorization of 6^n would contain the primes 2 and 5.

Prime factors of $6^n = (2\times3)^n = 2^n \times 3^n$

We can observe clearly, 5 is not in the prime factors of 6^n .

That means 6^n will not be divisible by 5.

∴ 6^n cannot end with the digit 0 for any natural number n.

6. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers. (IMP)

Solution : 7 x 11 x 13 + 13 = 13(7 x 11 + 1)

= 13(77 + 1)

= 13(78)

= 13 x 13 x 2 x 3 x 1

The given number has 2,3,13 and 1 as its factors.

Therefore, it is a composite number.

7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5(7 x 6 x 4 x 3 x 2 x 1 + 1)

= 5(1008 + 1)

= 5 x 1009 x 1

The given expression has 5, 1009 and 1 as its factors.

Therefore, it is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution : To do this we have to find LCM of both the numbers.

Prime factors of 12 = 2 x 2 x 3 = $2^2 \times 3^1$

Prime factors of 18 = 2 x 3 x 3 = $2^1 \times 3^2$

LCM of 18 and 12 = $2^2 \times 3^2$

= 4 x 9

= 36

∴ Ravi and Sonia will meet together at starting point after 36 minutes.

Irrational Numbers :-

A number ‘s’ is called irrational if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. (or) Non- terminating non-recurring decimals are known as irrational numbers.

Example : $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π, 0.10110111011110 . . ., etc.,

Theorem 1.2 : Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer. (*Proof not needed for examination*)

Theorem 1.3 : $\sqrt{2}$ is irrational. (IMP)

Proof : Let us assume that $\sqrt{2}$ is rational.

⇨ $\sqrt{2} = \frac{a}{b}$ (Where a and b are integers, b ≠ 0 and a, b are co-primes)

Now square both the sides, $\left(\sqrt{2}\right)^2 = \left(\frac{a}{b}\right)^2$

⇨ $2 = \frac{a^2}{b^2}$

⇨ $2b^2 = a^2$

if a^2 is divisible by 2 that means a is also divisible by 2. [*By Theorem-1.2*]

So, we can write a = 2c

Substituting for a, we get $2b^2 = (2c)^2$

⇨ $2b^2 = 4c^2$

⇨ $b^2 = 2c^2$

This means b^2 is divisible by 2 and so b is also divisible by 2.

Therefore, a and b have 2 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is a rational number.

So, we conclude that $\sqrt{2}$ is an irrational number.

Example 5 : Prove that $\sqrt{3}$ is irrational. (IMP)

Solution : Let us assume that $\sqrt{3}$ is rational.

⇨ $\sqrt{3} = \frac{a}{b}$ (Where a and b are integers, b ≠ 0 and a, b are co-primes)

Now square both the sides, $\left(\sqrt{3}\right)^2 = \left(\frac{a}{b}\right)^2$

⇨ $3 = \frac{a^2}{b^2}$

⇨ $3b^2 = a^2$

if a^2 is divisible by 3 that means a is also divisible by 3. [*By Theorem-1.2*]

So, we can write a = 3c

Substituting for a, we get $3b^2 = (3c)^2$

⇨ $3b^2 = 9c^2$

⇨ $b^2 = 3c^2$

This means b^2 is divisible by 3 and so b is also divisible by 3.

Therefore, a and b have 3 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is a rational number.

So, we conclude that $\sqrt{3}$ is an irrational number.

Example 6 : Show that $5 - \sqrt{3}$ is irrational. (IMP)

Solution : Let us assume that $5 - \sqrt{3}$ is rational.

$\Rightarrow 5 - \sqrt{3} = \frac{a}{b}$ (Where a and b are integers, $b \neq 0$)

$\Rightarrow 5 - \frac{a}{b} = \sqrt{3}$

$\Rightarrow \sqrt{3} = 5 - \frac{a}{b}$ [By rearranging the equation]

$\Rightarrow \sqrt{3} = \frac{5b - a}{b}$

Since a and b are integers, we get $\frac{5b-a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

Example 7 : Show that $3\sqrt{2}$ is irrational. (IMP)

Solution : Let us assume that $3\sqrt{2}$ is rational.

$\Rightarrow 3\sqrt{2} = \frac{a}{b}$ (Where a and b are integers, $b \neq 0$)

$\Rightarrow \sqrt{2} = \frac{a}{3b}$

Since a and b are integers, we get $\frac{a}{3b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3\sqrt{2}$ is rational.

So, we conclude that $3\sqrt{2}$ is irrational.

EXERCISE 1.2

1. Prove that $\sqrt{5}$ is irrational. (IMP)

Solution : Let us assume that $\sqrt{5}$ is rational.

$\Rightarrow \sqrt{5} = \frac{a}{b}$ (Where a and b are integers, $b \neq 0$ and a, b are co-primes)

Now square both the sides, $\left(\sqrt{5}\right)^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 5 = \frac{a^2}{b}$

$\Rightarrow 5b^2 = a^2$

if a^2 is divisible by 5 that means a is also divisible by 5. [*By Theorem-1.2*]

So, we can write a = 5c

Substituting for a, we get $5b^2 = (5c)^2$

$\Rightarrow 5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

This means b^2 is divisible by 5 and so b is also divisible by 5.

Therefore, a and b have 5 as a common factor.

But this contradicts the fact that a and b are co-prime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{5}$ is a rational number.

So, we conclude that $\sqrt{5}$ is an irrational number.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

Solution : Let us assume that $3 + 2\sqrt{5}$ is rational.

$\Rightarrow 3 + 2\sqrt{5} = \frac{a}{b}$ (Where a and b are integers, $b \neq 0$)

$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow 2\sqrt{5} = \frac{a-3b}{b}$

$\Rightarrow \sqrt{5} = \frac{a-3b}{2b}$

Since a and b are integers, we get $\frac{a-3b}{2b}$ is rational, and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$

Solution : Let us assume that $\frac{1}{\sqrt{2}}$ is rational.

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a}{b}$ (Where a and b are integers, $b \neq 0$)

$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{2a}{b}$

Since a and b are integers, we get $\frac{2a}{b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $\frac{1}{\sqrt{2}}$ is rational.

So, we conclude that $\frac{1}{\sqrt{2}}$ is irrational.

(iii) 6 + $\sqrt{2}$

Solution : Let us assume that 6 + $\sqrt{2}$ is rational.

$\Rightarrow 6 + \sqrt{2} = \frac{a}{b}$ (Where a and b are integers, b $\neq$ 0)

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$

$\Rightarrow \sqrt{2} = \frac{a-6b}{b}$

Since a and b are integers, we get $\frac{a-6b}{b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that 6 + $\sqrt{2}$ is rational.

So, we conclude that 6 + $\sqrt{2}$ is irrational.

(ii) → Home Work

Practice Questions

1. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is

(A) 4 (B) 2 (C) 1 (D) 3

2. If two positive integers a and b are written as a = $x^3 y^2$ and b = xy^3 ; x, y are prime numbers, then HCF (a, b) is

(A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2

3. If two positive integers p and q can be expressed as p = ab^2 and q = a^3b; a, b being prime numbers, then LCM (p, q) is

(A) ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3

4. The product of a non-zero rational and an irrational number is

(A) always irrational (B) always rational (C) rational or irrational (D) none

5. Which of the following is a rational number ?

(A) $\sqrt{2}$ (B) π (C) 0.346666... (D) 0.35214067...

6. Which of the following is not an irrational number ?

(A) $\sqrt{5}$ (B) $\sqrt{9}$ (C) $\frac{2}{\sqrt{3}}$ (D) 1.1010010001...

7. Assertion (A) : $\sqrt{11}$ is an irrational number.

Reason (R) : $\sqrt{p}$ is an irrational when 'p' is a prime.

(A) Both A and R are true and R is correct explanation of A.

(B) Both A and R are correct but R is not correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

8. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

9. Show that 12^n cannot end with the digit 0 for any natural number n.

10. Find the LCM and HCF of 24, 32 and 48 by applying the prime factorisation method.

11. Find the LCM and HCF of 18 and 24 by the prime factorisation method.

12. Show that $2 - 5\sqrt{3}$ is irrational.

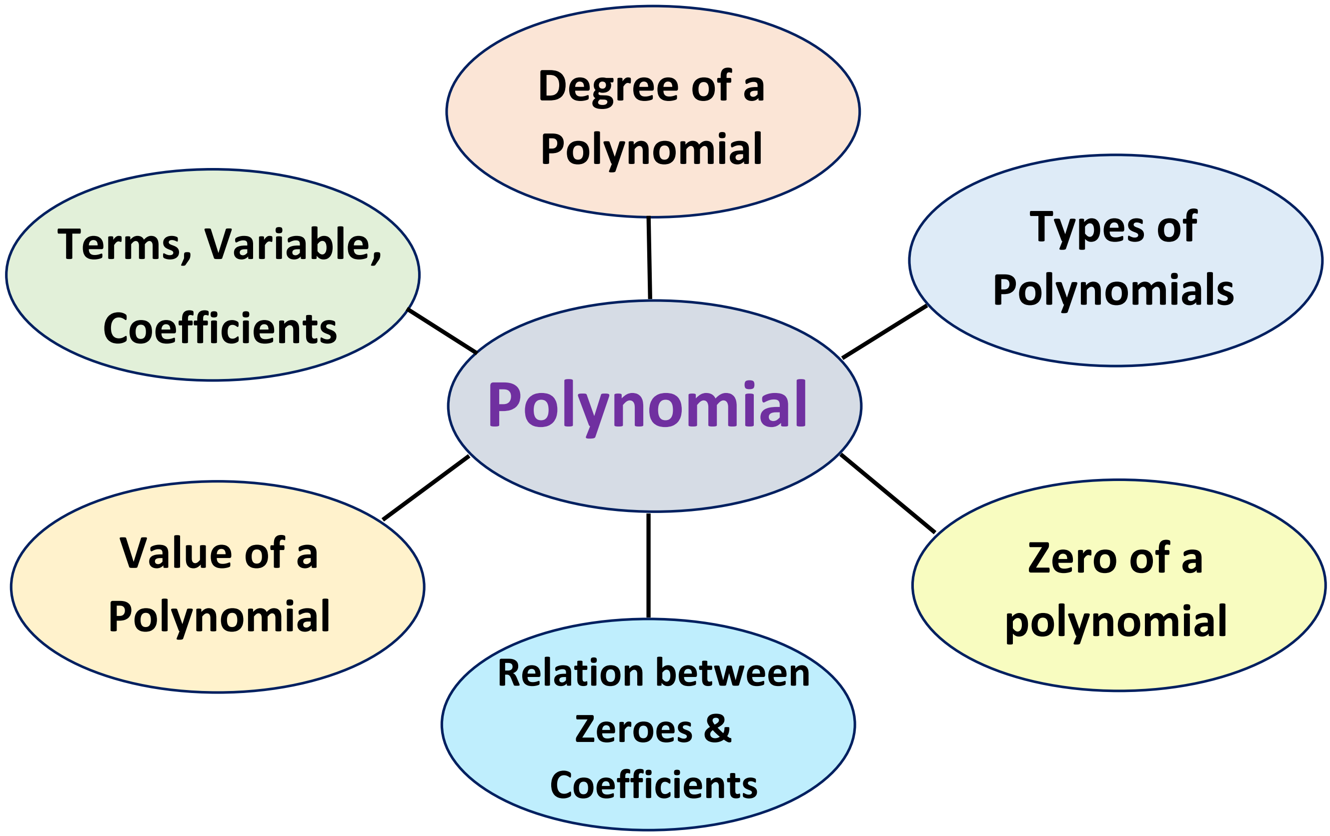
13. Show that $\sqrt{7}$ is irrational.

14. If xy = 180 and HCF (x, y) = 3, then find LCM (x, y).

15. Three bells ring at an interval of 4, 7 and 14 minutes respectively. All the three bells rang at 6 a.m. when the three bells will be ring together next ?

✦✦✦

2. Polynomials



Key Concepts :

- A mathematical expression dealing with variables is called an algebraic expression, Example : $3x + 2y + 5$, $3x^2y$, $4y - \frac{1}{y}$, …

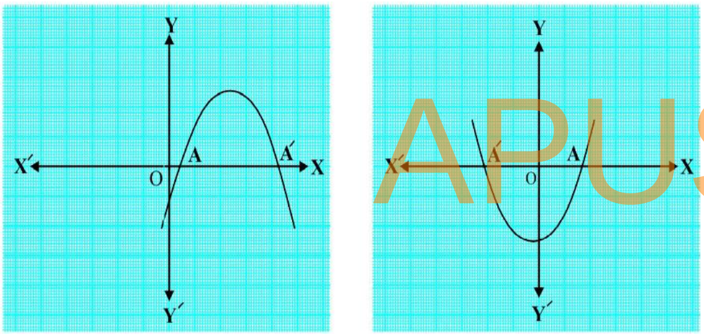
- A variable is a letter or symbol we don't know yet. (ex: x, y, z, t, u, …)
- A number on its own is called constant. (ex: 3, -2, $\frac{1}{5}$, …)
- A number multiplied by a variable is called coefficient. (ex: In $3x^2$, 3 is coefficient)
- An algebraic expression in which the variable(s) is/are raised to non-negative integral exponents is called a polynomial.

  Example : $3x + 2$, $x^2 + \frac{1}{2}x - 3$, $5y - y^3$, ….
- The Degree of a Polynomial p(x) is the highest exponent to which x is raised.
- Types of Polynomials (based on Degree):
  - *Linear Polynomial* : Has the highest exponent (degree) is 1 on the variable.
  - *Quadratic Polynomial* : Has highest exponent (degree) is 2.
  - *Cubic Polynomial* : Has highest exponent (degree) is 3.

Polynomial	Terms	Variable	Coefficient	Constant	Degree	Name
$3x - 1$	$3x$, -1	x	3	-1	1	Linear polynomial
$2y^2 - 5y + 7$	$2y^2$, $-5y$, 7	y	2, -5	7	2	Quadratic
$4u^2 + 3u^3 + 2$	$4u^2$, $3u^3$, 2	u	4, 3	2	3	Cubic polynomial

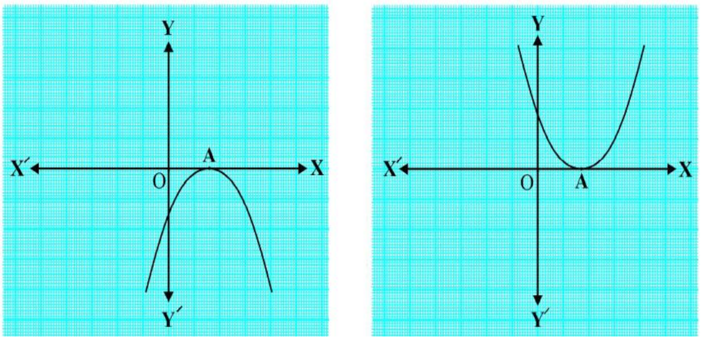
- Number of zeroes of a polynomial depends on its degree.
- A polynomial p(x) of degree n has at most n zeroes.
- The Value of a Polynomial p(x) at x = k is obtained by replacing x = k in the polynomial expression.
- A real number 'a' is a Zero of a Polynomial p(x) if p(a) = 0.
- Polynomials can be visualized as graphs. The shape of the graph depends on the degree of the polynomial.
    - The graph of a linear polynomial is a straight line.
    - The graph of a quadratic polynomial is a parabola.
- If the graph of a polynomial intersect x – axis in n points, then the polynomial has 'n' zeroes.
- The shape of the graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is depends on the value of 'a'. If a > 0, then it is open upwards like ∪. If a < 0, then open downwards like ∩.
- The graph of $ax^2 + bx + c$ can be seen in the following three cases.

Case (i) :



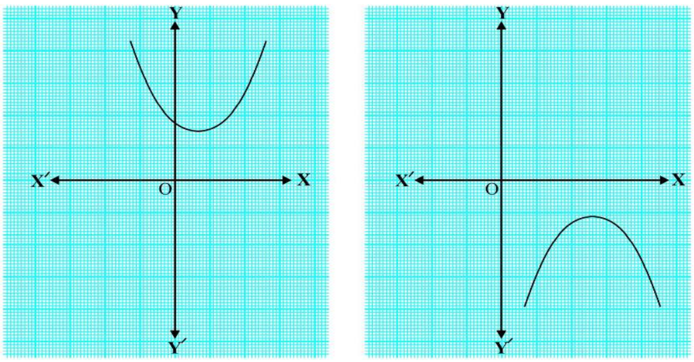
➔ In this case the given polynomial has two zeroes.

Case (ii) :



➔ In this case the given polynomial has only one zero.

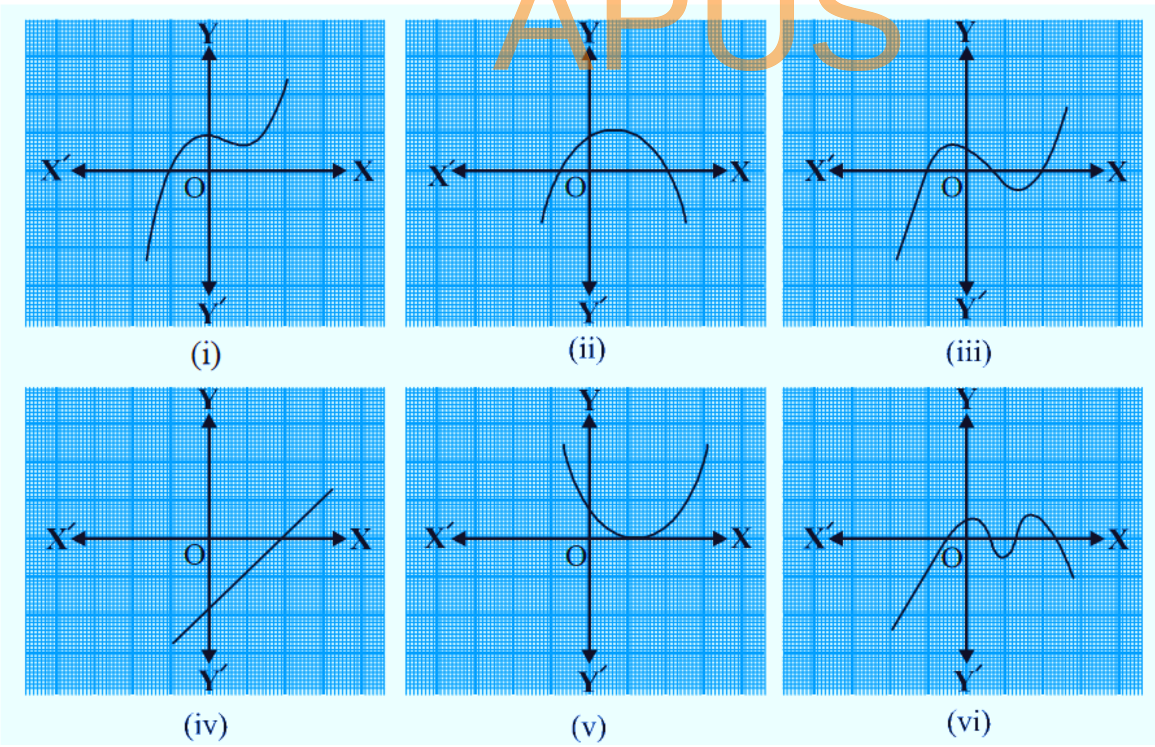
Case (iii) :



➔ In this case the given polynomial has no zeroes.

- There is a connection between the zeros of a polynomial and its coefficients.
- Quadratic Polynomial
  - General form: $p(x) = ax^2 + bx + c$
  - Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = -\frac{b}{a}$
  - Product of zeroes = $\boldsymbol{\alpha\beta} = \frac{c}{a}$
- Cubic Polynomial
  - General form: $p(x) = ax^3 + bx^2 + cx + d$
  - Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} + \boldsymbol{\gamma} = -\frac{b}{a}$
  - Sum of product of zeroes taken two at a time = $\boldsymbol{\alpha\beta} + \boldsymbol{\beta\gamma} + \boldsymbol{\gamma\alpha} = \frac{c}{a}$
  - Product of zeroes = $\boldsymbol{\alpha\beta\gamma} = -\frac{d}{a}$
- The quadratic polynomial whose zeroes are $\boldsymbol{\alpha}, \boldsymbol{\beta}$ is $k[x^2 - (a + b)x + ab]$ where k is a constant.
- $\boldsymbol{\alpha}, \boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are Greek letters pronounced as 'alpha', 'beta' and 'gamma' respectively.

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of y = p(x), where p(x) is a polynomial. For each of the graphs, find the number of zeroes of p(x).



Solution :

(i) Since the graph intersects the x-axis at one point only, the number of zeroes is 1.

(ii) Since the graph intersects the x-axis at two points, the number of zeroes is 2.

(iii) Since the graph intersects the x-axis at three points, the number of zeroes is 3.

(iv) Since the graph intersects the x-axis at one point only, the number of zeroes is 1.

(v) Since the graph intersects the x-axis at one point only, the number of zeroes is 1.

(vi) Since the graph intersects the x-axis at four points, the number of zeroes is 4.

EXERCISE 2.1

1. The graphs of y = p(x) are given in Fig. 2.10 below, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

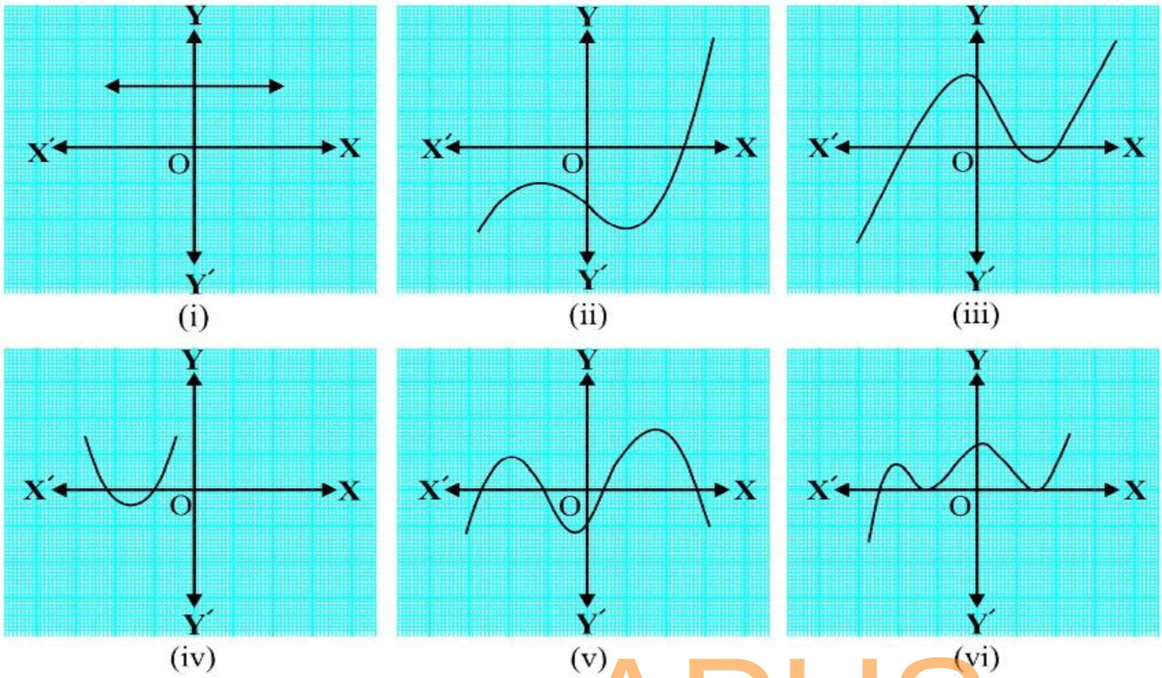


Fig. 2.10

Solution :

(i) Since the graph does not cut the x-axis at any point, the number of zeroes is 0.

(ii) Since the graph intersects the x-axis at only one point, the number of zeroes is 1.

(iii) Since the graph intersects the x-axis at three points, the number of zeroes is 3.

(iv) Since the graph intersects the x-axis at two points, the number of zeroes is 2.

(v) Since the graph intersects the x-axis at four points, the number of zeroes is 4.

(vi) Since the graph intersects the x-axis at three points, the number of zeroes is 3.

Relationship between Zeroes and Coefficients of a Polynomial :-

Quadratic Polynomial :- If $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ are zeroes of the quadratic polynomial $p(x)$ =

$ax^2 + bx + c$, then

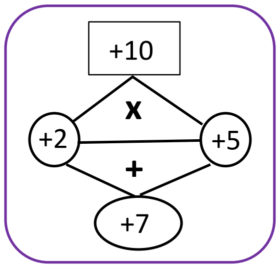
- Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$
- Product of zeroes = $\boldsymbol{\alpha\beta} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

Example 2 : Find the zeroes of the quadratic polynomial x2 + 7x + 10, and verify the relationship between the zeroes and the coefficients. [IMP]

Solution : We have given $x^2 + 7x + 10$

First we have to find zeroes of the given polynomial. To do this,

Let $x^2 + 7x + 10 = 0$



⇨ $x^2 + 2x + 5x + 10 = 0$

⇨ $x(x + 2) + 5(x + 2) = 0$

⇨ $(x + 2)(x + 5) = 0$

⇨ $x + 2 = 0$ (or) $x + 5 = 0$

⇨ $x = -2$ (or) $x = -5$

Therefore, the zeroes of $x^2 + 7x + 10$ are – 2 and – 5.

Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = (-2) + (-5) = -7 = \frac{-7}{1} \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\boldsymbol{\alpha\beta} = (-2)(-5) = 10 = \frac{10}{1} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

Solution : We have given $x^2 - 3$

First we have to find zeroes of the given polynomial. To do this,

Let $x^2 - 3 = 0$

⇨ $x^2 - (\sqrt{3})^2 = 0$

⇨ $(x + \sqrt{3})(x - \sqrt{3}) = 0$

$$a^2 - b^2 = (a + b)(a - b)$$

⇨ $x + \sqrt{3} = 0$ (or) $x - \sqrt{3} = 0$

⇨ $x = -\sqrt{3}$ (or) $x = \sqrt{3}$

Therefore, the zeroes of $x^2 - 3$ are $-\sqrt{3}$ and $\sqrt{3}$.

Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = (-\sqrt{3}) + (\sqrt{3}) = 0 = \frac{-0}{1} \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\boldsymbol{\alpha\beta} = (-\sqrt{3})(\sqrt{3}) = -3 = \frac{-3}{1} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2 respectively.

Solution : Let the zeroes of the quadratic polynomial be $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

Given that, $\boldsymbol{\alpha} + \boldsymbol{\beta} = -3$

$\boldsymbol{\alpha\beta} = 2$

Quadratic polynomial = $k[x^2 – (a + b)x + ab]$

Quadratic polynomial = $k[x^2 – (– 3)x + 2]$

$= k[x^2 + 3x + 2]$

If the value of k = 1, then the quadratic polynomial is $x^2 + 3x + 2$

Cubic Polynomial:-

- If α, β and γ are zeroes of the cubic polynomial, $p(x) = ax^3 + bx^2 + cx + d$, then
- Sum of zeroes = $\alpha + \beta + \gamma = -\frac{b}{a}$
- Sum of product of zeroes taken two at a time = $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
- Product of zeroes = $\alpha\beta\gamma = -\frac{d}{a}$

Example 5 : Verify that 3, –1, $-\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 – 5x^2 – 11x – 3$, and then verify the relationship between the zeroes and the coefficients. (Not from the examination point of view)

Solution : Given $p(x) = 3x^3 – 5x^2 – 11x – 3$

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$,

we get a = 3, b = – 5, c = –11, d = – 3.

$p(3) = 3(3)^3 – 5(3)^2 – 11(3) – 3$

$= 3(27) – 5(9) – 11(3) – 3$

$= 81 – 45 – 33 – 3$

$= 81 – 81$

$= 0$

$p(–1) = 3(–1)^3 – 5(–1)^2 – 11(–1) – 3$

$= 3(- 1) – 5(1) – 11(- 1) - 3$

$= –3 – 5 + 11 – 3$

$= 11 - 11$

$= 0$

$p(–1) = 3\left(-\frac{1}{3}\right)^3 – 5\left(-\frac{1}{3}\right)^2 – 11\left(-\frac{1}{3}\right) - 3$

$= 3\left(-\frac{1}{27}\right) – 5\left(\frac{1}{9}\right) – 11\left(-\frac{1}{3}\right) - 3$

$= -\frac{1}{9} – \frac{5}{9} + \frac{11}{3} – 3$

$= \frac{-1-5+33-27}{9}$

$= \frac{33-33}{9} = \frac{0}{9} = 0$

Therefore, 3, –1 and $-\frac{1}{3}$ are the zeroes of $3x^3 – 5x^2 – 11x – 3$.

Sum of zeroes = $\alpha + \beta + \gamma = 3 + (-1) + \left(-\frac{1}{3}\right) = 2 - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = -\frac{b}{a}$

Sum of product of zeroes taken two at a time = $\alpha\beta + \beta\gamma + \gamma\alpha$

$$= (3)(-1) + (-1)\left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)(3)$$

$$= -3 + \frac{1}{3} - 1 = -4 + \frac{1}{3} = \frac{-12+1}{3} = \frac{-11}{3} = \frac{c}{a}$$

Product of zeroes = $\alpha\beta\gamma = (3)(-1)\left(-\frac{1}{3}\right) = \frac{3}{3} = \frac{-(-3)}{3} = -\frac{d}{a}$

EXERCISE 2.2

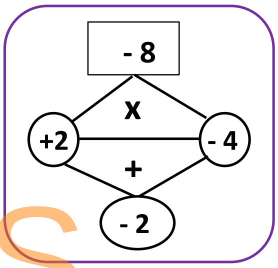
1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

Solution : We have given $x^2 - 2x - 8$

First we have to find zeroes of the given polynomial. To do this,

Let $x^2 - 2x - 8 = 0$



⇨ $x^2 + 2x - 4x - 8 = 0$

⇨ $x(x + 2) - 4(x + 2) = 0$

⇨ $(x + 2)(x - 4) = 0$

⇨ $x + 2 = 0$ (or) $x - 4 = 0$

⇨ $x = -2$ (or) $x = 4$

Therefore, the zeroes of $x^2 - 2x - 8$ are -2 and 4.

Sum of zeroes = $\alpha + \beta = (-2) + (4) = 2 = \frac{-(-2)}{1} \ \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\alpha\beta = (-2)(4) = -8 = \frac{-8}{1} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

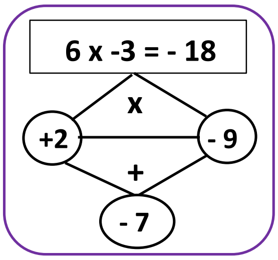
(iii) $6x^2 - 3 - 7x$

Solution : We have given $6x^2 - 3 - 7x$

First we have to find zeroes of the given polynomial. To do this,

Let $6x^2 - 3 - 7x = 0$

⇨ $6x^2 - 7x - 3 = 0$ [when writing in an order]



⇨ $6x^2 + 2x - 9x - 3 = 0$

⇨ $2x(3x + 1) - 3(3x + 1) = 0$

⇨ $(3x + 1)(2x - 3) = 0$

⇨ 3x + 1 = 0 (or) 2x - 3 = 0

⇨ 3x = - 1 (or) 2x = 3

⇨ x = $\frac{-1}{3}$ (or) x = $\frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = \left(-\frac{1}{3}\right) + \left(\frac{3}{2}\right) = \frac{-2+9}{6} = \frac{7}{6} = \frac{-(-7)}{6}$

$= \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\boldsymbol{\alpha\beta} = \left(-\frac{1}{3}\right)\left(\frac{3}{2}\right) = \frac{-3}{6} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

(iv) $4u^2 + 8u$

Solution : We have given $4u^2 + 8u$

First we have to find zeroes of the given polynomial. To do this,

Let $4u^2 + 8u = 0$

⇨ 4u(u + 2) = 0

⇨ 4u = 0 (or) u + 2 = 0

⇨ u = $\frac{0}{4}$ (or) u = - 2

⇨ u = 0 (or) u = - 2

Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta}$ = (0) + (- 2) = - 2 = - 2 x $\frac{4}{4} = \frac{-8}{4}$

$= \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\boldsymbol{\alpha\beta}$ = (0)(- 2) = 0 = $\frac{0}{4} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

(v) $t^2 - 15$

Solution : We have given $t^2 - 15$

First we have to find zeroes of the given polynomial. To do this,

Let $t^2 - 15 = 0$

⇨ $t^2 - (\sqrt{15})^2 = 0$

⇨ $(t + \sqrt{15})(t - \sqrt{15}) = 0$

$$a^2 - b^2 = (a+b)(a-b)$$

⇨ $t + \sqrt{15} = 0$ (or) $t - \sqrt{15} = 0$

⇨ $t = -\sqrt{15}$ (or) $t = \sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$.

Sum of zeroes = $\boldsymbol{\alpha} + \boldsymbol{\beta} = (-\sqrt{15}) + (\sqrt{15}) = 0 = \frac{-0}{1}\ \frac{-(Coefficient\ of\ x)}{Coefficient\ of\ x^2} = \frac{-b}{a}$

Product of zeroes = $\boldsymbol{\alpha\beta} = (-\sqrt{15})(\sqrt{15}) = -15 = \frac{-15}{1} = \frac{Constant\ term}{Coefficient\ of\ x^2} = \frac{c}{a}$

(ii) & (vi) → Home Work

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}$, - 1

Solution : Let the zeroes of the quadratic polynomial be $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

Given that, $\boldsymbol{\alpha} + \boldsymbol{\beta} = \frac{1}{4}$

$\boldsymbol{\alpha\beta} = -1$

Quadratic polynomial = $k[x^2 - (a + b)x + ab]$

Quadratic polynomial = $k[x^2 - \left(\frac{1}{4}\right)x + (-1)]$

$= k[x^2 - \frac{1}{4}x - 1]$

$= k\left[\frac{4x^2 - x - 4}{4}\right]$

If the value of k = 4, then the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Solution : Let the zeroes of the quadratic polynomial be $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

Given that, $\boldsymbol{\alpha} + \boldsymbol{\beta} = \sqrt{2}$

$\boldsymbol{\alpha\beta} = \frac{1}{3}$

Quadratic polynomial = $k[x^2 - (a + b)x + ab]$

Quadratic polynomial = $k\left[x^2 - (\sqrt{2})x + \left(\frac{1}{3}\right)\right]$

$= k\left[x^2 - \sqrt{2}x + \frac{1}{3}\right]$

$= k\left[\frac{3x^2 - 3\sqrt{2}x + 1}{3}\right]$

If the value of k = 3, then the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Solution : Let the zeroes of the quadratic polynomial be $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.

Given that, $\boldsymbol{\alpha} + \boldsymbol{\beta} = -\frac{1}{4}$

$\boldsymbol{\alpha\beta} = \frac{1}{4}$

Quadratic polynomial = $k[x^2 - (a + b)x + ab]$

Quadratic polynomial = $k\left[x^2 - \left(-\frac{1}{4}\right)x + \left(\frac{1}{4}\right)\right]$

$= k\left[x^2 + \frac{1}{4}x + \frac{1}{4}\right]$

$= k\left[\frac{4x^2 + x + 1}{4}\right]$

If the value of k = 4, then the quadratic polynomial is $4x^2 + x + 1$.

(iii), (iv) & (vi) → Home Work

Practice Questions

1. If one of the zeroes of the quadratic polynomial (k–1) x2 + k x + 1 is –3, then the value of k is

   (A) $\frac{4}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

2. A quadratic polynomial, whose zeroes are –3 and 4, is

   (A) $x^2 - x + 12$ (B) $x^2 + x + 12$ (C) $\frac{x^2}{2} - \frac{x}{2} - 6$ (D) $2x^2 + 2x - 24$

3. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

   (A) both positive (B) both negative

   (C) one positive and one negative (D) both equal

4. The quadratic polynomial whose product and sum are 5 and 8 respectively is

   (A) $k[x^2 - 5x + 8]$ (B) $k[x^2 - 8x + 5]$ (C) $k[x^2 + 5x - 8]$ (D) $k[x^2 + 8x - 5]$

5. Find the zeroes of the polynomial $3x^2 + 4x - 4$, and verify the relation between the coefficients and the zeroes of the polynomial.

6. Find the zeroes of the polynomial $x^2 - 4$, and verify the relation between the coefficients and the zeroes of the polynomial.

7. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$ respectively. Also find its zeroes.

8. Find a quadratic polynomial, the sum and product of whose zeroes are $-\frac{8}{3}$ and $\frac{4}{3}$ respectively.

9. Find a quadratic polynomial, whose zeroes are 3 and - 4 respectively.

10. Find a quadratic polynomial, whose zeroes are $-\frac{1}{3}$ and 1 respectively.

✦✦✦

CHAPTER 3

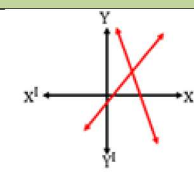
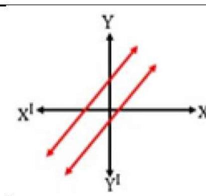
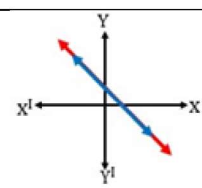
X-MATHEMATICS-NCERT-2024-25

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

PREPARED BY: BALABHADRA SURESH

https://sureshmathsmaterial.com

1. An equation of the form $ax + b = 0$ where a, b are real numbers and $a \neq 0$ is called **linear equation in one variable** x.
2. An equation of the form $ax + by + c = 0$ where a, b, c are real numbers and where at least one of a or b is not zero (i.e. $a^2 + b^2 \neq 0$), is called **a linear equation in two variables x and y**
3. Two linear equations in the same two variables are called a pair of linear equations in two variables
   $a_1x + b_1y + c_1 = 0({a_1}^2 + {b_1}^2 \neq 0)$
   $a_2x + b_2y + c_2 = 0({a_2}^2 + {b_2}^2 \neq 0)$
4. The values of x and y satisfying each one of given pair of linear equations are called their **solution**.
5. **Consistent and Inconsistent Systems:** A pair of linear equations can have at least one common solution; it's called a consistent system. If there are no common solutions, it's an inconsistent system.
6. The graph of linear equation is a straight line.

Comparison of ratios	Graphical representation	Algebraic interpretation	Solution	Graph
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Consistent	Unique solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	In consistent	No solution	
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Consistent	Infinite number of solutions	

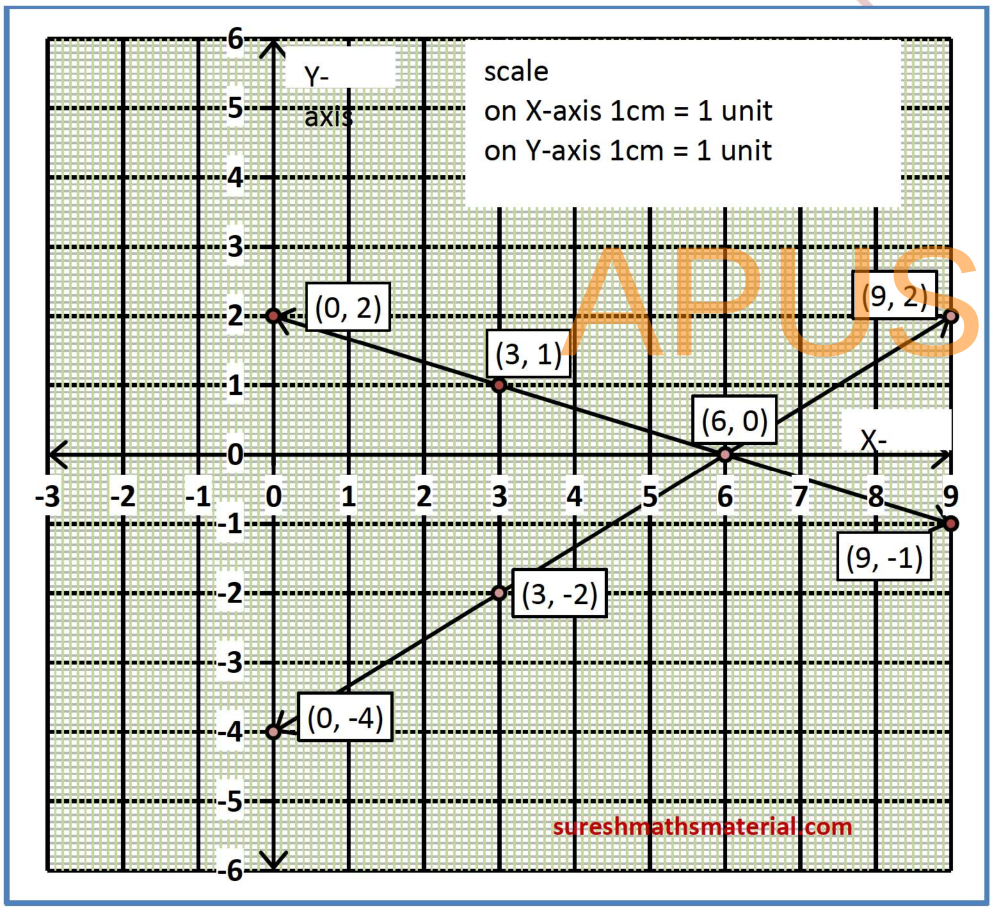
3.2 Graphical Method of Solution of a Pair of Linear Equations.

Example 1 : Check graphically whether the pair of equations
$x + 3y = 6$ and $2x - 3y = 12$ is consistent. If so, solve them graphically.

Sol:

$x + 3y = 6 \Rightarrow y = \frac{6-x}{3}$		
x	$y = \frac{6-x}{3}$	(x, y)
0	$y = \frac{6-0}{3} = \frac{6}{3} = 2$	$(0,2)$
3	$y = \frac{6-3}{3} = \frac{3}{3} = 1$	$(3,1)$
6	$y = \frac{6-6}{3} = \frac{0}{3} = 0$	$(6,0)$
9	$y = \frac{6-9}{3} = \frac{-3}{3} = -1$	$(9,-1)$

$2x - 3y = 12 \Rightarrow y = \frac{2x-12}{3}$		
x	$y = \frac{2x-12}{3}$	(x, y)
0	$y = \frac{2(0)-12}{3} = \frac{-12}{3} = -4$	$(0,-4)$
3	$y = \frac{2(3)-12}{3} = \frac{-6}{3} = -2$	$(3,-2)$
6	$y = \frac{2(6)-12}{3} = \frac{0}{3} = 0$	$(6,0)$
9	$y = \frac{2(9)-12}{3} = \frac{6}{3} = 2$	$(9,2)$



Both the lines intersect at (6,0)

So, the solution of the pair of linear equations is x = 6 and y = 0

i.e., the given pair of equations is consistent.

Example 2 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:

$5x - 8y + 1 = 0$ *and* $3x - \frac{24}{5}y + \frac{3}{5} = 0$

Sol:

$5x - 8y + 1 = 0 \rightarrow (1)$

$3x - \frac{24}{5}y + \frac{3}{5} = 0 \rightarrow (2)$

Multiply by $\frac{5}{3}$ *, we get*

$\Rightarrow \frac{5}{3} \times 3x - \frac{5}{3} \times \frac{24}{5}y + \frac{5}{3} \times \frac{3}{5} = \frac{5}{3} \times 0$

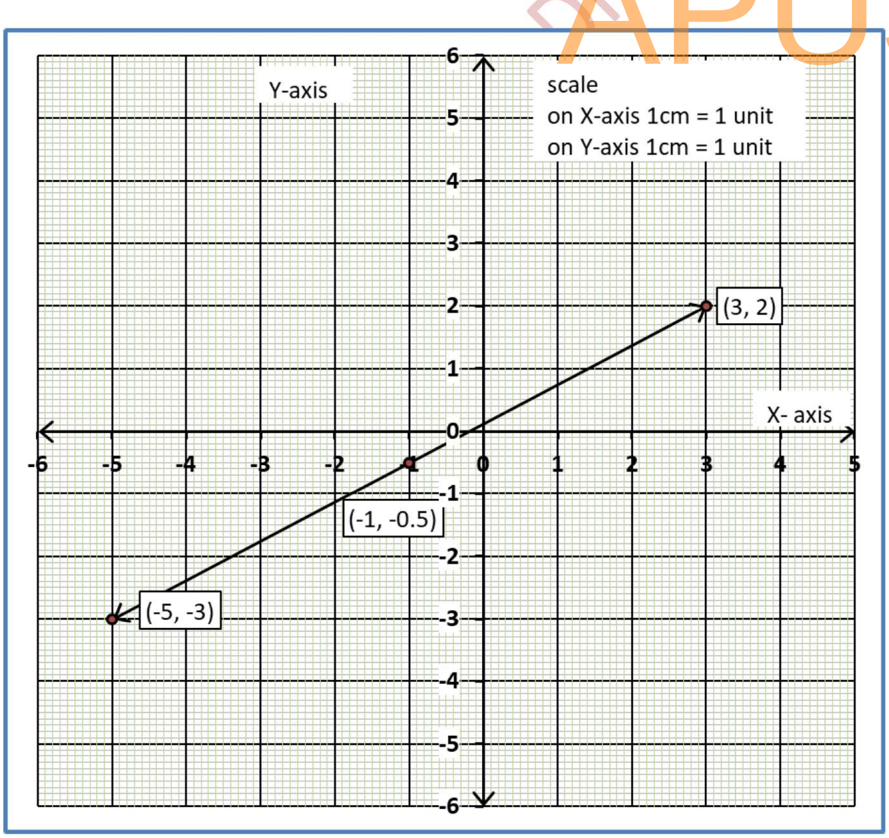
$\Rightarrow 5x - 8y + 1 = 0 \rightarrow (2)$

(1) and (2) are same equations.

Given equations are coincident.

Equations (1) and (2) have infinitely many solutions.

$5x - 8y = -1 \Rightarrow 8y = 5x + 1 \Rightarrow y = \frac{5x+1}{8}$		
x	$y = \frac{5x+1}{8}$	(x, y)
-5	$y = \frac{5(-5)+1}{8} = \frac{-25+1}{8} = \frac{-24}{8} = -3$	$(-5, -3)$
-1	$y = \frac{5(-1)+1}{8} = \frac{-5+1}{8} = \frac{-4}{8} = -0.5$	$(-1, -0.5)$
3	$y = \frac{5(3)+1}{8} = \frac{15+1}{8} = \frac{16}{8} = 2$	(3,2)



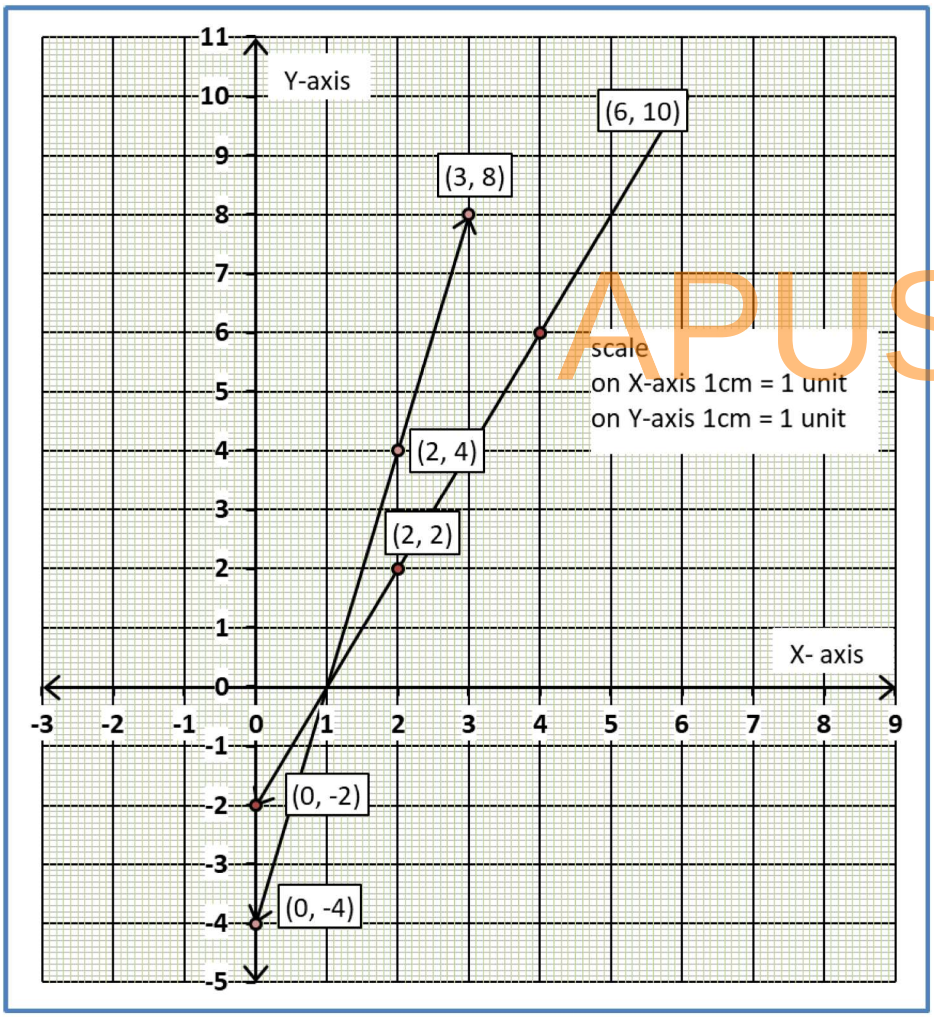
Example 3 : Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Solution : Let the number of pants =x and the number of skirts = y.

Then the equations formed are : y = 2x – 2 and y = 4x – 4

$\boldsymbol{y = 2x - 2}$		
x	$y = 2x - 2$	(x, y)
0	$y = 2(0) - 2 = 0 - 2 = -2$	$(0, -2)$
2	$y = 2(2) - 2 = 4 - 2 = 2$	$(2,2)$
4	$y = 4(2) - 2 = 8 - 2 = 6$	$(4,6)$
6	$y = 6(2) - 2 = 12 - 2 = 10$	$(6,10)$

$\boldsymbol{y = 4x - 4}$		
x	$y = 4x - 4$	(x, y)
0	$y = 4(0) - 4 = 0 - 4 = -4$	$(0, -4)$
1	$y = 4(1) - 4 = 4 - 4 = 0$	$(1,0)$
2	$y = 4(2) - 4 = 8 - 4 = 4$	$(2,4)$
3	$y = 4(3) - 4 = 12 - 4 = 8$	$(3,8)$



The two lines intersect at the point (1, 0)

So, x = 1, y = 0 is the required solution of the pair of linear equations.

i.e., the number of pants she purchased is 1 and she did not buy any skirt.

EXERCISE 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.
(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Sol: Let the number of boys=x and number of girls=y

Total number of students=10

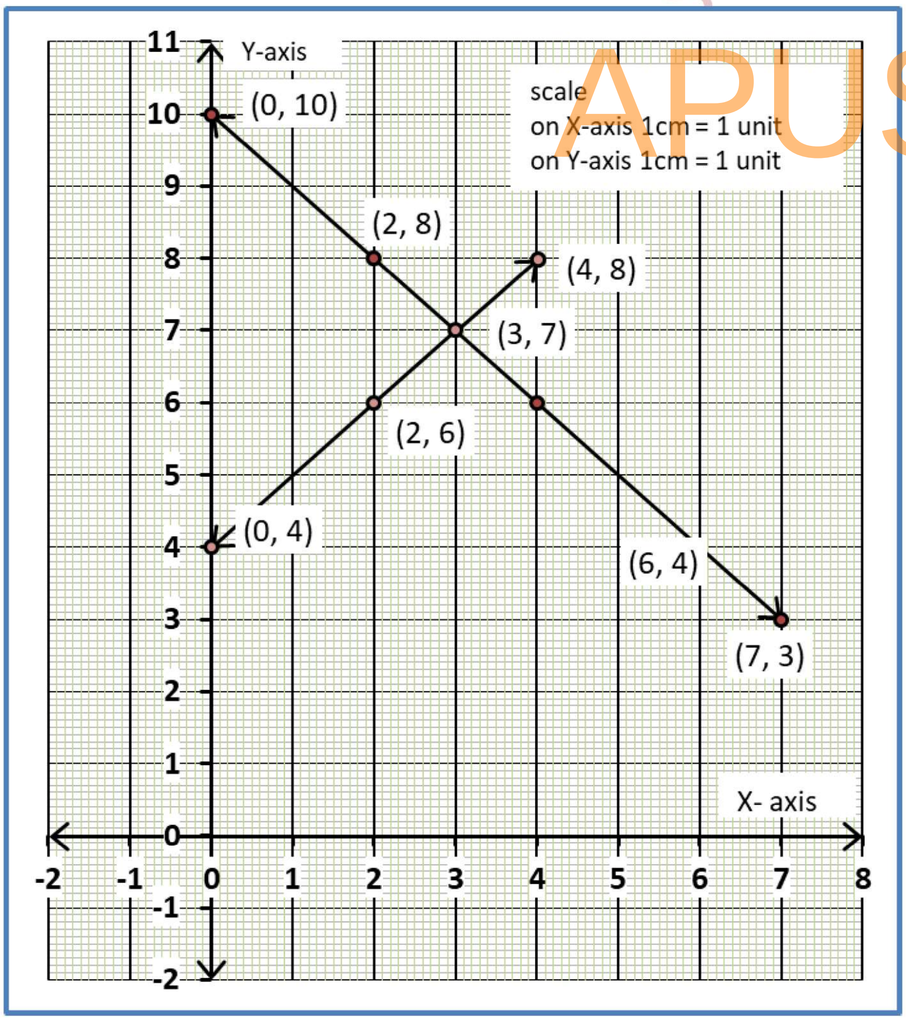
$\Rightarrow x + y = 10 \rightarrow (1)$

The number of girls is 4 more than the number of boys.

$\Rightarrow y = x + 4 \Rightarrow x - y = -4 \rightarrow (2)$

$x + y = 10 \Rightarrow y = 10 - x$		
x	$y = 10 - x$	(x, y)
2	$y = 10 - 2 = 8$	(2,8)
4	$y = 10 - 4 = 6$	(4,6)
6	$y = 10 - 6 = 4$	(6,4)
7	$y = 10 - 7 = 3$	(7,3)

$x - y = -4 \Rightarrow y = x + 4$		
x	$y = x + 4$	(x, y)
2	$y = 2 + 4 = 6$	(2,6)
3	$y = 3 + 4 = 7$	(3,7)
4	$y = 4 + 4 = 8$	(4,8)
6	$y = 6 + 4 = 10$	(6,10)



The two lines intersect at the point (3, 7)

So, $x = 3, y = 7$ is the required solution of the pair of linear equations.

i.e., the number of boys=3 and the number of girls=7.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

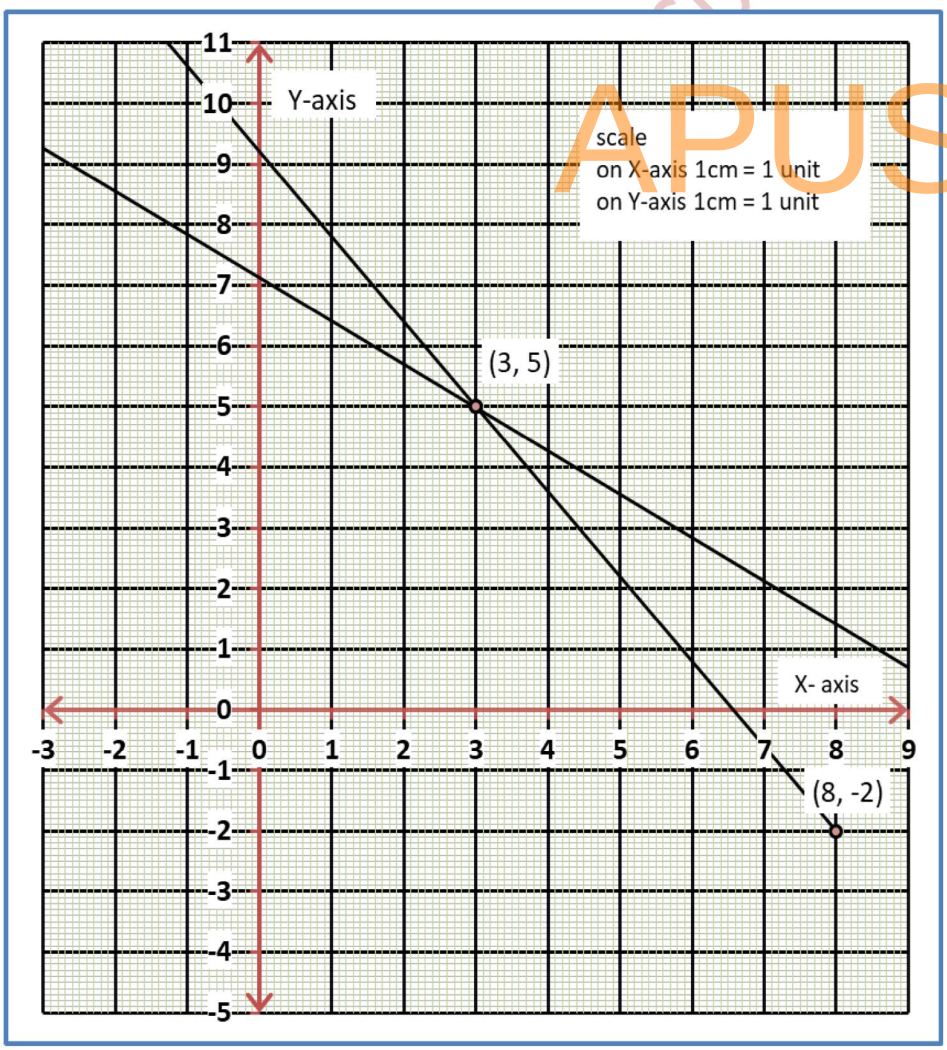
Sol: Let the cost of 1 pencil=₹ x and the cost of 1 pen=₹y

5 pencils + 7 pens = ₹ 50 $\Rightarrow 5x + 7y = 50 \rightarrow (1)$

7 pencils + 5 pens = ₹ 46 $\Rightarrow 7x + 5y = 46 \rightarrow (2)$

$5x+7y=50 \Rightarrow y=\frac{50-5x}{7}$		
x	$y=\frac{50-5x}{7}$	(x,y)
-4	$y=\frac{50-5(-4)}{7}=\frac{70}{7}=10$	$(-4,10)$
3	$y=\frac{50-5(3)}{7}=\frac{35}{7}=5$	$(3,5)$
10	$y=\frac{50-5(10)}{7}=\frac{0}{7}=0$	$(10,0)$

$7x+5y=46 \Rightarrow 5y=46-7x \Rightarrow y=\frac{46-7x}{5}$		
x	$y=\frac{46-7x}{5}$	(x,y)
-2	$y=\frac{46-7(-2)}{5}=\frac{60}{5}=12$	$(-2,12)$
3	$y=\frac{46-7(3)}{5}=\frac{25}{5}=5$	$(3,5)$
8	$y=\frac{46-7(8)}{5}=\frac{-10}{5}=-2$	$(8,-2)$



The two lines intersect at the point (3, 5)
So, x = 3, y = 7 is the required solution of the pair of linear equations.

i.e., the cost of 1 pencil=₹3 and the cost of 1 pen=₹5

2. **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines represented by the following pairs of linear equations equations intersect at a point, are parallel or are coincident.**

(i) $5x - 4y + 8 = 0$
$7x + 6y - 9 = 0$

Sol: $5x - 4y + 8 = 0$; $(a_1 = 5, \quad b_1 = -4, c_1 = 8)$

$7x + 6y - 9 = 0$; $(a_2 = 7, \ b_2 = 6, \ c_2 = -9)$

$\frac{a_1}{a_2} = \frac{5}{7}; \quad \frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}; \quad \frac{c_1}{c_2} = \frac{8}{-9} = \frac{-8}{9}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of linear equations intersect at a point.

(ii) $9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$

Sol: $9x + 3y + 12 = 0$; $a_1 = 9, \quad b_1 = 3, c_1 = 12$

$18x + 6y + 24 = 0$; $a_2 = 18, \ b_2 = 6, \ c_2 = 24$

$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are coincident

(iii) $6x - 3y + 10 = 0$
$2x - y + 9 = 0$

Sol: $6x - 3y + 10 = 0$; $a_1 = 6, \quad b_1 = -3, c_1 = 10$

$2x - y + 9 = 0$; $a_2 = 2, \ b_2 = -1, \ c_2 = 9$

$\frac{a_1}{a_2} = \frac{6}{2} = 3; \quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3; \quad \frac{c_1}{c_2} = \frac{10}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are parallel

3. **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines represented by the following pairs of linear equations are consistent, or inconsistent**

(i) $3x + 2y = 5$; $2x - 3y = 7$

Sol: $3x + 2y - 5 = 0$; $a_1 = 3, \quad b_1 = 2, c_1 = -5$

$2x - 3y - 7 = 0$; $a_2 = 2, \ b_2 = -3, \ c_2 = -7$

$\frac{a_1}{a_2} = \frac{3}{2}; \quad \frac{b_1}{b_2} = \frac{2}{-3}; \quad \frac{c_1}{c_2} = \frac{-5}{-7} = \frac{5}{7}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of lines are intersecting and have one solution.

The pair of given equations are consistent.

(ii) $2x - 3y = 8$; $4x - 6y = 9$

Sol: $2x - 3y - 8 = 0$; ; $a_1 = 2$, $b_1 = -3, c_1 = -8$

$4x - 6y - 9 = 0$; $a_2 = 4$, $b_2 = -6$, $c_2 = -9$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ Given pairs of linear equations are parallel

The pair of given equations are inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

Sol: $\frac{3}{2}x + \frac{5}{3}y = 7 \Rightarrow 6 \times \frac{3}{2}x + 6 \times \frac{5}{3}y - 6 \times 7 = 0$

$9x + 10y - 42 = 0$; $(a_1 = 9,$ $b_1 = 10,$ $c_1 = -42)$

$9x - 10y - 14 = 0$; $(a_2 = 9,$ $b_2 = -10,$ $c_2 = -14)$

$\frac{a_1}{a_2} = \frac{9}{9} = 1; \quad \frac{b_1}{b_2} = \frac{10}{-10} = -1; \quad \frac{c_1}{c_2} = \frac{-42}{-14} = 3$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ Given pairs of linear equations are intersecting and have one solution

The pair of given equations are consistent

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

Sol: $5x - 3y - 11 = 0$; $a_1 = 5$, $b_1 = -3, c_1 = -11$

$-10x + 6y + 22 = 0$; $a_2 = -10$, $b_2 = 6$, $c_2 = 22$

$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}; \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}; \quad \frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow$ Given pairs of linear equations are coincident and have infinitely many solutions.

The pair of given equations are consistent

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

$\frac{4}{3}x + 2y - 8 = 0$; $a_1 = \frac{4}{3}$, $b_1 = 2, c_1 = -8$

$2x + 3y - 12 = 0$; $a_2 = 2$, $b_2 = 3$, $c_2 = -12$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{4}{3 \times 2} = \frac{2}{3}; \qquad \frac{b_1}{b_2} = \frac{2}{3}; \qquad \frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

⇒ Given pairs of linear equations are coincident and have infinitely many solutions.

The pair of given equations are consistent

4. Which of the following pairs of linear equations are consistent /inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, 2x + 2y = 10$

Sol: $x + y - 5 = 0$; $a_1 = 1, \quad b_1 = 1, c_1 = -5$

$2x + 2y - 10 = 0$; $a_2 = 2, \ b_2 = 2, \ c_2 = -10$

$$\frac{a_1}{a_2} = \frac{1}{2}; \qquad \frac{b_1}{b_2} = \frac{1}{2}; \qquad \frac{c_1}{c_2} = \frac{-5}{10} = \frac{1}{2}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ ⇒ lines are coincident and have infinitely many solutions.

The pair of given equations are consistent

$x + y = 5 \Rightarrow y = 10 - x$		
x	$y = x + 4$	(x, y)
2	$y = 10 - 2 = 8$	(2,8)
4	$y = 10 - 4 = 6$	(4,6)
6	$y = 10 - 6 = 4$	(6,4)
7	$y = 10 - 7 = 3$	(7,3)

(ii) $x - y = 8, 3x - 3y = 16$

Sol: $x - y - 8 = 0$; $a_1 = 1, \quad b_1 = -1, c_1 = -8$

$3x - 3y - 16 = 0$; $a_2 = 3, \ b_2 = -3, \ c_2 = -16$

$$\frac{a_1}{a_2} = \frac{1}{3}; \qquad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}; \qquad \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ ⇒ lines are parallel and have no solution.

The pair of given equations are inconsistent.

(iii) $2x + y - 6 = 0, 4x - 2y - 4 = 0$

Sol: $2x + y - 6 = 0$; $a_1 = 2, \quad b_1 = 1, c_1 = -6$

$4x - 2y - 4 = 0$; $a_2 = 4, \ b_2 = -2, \ c_2 = -4$

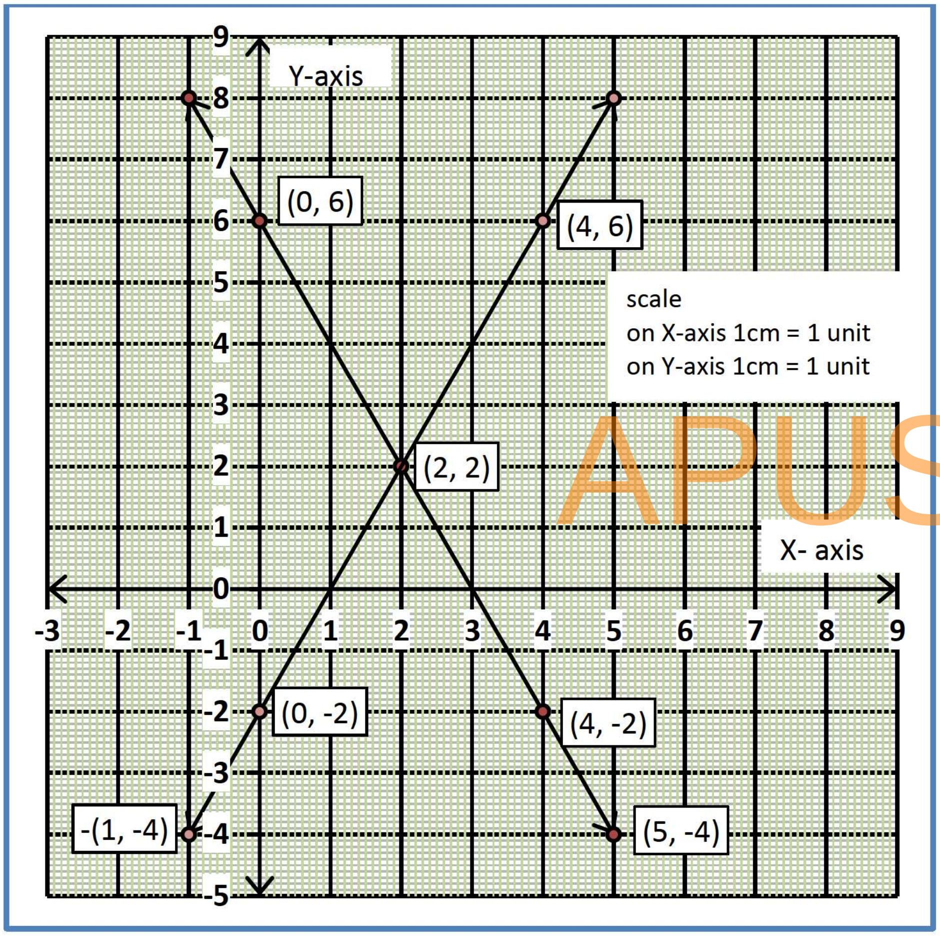
$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \qquad \frac{b_1}{b_2} = \frac{1}{-2} = \frac{-1}{2}; \qquad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow$ lines are intersecting and have one solution.

The pair of given equations are consistent

$2x + y - 6 = 0 \Rightarrow y = 6 - 2x$		
x	$y = 6 - 2x$	(x, y)
0	$y = 6 - 2 \times 0 = 6 - 0 = 6$	(0,6)
2	$y = 6 - 2 \times 2 = 6 - 4 = 2$	(2,2)
4	$y = 6 - 2 \times 4 = 6 - 8 = -2$	$(4, -2)$

$4x - 2y - 4 = 0 \Rightarrow y = 2x - 2$		
x	$y = 2x - 2$	(x, y)
0	$y = 2 \times 0 - 2 = 0 - 2 = -2$	$(0, -2)$
2	$y = 2 \times 2 - 2 = 4 - 2 = 2$	(2,2)
4	$y = 2 \times 4 - 2 = 8 - 2 = 6$	(4,6)



Graphs intersect at (2,2)

Solution:x=2 and y=2

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Sol: $2x - 2y - 2 = 0$; $a_1 = 2$, $b_1 = -2, c_1 = -2$

$4x - 4y - 5 = 0$; $a_2 = 4$, $b_2 = -4$, $c_2 = -5$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}; \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}; \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ lines are parallel and have no solution.

The pair of given equations are inconsistent.

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Sol: Let length of the garden $= x$ and breadth $= y$

Given: length=width+4

$x = y + 4$

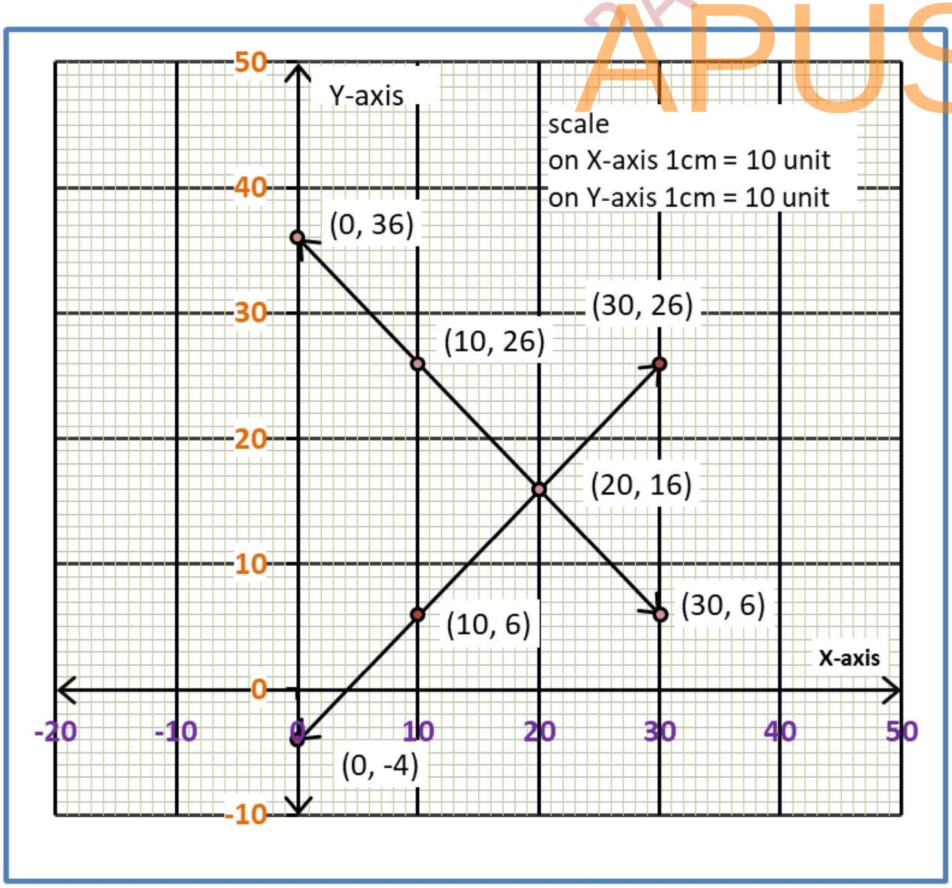
$x - y = 4 \rightarrow (1)$

Half the perimeter of the rectangle =36 m

$x + y = 36 \rightarrow (2)$

$x - y = 4 \Rightarrow y = x - 4$		
x	$y = x - 4$	(x, y)
0	$y = 0 - 4 = -4$	$(0, -4)$
10	$y = 10 - 4 = 6$	(10,6)
20	$y = 20 - 4 = 16$	(20,16)
30	$y = 30 - 4 = 26$	(30,26)

$x + y = 36 \Rightarrow y = 36 - x$		
x	$y = 36 - x$	(x, y)
0	$y = 36 - 0 = 36$	(0,36)
10	$y = 36 - 10 = 26$	(10,26)
20	$y = 36 - 20 = 16$	(20,16)
30	$y = 36 - 30 = 6$	(30,6)



The two lines intersect at the point (20, 16)

So, x = 20, y = 16 is the required solution of the pair of linear equations.

i.e., Length=20m and the breadth=16m.

**6. Given the linear equation 2x + 3y – 8 = 0, write another linear equation in two variables such that the geometrical representation of the pair so formed is:
(i) intersecting lines (ii) parallel lines (iii) coincident lines**

Sol: Given the linear equation 2x + 3y – 8 = 0

(i) Intersecting lines:

Condition: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$4x - 5y + 7 = 0$

(ii) Parallel lines:

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$4x + 6y + 7 = 0$

(iii) Coincident lines

Condition: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

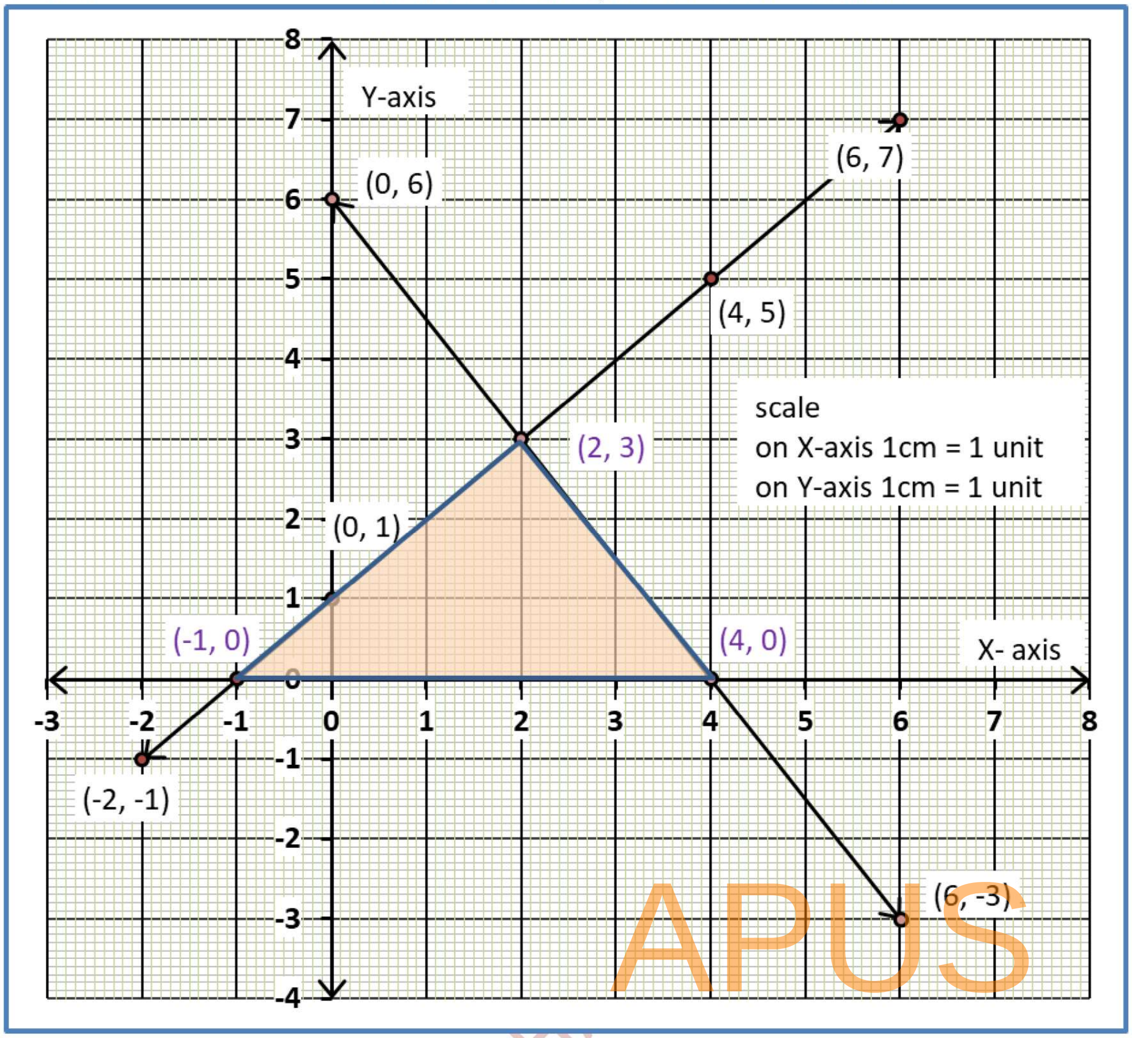
$8x + 12y - 32 = 0$

7. Draw the graphs of the equations x – y + 1 = 0 and 3x + 2y – 12 = 0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol:

$x - y + 1 = 0 \Rightarrow y = x + 1$		
x	$y = x + 1$	(x, y)
-2	$y = -2 + 1 = -1$	$(-2, -1)$
-1	$y = -1 + 1 = 0$	$(-1, 0)$
0	$y = 0 + 1 = 1$	$(0, 1)$
4	$y = 4 + 1 = 5$	$(4, 5)$
6	$y = 6 + 1 = 7$	$(6, 7)$

$x - y + 1 = 0 \Rightarrow y = x + 1$		
x	$y = x + 1$	(x, y)
-2	$y = -2 + 1 = -1$	$(-2, -1)$
-1	$y = -1 + 1 = 0$	$(-1, 0)$
0	$y = 0 + 1 = 1$	$(0, 1)$
4	$y = 4 + 1 = 5$	$(4, 5)$
6	$y = 6 + 1 = 7$	$(6, 7)$



Vertices of the triangle are A(-1,0), B(4,0) and C(2,3)

ALGEBRAIC METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS

3.3.1 Substitution Method

Example 4 : Solve the following pair of equations by substitution method: 7x – 15y = 2, x + 2y = 3

Sol: $7x - 15y = 2 \rightarrow (1)$

$x + 2y = 3 \rightarrow (2)$

From (2): $x = 3 - 2y \rightarrow (3)$

Substitute the value of $x = 3 - 2y$ in Equation (1).

$7(3 - 2y) - 15y = 2$

$21 - 14y - 15y = 2$

$-29y = -19$

$y = \frac{19}{29}$

Substituting this value of y in Equation (3)

$x = 3 - 2\left(\frac{19}{29}\right) = 3 - \frac{38}{29} = \frac{87 - 38}{29} = \frac{49}{29}$

The solution is $x = \frac{49}{29}, y = \frac{19}{29}$

Example 5 : Solve the following question—Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically by the method of substitution.

Sol:

	Aftab's age	daughter's age
Present	x	y
Seven years ago	$x - 7$	$y - 7$
Three years from now	$x + 3$	$y + 3$

Seven years ago:

Aftab's age=7× daughter's age

$x - 7 = 7(y - 7)$

$x - 7 = 7y - 49$

$x - 7y = -42 \longrightarrow (1)$

Three years from now:

Aftab's age=3× daughter's age

$x + 3 = 3(y + 3)$

$x + 3 = 3y + 9$

$x - 3y = 6 \rightarrow (2)$

From(2): $x = 6 + 3y \rightarrow (3)$

Substitute the value of x in Equation (1).

$6 + 3y - 7y = -42$

$6 - 4y = -42$

$-4y = -42 - 6 = -48$

$y = \frac{-48}{-4} = 12$

Substitute the value of y in equation (3)

$x = 6 + 3y = 6 + 3 \times 12 = 6 + 36 = 42$

So, Aftab and his daughter are 42 and 12 years old, respectively.

Example 6 : In a shop the cost of 2 pencils and 3 erasers is ₹9 and the cost of 4 pencils and 6 erasers is ₹18. Find the cost of each pencil and each eraser.

Sol: Let cost of 1 pencil $= x$ and cost of 1 eraser $= y$

2 pencils + 3 erasers = ₹9 $\Rightarrow 2x + 3y = 9 \longrightarrow (1)$

4 pencils + 6 erasers = ₹18 $\Rightarrow 4x + 6y = 18 \longrightarrow (2)$

From the equ(1): $x = \dfrac{9-3y}{2}$

Substitute the value of x in Equation (2)

$4\left(\dfrac{9-3y}{2}\right) + 6y = 18$

$18 - 6y + 6y = 18$

$18 = 18$

This statement is true for all values of y, the given equations are the same

Therefore, Equations (1) and (2) have infinitely many solutions.

We cannot find a unique cost of a pencil and an eraser,

Example 7 : Two rails are represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Will the rails cross each other?

Solution : The pair of linear equations formed were:

$x + 2y - 4 = 0 \longrightarrow (1)$

$2x + 4y - 12 = 0 \longrightarrow (2)$

From Equ(1): $x = 4 - 2y$

Substitute this value of x in Equ (2)

$2(4-2y) + 4y - 12 = 0$

$8 - 4y + 4y - 12 = 0$

$-4 = 0$

This is a false statement.

Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

EXERCISE 3.2

1. **Solve the following pair of linear equations by the substitution method.**

(i) $x + y = 14$; $x - y = 4$

Sol: $x + y = 14 \longrightarrow (1)$

$x - y = 4 \longrightarrow (2)$

From (1): $x = 14 - y \longrightarrow (3)$

Substitute this value of x in Equ (2)

$14 - y - y = 4$

$-2y = 4 - 14$

$-2y = -10$

$y = \frac{-10}{-2} = 5$

Substitute the value of y in equation (3)

$x = 14 - 5 = 9$

The solution is $x = 9, y = 5$

(ii) $\boldsymbol{s - t = 3; \frac{s}{3} + \frac{t}{2} = 6}$

Sol: $s - t = 3 \rightarrow (1)$

$\frac{s}{3} + \frac{t}{2} = 6 \Longrightarrow 6 \times \frac{s}{3} + 6 \times \frac{t}{2} = 6 \times 6$

$2s + 3t = 36 \rightarrow (2)$

From equ (1): $s = 3 + t \rightarrow (3)$

Substitute this value of s in Equ (2)

$2(3 + t) + 3t = 36$

$6 + 2t + 3t = 36$

$5t = 36 - 6 = 30$

$t = \frac{30}{5} = 6$

Substitute the value of t in equation (3)

$s = 3 + 6 = 9$

$The\ solution\ is\ s = 9, t = 6$

(iii) $\boldsymbol{3x - y = 3;\ 9x - 3y = 9}$

Sol: $3x - y = 3 \rightarrow (1)$

$9x - 3y = 9 \rightarrow (2)$

$From (1): y = 3x - 3$

$Substitute\ this\ value\ of\ y\ in\ Equ\ (2)$

$9x - 3(3x - 3) = 9$

$9x - 9x + 9 = 9$

$9 = 9$

This statement is true for all values of x, the given equations are the same

Therefore, Equations (1) and (2) have infinitely many solutions

$(iv) 0.2x + 0.3y = 1.3;\ 0.4x + 0.5y = 2.3$

$Sol: 0.2x + 0.3y = 1.3$

Multiply with 10

$2x + 3y = 13 \rightarrow (1)$

$0.4x + 0.5y = 2.3$

Multiply with 10

$4x + 5y = 23 \rightarrow (2)$

From(1): $2x = 13 - 3y$

$\Rightarrow x = \frac{13 - 3y}{2} \rightarrow (3)$

Substitute the value of x in equation (2) we get

$4\left(\frac{13 - 3y}{2}\right) + 5y = 23$

$26 - 6y + 5y = 23$

$-y = 23 - 26 = -3$

$y = 3$

Substitute $y = 3$ in (3)

$x = \frac{13 - 3y}{2} = \frac{13 - 3 \times 3}{2} = \frac{13 - 9}{2} = \frac{4}{2} = 2$

The required solution is $x = 2$ *and* $y = 3$

$(v) \sqrt{2}x + \sqrt{3}y = 0;\ \sqrt{3}x - \sqrt{8}y = 0$

$Sol: \sqrt{2}x + \sqrt{3}y = 0 \rightarrow (1)$

$\sqrt{3}x - \sqrt{8}y = 0 \rightarrow (2)$

From equ(1): $\sqrt{3}y = -\sqrt{2}x$

$y = \frac{-\sqrt{2}x}{\sqrt{3}} \rightarrow (3)$

Substituting the value of y in equation (2) we get

$\sqrt{3}x - \sqrt{8}\left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Longrightarrow \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0 \Longrightarrow \frac{3x + 4x}{\sqrt{3}} = 0$

$3x + 4x = 0$

$7x = 0$

$x = 0$

$$y = \frac{-\sqrt{2}x}{\sqrt{3}} = 0$$

The required solution is $x = 0$ *and* $y = 0$.

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2;\ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Sol: $\frac{3x}{2} - \frac{5y}{3} = -2 \Rightarrow 6 \times \frac{3x}{2} - 6 \times \frac{5y}{3} = 6 \times (-2)$

$9x - 10y = -12 \to (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \Rightarrow 6 \times \frac{x}{3} + 6 \times \frac{y}{2} = 6 \times \frac{13}{6}$

$2x + 3y = 13 \to (2)$

$From\ (1): x = \frac{10y - 12}{9} \to (3)$

Substitute this value of x $in\ Equ$ (2)

$$2\left(\frac{10y - 12}{9}\right) + 3y = 13$$

$$\frac{20y - 24}{9} + 3y = 13$$

$$20y - 24 + 27y = 117$$

$$47y = 117 + 24 = 141$$

$$y = \frac{141}{47} = 3$$

Substituting the value of y in equation (3)

$$x = \frac{10(3) - 12}{9} = \frac{30 - 12}{9} = \frac{18}{9} = 2$$

The solution is $x = 2, y = 3$

2. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.

Sol: $2x + 3y = 11 \to (1)$

$2x - 4y = -24 \to (2)$

From (1): $x = \frac{11 - 3y}{2} \to (3)$

Substituting the value of y in equation (2)

$$2\left(\frac{11 - 3y}{2}\right) - 4y = -24$$

$11 - 3y - 4y = -24$

$-7y = -24 - 11$

$-7y = -35$

$y = 5$

Substituting the value of y in equation (3)

$$x = \frac{11 - 3(5)}{2} = \frac{11 - 15}{2} = \frac{-4}{2} = -2$$

The solution is $x = -2, y = 5$

Substituting $x = -2, y = 5$ *in* $y = mx + 3$

$5 = m(-2) + 3$

$5 = -2m + 3$

$2m = 3 - 5$

$2m = -2$

$m = -1$

The value of 'm' $= -1$

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

Sol: Let the two numbers are x and y(x>y)

From problem

$x - y = 26 \rightarrow (1)$

$x = 3y \rightarrow (2)$

Substituting the value of x in equation (1)

$3y - y = 26$

$2y = 26$

$y = 13$

Substitute y=13 in (2)

$x = 3 \times 13 = 39$

The required numbers are 39 and 13

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol: Let the two supplementary angles be x and y $(x > y)$

From problem

$x + y = 180^0 \rightarrow (1)$

$x - y = 18^0 \rightarrow (2)$

$From\ (1): x = 180^0 - y \rightarrow (3)$

Substitute x value in (2)

$180^0 - y - y = 18^0$

$2y = 180^0 - 18^0$

$2y = 162^0$

$y = 81^0$

Substitute $y = 81^0\ in\ equ\ (3)$

$x = 180^0 - 81^0 = 99^0$

The angles are 99^0 *and* 81^0

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

Sol: let the cost of each bat = ₹x and ball = ₹y

From problem

$7x + 6y = 3800 \rightarrow (1)$

$3x + 5y = 1750 \rightarrow (2)$

From (1): $y = \frac{3800 - 7x}{6} \rightarrow (3)$

Substituting the value of y in (2)

$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$

$3x + \frac{19000 - 35x}{6} = 1750$

$18x + 19000 - 35x = 10500$

$-17x = 10500 - 19000$

$-17x = -8500$

$x = \frac{-8500}{-17} = 500$

Substituting the value of x=500 in(3)

$y = \frac{3800 - 7(500)}{6} = \frac{3800 - 3500}{6} = \frac{300}{6} = 50$

The cost of each bat=₹ 500 and each ball=₹50

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

Sol: Let the fixed charge = ₹x and the charge for 1km = ₹y

The charge paid for 10 km=₹105

$x + 10y = 105 \rightarrow (1)$

The charge paid for 15 km=₹155

$x + 15y = 155 \rightarrow (2)$

$from\ (1): x = 105 - 10y \rightarrow (3)$

Substituting the value of y in (3)

$105 - 10y + 15y = 155$

$105 + 5y = 155$

$5y = 155 - 105 = 50$

$y = \frac{50}{5} = 10$

Substitute y=10 in (3)

$x = 105 - 10 \times 10$

$x = 105 - 100$

$x = 5$

∴ Fixed charge= $x = ₹5$

Charge for 1 km= $y = ₹10$

Charge for 25 km $= \boldsymbol{x + 25y} = 5 + 25 \times 10 = 5 + 250 = ₹255$

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

Sol: Let the fraction $= \frac{x}{y}$

From the problem

$\frac{x+2}{y+2} = \frac{9}{11}$ and $\frac{x+3}{y+3} = \frac{5}{6}$

$11(x+2) = 9(y+2)$ and $6(x+3) = 5(y+3)$

$11x + 22 = 9y + 18$ and $6x + 18 = 5y + 15$

$11x - 9y = 18 - 22$ and $6x - 5y = 15 - 18$

$11x - 9y = -4 \rightarrow (1)$ and $6x - 5y = -3 \rightarrow (2)$

From(1): $x = \frac{9y-4}{11} \rightarrow (3)$

Substituting the value of x in (2)

$6\left(\frac{9y-4}{11}\right) - 5y = -3$

$\frac{54y - 24}{11} - 5y = -3$

$54y - 24 - 55y = -33$

$-y = -33 + 24$

$-y = -9$

$y = 9$

Substitute y=9 in (3)

$x = \frac{9(9)-4}{11} = \frac{81-4}{11} = \frac{77}{11} = 7$

The required fraction $= \frac{7}{9}$

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Sol: Let the age of Jacob $= x$ and his son $= y$

From problem

$x + 5 = 3(y + 5)$

$x + 5 = 3y + 15$

$x - 3y = 15 - 5$

$x - 3y = 10 \rightarrow (1)$

$x - 5 = 7(y - 5)$

$x - 5 = 7y - 35$

$x - 7y = -35 + 5$

$x - 7y = -30 \rightarrow (2)$

$From\ (1): x = 10 + 3y \rightarrow (3)$

Substituting the value of x in (2)

$10 + 3y - 7y = -30$

$10 - 4y = -30$

$-4y = -30 - 10$

$-4y = -40$

$y = 10$

Substituting y=10 in (3)

$x = 10 + 3(10) = 10 + 30 = 40$

The presen age of Jacob's $= 40$ *years* and his son is 10 *years*

Elimination Method

Example 8 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

Sol: The ratio of incomes of two persons = 9 : 7

Let their incomes be $9x$ and $7x$

The ratio of their expenditures = 4 : 3

Let their expenditures be $4y$ and $3y$

Given each of them manages to save ₹2000 per month

$9x - 4y = 2000 \rightarrow (1)$

$7x - 3y = 2000 \rightarrow (2)$

$3 \times (1) \Rightarrow 27x - 12y = 6000$

$4 \times (2) \Rightarrow 28x - 12y = 8000$

$\quad (-) \quad (+) \quad (-)$

$-x = -2000$

$x = 2000$

Substitute $x = 2000$ in (1)

$9(2000) - 4y = 2000$

$18000 - 4y = 2000$

$-4y = 2000 - 18000$

$-4y = -16000$

$4y = 16000$

$y = \frac{16000}{4} = 4000$

Their incomes are $9 \times 2000 \quad and \quad 7 \times 2000$

$\Rightarrow$ ₹18000 $\quad and \quad$ ₹ 14000

Example 9 : Use elimination method to find all possible solutions of the following pair of linear equations :$2x + 3y = 8; 4x + 6y = 7$

Sol: $2x + 3y = 8 \rightarrow (1)$

$4x + 6y = 7 \rightarrow (2)$

$2 \times (1) \Rightarrow 4x + 6y = 16$

$1 \times (2) \Rightarrow 4x + 6y = 7$

$(-) \quad (+) \quad (-)$

$Subtract$ $0 = 9$ it is not possible.

So, the given pair of equations has no solutions.

Example 10 : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

Sol: Let the unit place digit be x and tens place digit be y

The number$=10y + x$

The number obtained by reversing the digits$=10x + y$

From the problem

$(10y + x) + (10x + y) = 66$

$11x + 11y = 66$

$x + y = 6 \rightarrow (1)$

Given the digits of the number differ by 2

$x - y = 2 \rightarrow (2)$

$(1) \Rightarrow x + y = 6$

$(2) \Rightarrow x - y = 2$

$Adding \quad 2x = 8$

$x = \frac{8}{2} = 4$

Substitute $x = 4$ in (1)

$4 + y = 6$

$y = 6 - 4 = 2$

The number is $10y + x$ and $10x + y$

$\Rightarrow 10 \times 2 + 4 \quad and \quad 10 \times 4 + 2$

$\Rightarrow 24 \ and \ 42$

EXERCISE 3.3

1. **Solve the following pair of linear equations by the elimination method and the substitution method :**

$(i) x + y = 5$ and $2x - 3y = 4$

Sol: $x + y = 5 \rightarrow (1)$

$2x - 3y = 4 \rightarrow (2)$

$From\ (1): x = 5 - y \rightarrow (3)$

Substitute $x = 5 - y$ in equation (2)

$2(5 - y) - 3y = 4$

$10 - 2y - 3y = 4$

$-5y = 4 - 10 = -6$

$y = \frac{-6}{-5} = \frac{6}{5}$

Substitute $x = \frac{6}{5}$ in equation (1)

$x = 5 - \frac{6}{5} = \frac{25 - 6}{5} = \frac{19}{5}$

$\therefore$ The solution is $x = \frac{19}{5}$ and $y = \frac{6}{5}$

$(ii) 3x + 4y = 10$ and $2x - 2y = 2$

Sol: $3x + 4y = 10 \rightarrow (1)$

$2x - 2y = 2 \rightarrow (2)$

$From\ (2): 2x = 2 + 2y$

$x = 1 + y \rightarrow (3)$

Substitute $x = 1 + y$ in equation (1)

$3(1 + y) + 4y = 10$

$3 + 3y + 4y = 10$

$3 + 7y = 10$

$7y = 7$

$y = 1$

Substitute $y = 1$ in equation (3)

$x = 1 + 1 = 2$

$\therefore$ The solution is $x = 2$ and $y = 1$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

Sol: $3x - 5y = 4 \rightarrow (1)$

$9x - 2y = 7 \rightarrow (2)$

From (1): $3x = 4 + 5y$

$$x = \frac{4 + 5y}{3} \rightarrow (3)$$

Substitute x *value* in equation (2)

$$9\left(\frac{4 + 5y}{3}\right) - 2y = 7$$

$$12 + 15y - 2y = 7$$

$$12 + 13y = 7$$

$$13y = 7 - 12 = -5$$

$$y = \frac{-5}{13}$$

Substitute $y = \frac{-5}{13}$ in equation (3)

$$x = \frac{4 + 5y}{3} = \frac{4 + 5\left(\frac{-5}{13}\right)}{3} = \frac{52 - 25}{3 \times 13} = \frac{27}{3 \times 13} = \frac{9}{13}$$

$\therefore$ The solution is $x = \frac{9}{13}$ *and* $y = \frac{-5}{13}$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

Sol: $\frac{x}{2} + \frac{2y}{3} = -1 \Rightarrow 3x + 4y = -6 \rightarrow (1)$

$x - \frac{y}{3} = 3 \Rightarrow 3x - y = 9 \rightarrow (2)$

Fropm (2): $y = 3x - 9$

Substitute $y = 3x - 9$ in equation (1)

$$3x + 4(3x - 9) = -6$$

$$3x + 12x - 36 = -6$$

$$15x = -6 + 36 = 30$$

$$x = 2$$

$\therefore$ The solution is $x = 2$ *and* $y = -3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Sol: Let the fraction $= \frac{x}{y}$

From problem

$\frac{x+1}{y-1} = 1$ and $\frac{x}{y+1} = \frac{1}{2}$

$x + 1 = y - 1$ and $2x = y + 1$

$x - y = -1 - 1$ and $2x - y = 1$

$x - y = -2 \rightarrow (1)$ and $2x - y = 1 \rightarrow (2)$

$(1) \Rightarrow \quad x - y = -2$

$-1 \times (2) \Rightarrow -2x + y = -1$

$Adding \quad -x \quad = -3$

$x = 3$

Substitute x=3 in (1)

$3 - y = -2$

$-y = -2 - 3$

$y = 5$

Required fraction $= \frac{3}{5}$

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

Sol:

	Nuri's age	Sonu's age
Present	x	y
5 years ago	$x - 5$	$y - 5$
10 years later	$x + 10$	$y + 10$

From problem

$x - 5 = 3(y - 5)$ and $x + 10 = 2(y + 10)$

$x - 5 = 3y - 15$ and $x + 10 = 2y + 20$

$x - 3y = -15 + 5$ and $x - 2y = 20 - 10$

$x - 3y = -10 \rightarrow (1)$ and $x - 2y = 10 \rightarrow (2)$

$2 \times (1) \Rightarrow \quad 2x - 6y = -20$

$-3 \times (2) \Rightarrow -3x + 6y = -30$

$Adding \quad -x \quad = -50$

$x = 50$

Substitute x=50in equ (1)

$50 - 3y = -10$

$-3y = -10 - 50$

$-3y = -60$

$y = 20$

Age of Nuri=50 years and age of Sonu=20 years

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Sol: Let the unit place of digit be x and tens place digit be y

The number=$10y + x$

The number obtained by reversing the digits=$10x + y$

From the problem

$x+y=9 \rightarrow (1)$
$9(10y+x)=2(10x+y)$
$90y+9x=20x+2y$
$20x+2y-90y-9x=0$
$11x-88y=0$
$11(x-8y)=0$
$x-8y=0 \rightarrow (2)$
From (1)-(2)
$x+y-x+8y=9-0$
$9y=9$
$y=1$
Substitute y=1 in (1)
$x+1=9$
$x=8$
The number=10y+x=10×1+8=18

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

Sol: Let the number of ₹50 notes $= x$

The number of ₹100 notes $= y$

Total notes=25

$x+y=25 \rightarrow (1)$

Value of notes=₹ 2000

$50x+100y=2000$

$x+2y=40 \rightarrow (2)$

$(2) \Rightarrow x+2y=40$

$(1) \Rightarrow x+\ \ y=25$

$\quad (-)\ (-)\ \ (-)$

$\qquad y=15$

Substitute y=15 in equ (1)

$x+15=25$

$x=25-15=10$

∴ Meena received ten ₹50 notes and fifteen ₹100 rupee notes.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Sol: Let the fixed charge for the first three days $=x$
Charge per extra day$=y$
Saritha paid ₹ 27 for a book kept for seven days.
$x+4y=27 \rightarrow (1)$Susy paid ₹ 21 for the book she kept for five days.
$x+2y=21 \rightarrow (2)$
From (1)-(2)
$x+4y-x-2y=27-21$

$2y = 6$
$y = 3$
Substituting y = 3 in equation (1)
$x + 4(3) = 27$
$x = 27 - 12$
$x = 15$
The fixed charge=₹ 15 and the charge for each extra day=₹3

Some more problems for brain boosting

1. For the pair of equations $\lambda x + 3y = -7$ and $2x + 6y = 14$ to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.
2. For all real values of c, the pair of equations $x - 2y = 8$ and $5x - 10y = c$ have a unique solution. Justify whether it is true or false.
3. For which value(s) of k will the pair of equations $kx + 3y = k - 3$ and $12x + ky = k$ have no solution?
4. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle
5. Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis
6. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.
7. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?
8. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.
9. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers
10. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.
11. Solve the following system of linear equations $7x - 2y = 5$ *and* $8x + 7y = 15$ and verify your answer [CBSE 2024]
12. Three years ago, Rashmi was thrice as old as Nazma. Ten years later, Rashmi will be twise as old as Nazma. How old are Rashmi and Nazma now? [CBSE 2024]

Answers

1. No
2. False
3. k=-6
4. (2,0), (0, 4), (0, −4); 8 sq. units.
5. 6 sq units.
6. Do not cross each other
7. Salim's age = 38 years, Daughter's age = 14 years
8. 40 years
9. 40,48

10. $4x + 4y = 100, 3x = y + 15$, where Rs x and Rs y are the costs of a pen and a pencil box respectively; Rs 10, Rs 15

11. x=1 and y=1

12. Rashmi=42 years and Nazma=16 years

MCQ

1. The pair of equations $5x - 15y = 8$ and $3x - 9y = 24/5$ has
(A) one solution (B) two solutions (C) infinitely many solutions (D) no solution

2. The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is
(A) 25 (B) 72 (C) 63 (D) 36

3. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have
(A) a unique solution (B) exactly two solutions (C) infinitely many solutions (D) no solution

4. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?
(A) 1 /2 (B) -1/ 2 (C) 2 (D) −2

5. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is
(A) −5/ 4 (B) 2/ 5 (C) 15/ 4 (D) 3/ 2

6. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is
(A) 3 (B) – 3 (C) −12 (D) no value

7. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be
(A) $10x + 14y + 4 = 0$ (B) $-10x - 14y + 4 = 0$ (C) $-10x + 14y + 4 = 0$ (D) $10x - 14y = -4$

8. If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively
(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) −1 and −3

9. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively
(A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25

10.

1) C	2) D	3)D	4)C	5)C	6)D	7)D	8)C	9)D	10)

Case Study-based Questions

1) A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct answer	Marks deducted for wrong answer
True/False	1	0.25

(i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?

(ii) How many questions did he guess?

(iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

(iv) If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

Sol: (i) Let correctly answered questions=x and wrong answered questions=y

Total number of questions=120

$x + y = 120 \rightarrow (1)$

Total marks=90

$1 \times x - 0.25 \times y = 90$

Multiplying with 4

$4x - y = 360 \rightarrow (2)$

From (1)+(2)

$x + y + 4x - y = 120 + 360$

$5x = 480$

$x = \frac{480}{5} = 96$

$Substitute\ x = 96\ in\ (1)$

$96 + y = 120$

$y = 120 - 96 = 24$

$\therefore x = 96\ and\ y = 24$

The number of questions student answered correctly=96

(ii) The number of questions by guessing were wrong=24

(iii) If student attempted by guessing were wrong and answered 80 correctly then

$student\ got\ the\ marks = 1 \times 80 - 0.25 \times 40 = 80 - 10 = 70$

(iv) Let correctly answered questions=x then wrong answered questions=120-x

$x - 0.25(120 - x) = 95$

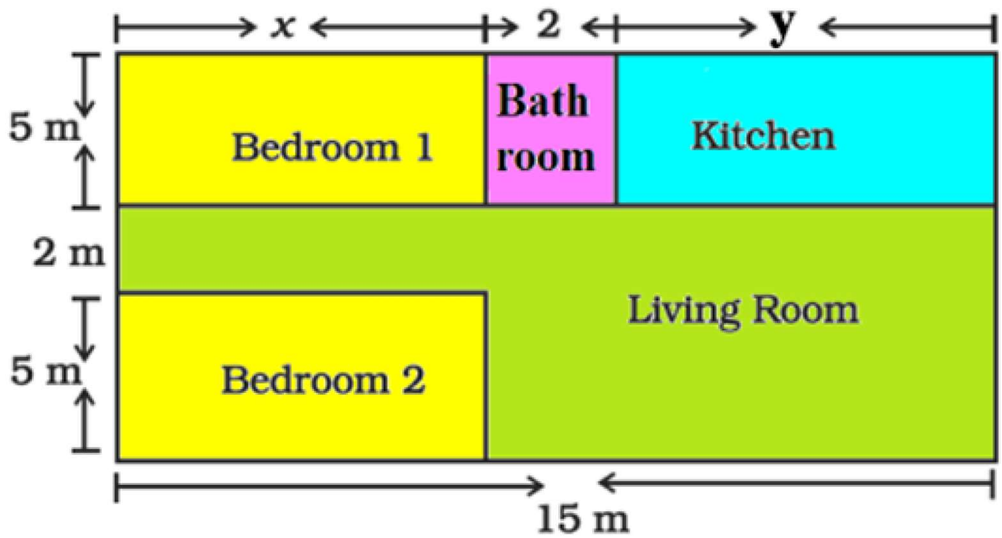
Multiply by 4

$4x - 120 + x = 380$

$5x = 500$

$x = 100$

2) Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m.



Based on the above information, answer the following questions:

(i) Form the pair of linear equations in two variables from this situation.

(ii) Find the length of the outer boundary of the layout.

(iii) Find the area of each bedroom and kitchen in the layout.

(iv) Find the area of living room in the layout.

(v) Find the cost of laying tiles in kitchen at the rate of ₹50 per sq.m

Sol: (i) Total length=15 m

$x + 2 + y = 15$

$x + y = 13 \rightarrow (1)$

The areas of two bedrooms and kitchen together is 95 sq.m

$2 \times (5x) + 5y = 95$

$2x + y = 19 \rightarrow (2)$

The required pair of linear equations are $x + y = 13$ *and* $2x + y = 19$

(ii) The length of the outer boundary of the layout=2(15 m+12 m)=2×27 m=54 m

(iii) From (2)-(1)

$2x + y - x - y = 19 - 13$

$x = 6$

Substitute $x = 6$ *in* (1)

$6 + y = 13$

$y = 13 - 6 = 7$

$\therefore x = 6$ *and* $y = 7$

$Area\ of\ each\ bedroom = 5x = 5 \times 6 = 30\ sq\ m$

$Area\ of\ kitchen = 5y = 5 \times 7 = 35\ sq.m$

(iv) Area of living room $= 15 \times 7 -$ Area of bedroom $= 105 - 30 = 75\ sq.m.$

(v) Area of kitchen $= 35\ sq.m$

The cost of laying tiles in kitchen at the rate of ₹ 50 per sq.m=₹50×35=₹1750

Previous year problems:

1. **A shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes etc .He takes a fixed charge for the first two days, and an additional charge for subsequent day. Amruta paid ₹22 for a book kept for six days, while Radhika paid ₹16 for the book kept for four days. Assume that the fixed charge be ₹ x and additional charge (per day) be ₹ y .**

   Based on the above information, answer any four of the following questions

   a) The situation of amount paid by Radhika, is algebraically represented by.

   $Sol: x + 2y = 16 \rightarrow (1)$

   b) The situation of amount paid by Amruta, is algebraically represented by

   $Sol: x + 4y = 22 \rightarrow (2)$

   c) What are the fixed charges of the book?

   $Sol: 2 \times (1) - (2) \Rightarrow 2x + 4y - x - 4y = 32 - 22$

   $\Rightarrow x = 10$

   The fixed charges of the book =₹10

   d) What are the additional charges for each subsequent day for a book?

   $Sol: Substitute\ x = 10\ in\ (1)$

   $10 + 2y = 16$

   $2y = 16 - 10 = 6$

   $y = \frac{6}{2} = 3$

   The additional charges for each subsequent day for a book=₹3

   e) What is the total amount paid by both, if both of them have kept the book for 2 more days?

   $Sol: (22 + 2 \times 3) + (16 + 2 \times 3) = 28 + 22 = ₹50$

2. **Two Schools 'P' and 'Q' decided to award prizes to their students for two games of Hockey ₹x per student and Cricket ₹y per student. School 'P' decided to award a total of 9,500 for the two games to 5 and 4 students respectively; while school 'Q' decided to award 7,370 for the two games to 4 and 3 students respectively. Based on the above information, answer the following questions**

   a) Represent the given information algebraically(in x and y)

   $Sol: 5x + 4y = 9500 \ \rightarrow (1)\ \ and\ 4x + 3y = 7{,}370 \rightarrow (2)$

   b) What is the prize amount for hockey?

   ***Sol*:**

   $4 \times (2) \Rightarrow 16x + 12y = 29{,}480$

$3 \times (1) \Rightarrow 15x + 12y = 28{,}500$

$On\ Subtracting : x = 980$

The prize amount for hockey=₹980

c) Prize amount of which game is more and by how much?

Sol: Substitute x=980 in (1)

$5 \times 980 + 4y = 9500$

$4900 + 4y = 9500$

$4y = 9500 - 4900 = 4600$

$y = \frac{4600}{4} = 1150$

The prize amount for cricket=₹1150

The prize amount of cricket is more and it is (1150-980)=₹170

d) What will be the total amount prize if there are 2 students each from 2 games?

$Sol: 2(x + y) = 2(980 + 1150) = 2 \times 2130 = ₹4260$

CHAPTER 4

X-MATHEMATICS-NCERT-2024-25

QUADRATIC EQUATIONS

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https://sureshmathsmaterial.com

1. The polynomial equation whose highest degree is two is called a quadratic equation.
2. Any equation of the form $p(x) = 0$ where p(x) is polynomial of degree 2, is a **quadratic equation**.
3. **Standard form of quadratic equation** in variable x is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $a \neq 0$. We can write it as $y = ax^2 + bx + c$

coefficient of x^2 — constant

$$ax^2 + bx + c = 0$$

coefficient of x

Example 1 : Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol: Let the number of marbles at John $= x$. Then the number of marbles at Jivanti $= 45 - x$.

The number of marbles left with John, when he lost 5 marbles $= x - 5$

The number of marbles left with Jivanti, when she lost 5 marbles $= 45 - x - 5 = 40 - x$

Therefore, their product $= 124$

$(x - 5)(40 - x) = 124$

$40x - x^2 - 200 + 5x = 124$

https://sureshmathsmaterial.com

$-x^2 + 45x - 200 = 124$

$-x^2 + 45x - 200 - 124 = 0$

$-x^2 + 45x - 324 = 0$

$x^2 - 45x + 324 = 0$

This is the required representation of the problem mathematically.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹750. We would like to find out the number of toys produced on that day.

Sol: Let the number of toys produced on that day $= x$.

The cost of production (in rupees) of each toy that day $= 55 - x$

So, the total cost of production (in rupees) that day $= x\,(55 - x)$

$x\,(55 - x) = 750$

$55x - x^2 = 750$

$-x^2 + 55x - 750 = 0$

$x^2 - 55x + 750 = 0$

This is the required representation of the problem mathematically.

Example 2 : Check whether the following are quadratic equations:

Standard form of quadratic equation in variable x is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b, c are real numbers and $a \neq 0$.

i. $\boldsymbol{(x-2)^2 + 1 = 2x - 3}$

Sol: $(x-2)^2 + 1 = 2x - 3$

$\Rightarrow x^2 - 4x + 4 + 1 - 2x + 3 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$

It is of the form $ax^2 + bx + c = 0\ (a = 1, b = -6, c = 8)$

The given equation is a quadratic equation.

ii. $\boldsymbol{x(x+1) + 8 = (x+2)(x-2)}$

Sol: $x(x+1) + 8 = (x+2)(x-2)$

$\Rightarrow x^2 + x + 8 = x^2 - 2^2$

$\Rightarrow x^2 + x + 8 - x^2 + 4 = 0$

$\Rightarrow x + 12 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

iii. $\boldsymbol{x(2x+3) = x^2 + 1}$

Sol: $x(2x+3) = x^2 + 1$

$\Rightarrow 2x^2 + 3x - x^2 - 1 = 0$

$\Rightarrow x^2 + 3x - 1 = 0$

It is of the form $ax^2 + bx + c = 0\ (a = 1, b = 3, c = -1)$

The given equation is a quadratic equation.

iv. $\boldsymbol{(x+2)^3 = x^3 - 4}$

Sol: $(x+2)^3 = x^3 - 4$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x+2) = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x(x+2) = x^3 - 4$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - x^3 + 4 = 0$

$\Rightarrow 6x^2 + 12x + 12 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 6, b = 12, c = 12)$

The given equation is a quadratic equation.

EXERCISE 4.1

1. **Check whether the following are quadratic equations :**

i. $\mathbf{(x+1)^2 = 2(x-3)}$

Sol: $(x+1)^2 = 2(x-3)$

$\Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 2x + 1 - 2x + 6 = 0$

$\Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = 0, c = 7)$

The given equation is a quadratic equation.

ii. $\mathbf{x^2 - 2x = (-2)(3-x)}$

Sol: $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x + 6 - 2x = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = -4, c = 6)$

The given equation is a quadratic equation.

iii. $\mathbf{(x-2)(x+1) = (x-1)(x+3)}$

Sol: $(x-2)(x+1) = (x-1)(x+3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3$

$\Rightarrow x^2 - x - 2 - x^2 - 2x + 3 = 0$

$\Rightarrow -3x + 1 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

iv. $(x - 3)(2x + 1) = x(x + 5)$

Sol: $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 - 5x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = -10, c = -3)$

The given equation is a quadratic equation.

v. $(2x - 1)(x - 3) = (x + 5)(x - 1)$

Sol: $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 - 4x + 5 = 0$

$\Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 1, b = -11, c = 8)$

The given equation is a quadratic equation.

vi. $x^2 + 3x + 1 = (x - 2)^2$

Sol: $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

$\Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

vii. $(x + 2)^3 = 2x(x^2 - 1)$

Sol: $(x+2)^3 = 2x(x^2-1)$

$\Rightarrow x^3 + 2^3 + 3 \times x \times 2(x+2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$

$\Rightarrow -x^3 + 6x^2 + 14x + 8 = 0$

It is not of the form $ax^2 + bx + c = 0$

The given equation is not a quadratic equation.

viii. $\boldsymbol{x^3 - 4x^2 - x + 1 = (x-2)^3}$

Sol: $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3 \times x \times 2(x-2)$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$

$\Rightarrow x^3 - 4x^2 - x + 1 - x^3 + 8 + 6x^2 - 12x = 0$

$\Rightarrow 2x^2 - 13x + 9 = 0$

It is of the form $ax^2 + bx + c = 0$ $(a = 2, b = -13, c = 9)$

The given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations :

i. The area of a rectangular plot is 528 m² . The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

Sol: Let breadth of rectangular plot $(b) = x\ m$

Length of rectangular plot $(l) = (2x+1)m$

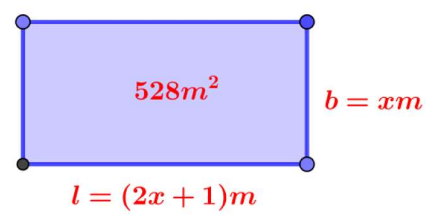
Given area of the rectangular plot $= 528\ m^2$

$l \times b = 528$

$(2x+1) \times x = 528$

$2x^2 + x - 528 = 0$

This is the required quadratic equation.



ii. The product of two consecutive positive integers is 306. We need to find the integers.

Sol: Let the two consecutive positive integers be $x, x+1$

Given the product of two consecutive positive integers $= 306$

$x \times (x+1) = 306 \Longrightarrow x^2 + x - 306 = 0$

This is the required quadratic equation

iii. **Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. We need to find Rohan's present age.**

Sol: Let Rohan's age=x years

Rohan's mother age=$(x+26) years$

After 3 years

Rohan's age=$x+3$ years

	Rohan	Rohan's mother
Present age(years)	x	$x+26$
Age after 3 years	$x+3$	$x+29$

Rohan's mother age=$(x+26+3) = (x+29) years$

Given the product of their ages after 3 years=360 years

$\Rightarrow (x+3)(x+29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

This is the required quadratic equation.

iv. **A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.**

Sol: Let the speed of the train= $x\ km/h$

Distance= 480 km

$\text{Time}(T_1) = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{x}\ h.$

If the speed had been 8 km/h less, then the speed=$(x-8)\ km/h$

$\text{Time}(T_2) = \frac{\text{Distance}}{\text{Speed}} = \frac{480}{x-8} h$

Difference of the times= $(T_2 - T_1)$ =3h

$$\frac{480}{x-8} - \frac{480}{x} = 3$$

$$480\left(\frac{1}{x-8} - \frac{1}{x}\right) = 3$$

$$\frac{x-(x-8)}{x(x-8)} = \frac{3}{480}$$

$$\frac{x - x + 8}{x^2 - 8x} = \frac{1}{160}$$

$$\frac{8}{x^2 - 8x} = \frac{1}{160}$$

$$x^2 - 8x = 160 \times 8$$

$$x^2 - 8x = 1280$$

$$x^2 - 8x - 1280 = 0$$

This is the required quadratic equation to find the speed of the train.

Solution of a Quadratic Equation by Factorisation

(i) A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\,\alpha + c = 0$ We also say that x = α is a solution of the quadratic equation. (i.e) the real value of for which the quadratic equation $ax^2 + bx + c = 0$is satisfied is called its solution.

(ii) The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Example-3. Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorisation.

Sol: $2x^2 - 5x + 3 = 0$

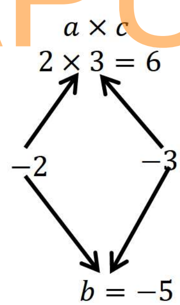
$2x^2 - 2x - 3x + 3 = 0$

$2x(x - 1) - 3(x - 1) = 0$

$(x - 1)(2x - 3) = 0$

$x - 1 = 0 \quad or \quad 2x - 3 = 0$

$x = 1 \quad or \quad x = \frac{3}{2}$

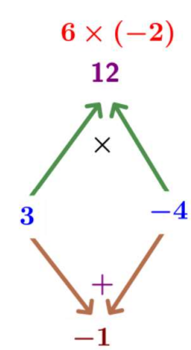


$\therefore 1\ and\ \frac{3}{2}$ are the roots of the equation $2x^2 - 5x + 3 = 0$

Example 4 : Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Sol: $6x^2 - x - 2 = 0$

$6 \times (-2) = -12$
$3 \times (-4) = -12\ and\ 3 + (-4) = -1$



$6x^2 + 3x - 4x - 2 = 0$

$3x(2x + 1) - 2(2x + 1) = 0$

$(2x + 1)(3x - 2) = 0$

$2x + 1 = 0\ or\ 3x - 2 = 0$

$x = -\frac{1}{2}\ or\ x = \frac{2}{3}$

The roots of $6x^2 - x - 2 = 0\ are\ -\frac{1}{2}\ and\ \frac{2}{3}$

Example 5 : Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$

Sol: $3x^2 - 2\sqrt{6}x + 2 = 0$

$3 \times 2 = 6$

$(-\sqrt{6})(-\sqrt{6}) = 6$

$-\sqrt{6} - \sqrt{6} = -2\sqrt{6}$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3} \times \sqrt{3}x^2 - \sqrt{3} \times \sqrt{2}x - \sqrt{3} \times \sqrt{2}x + \sqrt{2} \times \sqrt{2} = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\sqrt{3}x - \sqrt{2} = 0 \Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

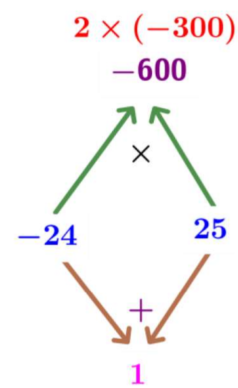
The roots of $3x^2 - 2\sqrt{6}x + 2 = 0\ are\ \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

Example 6 : Find the dimensions of the prayer hall having a carpet area of 300 square metres with its length one metre more than twice its breadth.

Sol: Let breadth of the hall = x metres

Length = $(2x + 1)$ metres.

Area of the hall = $(2x + 1).x\ m^2 = (2x^2 + x)m^2$



$2x^2 + x = 300\ (Given)$

$2x^2 + x - 300 = 0$

$2x^2 - 24x + 25x - 300 = 0$

$2x(x - 12) + 25(x - 12) = 0$

$i.e., (x - 12)(2x + 25) = 0$

$x - 12 = or\ 2x + 25 = 0$

$x = 12\ or\ x = \frac{-25}{2}$

Since x is the breadth of the hall, it cannot be negative

$\therefore x = 12$

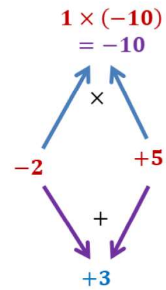
Length $= 2x + 1 = 2 \times 12 + 1 = 24 + 1 = 25m$ and breadth $= x = 12\ m$

The dimensions of the hall are $25\ m\ and\ 12m$

EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

2. $(i) x^2 - 3x - 10 = 0$



Sol: $x^2 - 3x - 10 = 0$v

$x^2 - 5x + 2x - 10 = 0$

$x(x - 5) + 2(x - 5) = 0$

$(x + 2)(x - 5) = 0$

$x + 2 = 0 \quad or \quad x - 5 = 0$

$x = -2 \ or \ x = 5$

The roots of $x^2 - 3x - 10 = 0$ are $-2\ and\ 5$

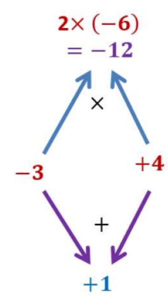
$(ii) 2x^2 + x - 6 = 0$

Sol: $2x^2 + x - 6 = 0$

$2x^2 - 3x + 4x - 6 = 0$

$x(2x - 3) + 2(2x - 3) = 0$

$(2x - 3)(x + 2) = 0$



$2x - 3 = 0 \quad or \quad x + 2 = 0$

$x = \frac{3}{2} \quad or \quad x = -2$

The roots of $2x^2 + x - 6 = 0\ are\ \frac{3}{2}\ and\ -2$

$(iii)\ \sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2} \times 5\sqrt{2} = 5 \times 2 = 10$
$2 \times 5 = 10$
$2 + 5 = 7$

Sol: $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$(x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$x + \sqrt{2} = 0 \quad or \quad \sqrt{2}x + 5 = 0$

$x = -\sqrt{2}$ or $x = \frac{-5}{\sqrt{2}}$

The roots of $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ are $-\sqrt{2}$ and $\frac{-5}{\sqrt{2}}$

$(iv)\ 2x^2 - x + \frac{1}{8} = 0$

Sol: $2x^2 - x + \frac{1}{8} = 0$

Multiply with '8'

$8 \times 2x^2 - 8 \times x + 8 \times \frac{1}{8} = 8 \times 0$

$16x^2 - 8x + 1 = 0$

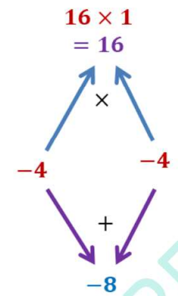
$16x^2 - 4x - 4x + 1 = 0$

$4x(4x - 1) - 1(4x - 1) = 0$

$(4x - 1)(4x - 1) = 0$

$4x - 1 = 0$ or $4x - 1 = 0$

$x = \frac{1}{4}$ or $x = \frac{1}{4}$



The roots of $2x^2 - x + \frac{1}{8} = 0$ are $\frac{1}{4}$ and $\frac{1}{4}$.

$(v)\ 100x^2 - 20x + 1 = 0$

Sol: $100x^2 - 20x + 1 = 0$

$100x^2 - 10x - 10x + 1 = 0$

$10x(10x - 1) - 1(10x - 1) = 0$

$(10x - 1)(10x - 1) = 0$

$10x - 1 = 0$ or $10x - 1 = 0$

$x = \frac{1}{10}$ or $x = \frac{1}{10}$

The roots of $100x^2 - 20x + 1 = 0$ are $\frac{1}{10}$ and $\frac{1}{10}$

3. (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Sol: Total marbles=45

	John	Jivanti
Number of marbles	x	$45 - x$
Number of marbles when he lost 5 marbles	$x - 5$	$45 - x - 5 = 40 - x$

Given that the product of marbles when they lost 5 marbles =124

$(x - 5)(40 - x) = 124$

$40x - x^2 - 200 + 5x = 124$

$-x^2 + 45x - 200 = 124$

$-x^2 + 45x - 200 - 124 = 0$

$-x^2 + 45x - 324 = 0$

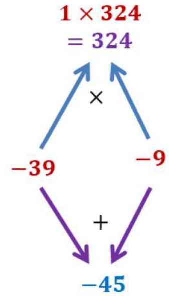
$x^2 - 45x + 324 = 0$

$x^2 - 36x - 9x + 324 = 0$

$x(x - 36) - 9(x - 36) = 0$

$(x - 36)(x - 9) = 0$

$x = 36 \text{ or } x = 9$



If $x = 36$ then John's marbles=36 and Jivanti's marbles=9

If $x = 9$ then John's marbles=9 and Jivanti's marbles=36

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Sol: Let the number of toys produced on that day be x. Therefore, the cost of production (in rupees) of each toy that day = $55 - x$

Given the total cost of production (in rupees) that day $= 750$

$x(55 - x) = 750$

$55x - x^2 = 750$

$-x^2 + 55x - 750 = 0$

$x^2 - 55x + 750 = 0$

$(x - 25)(x - 30) = 0$

$x = 25 \text{ or } x = 30$

$(-25) \times (-30) = 750$
$-25 - 30 = -55$

$\therefore$ The number of toys produced on that day = 25 or 30

4. Find two numbers whose sum is 27 and product is 182.

Sol: Let one number $= x$, The second number $= 27 - x$

Product of numbers=182

$x(27 - x) = 182$

$27x - x^2 = 182$

$-x^2 + 27x - 182 = 0$

$x^2 - 27x + 182 = 0$

$x^2 - 13x - 14x + 182 = 0$

$x(x - 13) - 14(x - 13) = 0$

$(x - 13)(x - 14) = 0$

$x - 13 = 0 \quad or \quad x - 14 = 0$

$x = 13 \quad or \quad x = 14$

If $x = 13$ the required numbers are 13 and 14.

If $x = 14$ the required numbers are 14 and 13.

5. Find two consecutive positive integers, sum of whose squares is 365.

Sol: Let the two consecutive positive integers be $x, x + 1$.

Sum of whose squares $= 365$

$x^2 + (x + 1)^2 = 365$

$x^2 + x^2 + 2x + 1 - 365 = 0$

$2x^2 + 2x - 364 = 0$

$x^2 + x - 182 = 0$

$x^2 - 13x + 14x - 182 = 0$

$x(x - 13) + 14(x - 17) = 0$

$(x - 13)(x + 14) = 0$

$x = 13 \quad or \quad x = -14$

$\therefore x = 13$ (since x is a positive integer so $x \neq -14$)

The required two consecutive positive integers are 13 and 14.

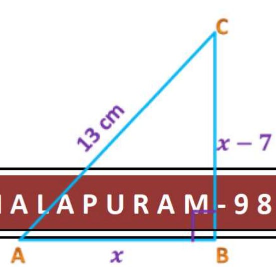
6. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol: Base of a right triangle (AB)$=x$

The altitude (BC) $= x - 7$ cm

The hypotenuse (AC)$=$ 13 cm

From Pythagoras theorem



$AB^2 + BC^2 = AC^2$

$x^2 + (x-7)^2 = 13^2$

$x^2 + x^2 - 14x + 49 - 169 = 0$

$2x^2 - 14x - 120 = 0$

$x^2 - 7x - 60 = 0$

$x^2 - 12x + 5x - 60 = 0$

$x(x-12) + 5(x-12) = 0$

$(x-12)(x+5) = 0$

$x - 12 = 0 \quad or \quad x + 5 = 0$

$x = 12 \quad or \quad x = -5$

$\therefore x = 12$ (since side of a triangle is positive integer so $x \neq -5$)

The other two sides are 12 cm, $(12-7)$ cm $i.e$ 12 cm , 5 cm.

7. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ` 90, find the number of articles produced and the cost of each article.

Sol: Let the number of articles produced$=x$

The cost of each article$=Rs\ (2x+3)$

Given the total cost of production on that day=Rs 90

$x(2x+3) = 90$

$2x^2 + 3x - 90 = 0$

$2x^2 - 12x + 15x - 90 = 0$

$2x(x-6) + 15(x-6) = 0$

$(x-6)(2x+15) = 0$

$x - 6 = 0 \quad or \quad 2x + 15 = 0$

$x = 6 \quad or \quad x = \frac{-15}{2}$

$\therefore\ x = 6$ (Number of articles is always can't be negative)

The number of articles produced= x =6

The cost of each article= $(2x+3)$= $(2\times 6+3)$ = ₹ 15.

Quadratic formula (formula for finding the roots of a quadratic equation)

The the roots of quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Nature of Roots

The nature of roots of a quadratic equation $ax^2+bx+c=0$ depends on b^2-4ac

b^2-4ac is called the discriminant of this quadratic equation.

A quadratic equation $ax^2+bx+c=0$ has

(i) two distinct real roots, if $b^2-4ac>0$.

(ii) two equal real roots, if $b^2-4ac=0$

(iii) no real roots, if $b^2-4ac<0$.

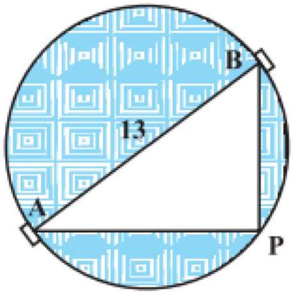
Example 7: Find the discriminant of the quadratic equation $2x^2-4x+3=0$, and hence find the nature of its roots.

Sol: Given Q. E is $2x^2-4x+3=0$; $a=2, b=-4, c=3$

$b^2-4ac=(-4)^2-4\times 2\times 3=16-24=-8<0$

So, the given equation has no real roots.

Example 8 : A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?



Sol: Let P be the required location of the pole.

Let BP = x m then AP = $(x+7)$m and AB = 13 m

We know that angle in semicircle=90^0. So $\angle APB=90^0$

$AP^2+BP^2=AB^2$ (By Pythagoras theorem)

$(x+7)^2+x^2=13^2$

$x^2+14x+49+x^2-169=0$

$2x^2+14x-120=0$

$-5\times 12=-60$
$-5+12=7$

$x^2+7x-60=0$

$(x-5)(x+12)=0$

$x-5=0$ or $x+12=0$

$x = 5 \text{ or } x = -12$

$\therefore x = 5$ (distance can't be negative)

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12m from the gate A.

Example-9. Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

Sol: Given Q.E is $3x^2 - 2x + \frac{1}{3} = 0$: $a = 3, b = -2, c = \frac{1}{3}$

$b^2 - 4ac = (-2)^2 - 4 \times 3 \times \frac{1}{3} = 4 - 4 = 0$

So, the roots are equal and real.

The roots are $\frac{-b}{2a}, \frac{-b}{2a} \Rightarrow \frac{2}{2\times3}, \frac{2}{2\times3} \Rightarrow \frac{1}{3}, \frac{1}{3}$.

EXERCISE 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

Sol: $a = 2, b = -3, c = 5$

$b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$

So, the Q.E has no real roots.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Sol: $a = 3, b = -4\sqrt{3}, c = 4$

$b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

So, the roots are real and equal

The roots are $\frac{-b}{2a}, \frac{-b}{2a} \Rightarrow \frac{4\sqrt{3}}{2\times3}, \frac{4\sqrt{3}}{2\times3} \Rightarrow \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Sol: $a = 2, b = -6, c = 3$

$b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

So, the roots are real and distinct.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{2(3 \pm \sqrt{3})}{4} = \frac{3 \pm \sqrt{3}}{2}$$

The roots are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

Sol: $a = 2, b = k, c = 3$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$k^2 - 4 \times 2 \times 3 = 0$

$k^2 = 24 \Rightarrow k = \pm\sqrt{24} = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x-2) + 6 = 0$

Sol: $kx^2 - 2kx + 6 = 0$

$a = k, b = -2k, c = 6$

If the Q.E has equal roots then $b^2 - 4ac = 0$

$(-2k)^2 - 4 \times k \times 6 = 0$

$4k^2 - 24k = 0$

$4k(k-6) = 0$

$4k = 0 \ or \ k - 6 = 0$

$k = 0 \ \ or \ \ k = 6$

$\therefore k = 6 \ (if \ k = 0 \ then \ a = 0, b = 0 \ it \ is \ not \ a \ Q.E)$

3. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m2 ? If so, find its length and breadth.

Sol: Let the breadth $(b) = x\ m$

Length $(l) = 2x\ m$

Given area of the rectangular grove=800m^2

$x \times 2x = 800$

$x^2 = \frac{800}{2} = 400 \Rightarrow x = \sqrt{400} = 20$

Yes , it is possible

Length of the mango grove=$2 \times 20 = 40m$

Breadth of the mango grove=20 m.

4. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Sol: The sum of ages of two friends=20 years

	First friend	Second friend
Present age(in years)	x	$20 - x$
Four years ago age	$x - 4$	$20 - x - 4 = 16 - x$

Four years ago, the product of their ages=48

$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x - 48 = 0$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

$a = 1, b = -20, c = 112$

$b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112 = 400 - 448 = -48 < 0$

The roots are not real. So, the situation is not possible.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m² ? If so, find its length and breadth.

Sol: Let length of rectangular park (l)=x m

Perimeter of the park=80 m

$2(l + b) = 80$

$x + b = \frac{80}{2} = 40 \Rightarrow b = 40 - x$

Area of park=400 m^2

$x(40 - x) = 400$

$40x - x^2 - 400 = 0$

$x^2 - 40x + 400 = 0$

$(x - 20)(x - 20) = 0$

$x - 20 = 0 \Rightarrow x = 20$

Length of the rectangular park=20 m

Breadth of the rectangular park=$40 - 20$ =20m

CASE STUDY BASED QUESTINS

1) **Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400**

(i) What will be the distance covered by Ajay's car in two hours?

a) 2(x +5)km b) (x – 5)km c) 2(x + 10)km d) (2x + 5)km

(ii) Which of the following quadratic equation describe the speed of Raj's car?

a) $x^2 - 5x - 500 = 0$ b) $x^2 + 4x - 400 = 0$ c) $x^2 + 5x - 500 = 0$ d) $x^2 - 4x + 400 = 0$

(iii) What is the speed of Raj's car?

a) 20 km/hour b) 15 km/hour c) 25 km/hour d) 10 km/hour

(iv) How much time took Ajay to travel 400 km?

a) 20 hour b) 40 hour c) 25 hour d) 16 hour

Sol:

Speed of Raja's car= $x\ km/h$

Speed of Ajay's car= $(x + 5)\ km/h$

(i) a

Sol: The distance covered by Ajay's car in two hours=$2 \times (x + 5)\ km = 2(x + 5)\ km$

(ii) c

Sol: Distance=400km

$$Time\ taken\ by\ Raj's\ car(T_1) = \frac{Distance}{Speed} = \frac{400}{x}\ h$$

$$Time\ taken\ by\ Ajay's\ car(T_2) = \frac{Distance}{Speed} = \frac{400}{x+5}\ h$$

$$According\ to\ problem : T_1 - T_2 = 4$$

$$\frac{400}{x} - \frac{400}{x+5} = 4$$

$$400\left(\frac{1}{x} - \frac{1}{x+5}\right) = 4$$

$$\frac{x+5-x}{x(x+5)} = \frac{4}{400}$$

$$\frac{5}{x^2+5x} = \frac{1}{100}$$

$$x^2 + 5x = 500$$

$$x^2 + 5x - 500 = 0$$

(iii) a

Sol:

$$x^2 + 5x - 500 = 0$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x+25) - 20(x+25) = 0$$

$$(x+25)(x-20) = 0$$

$$x = 20\ or - 25$$

$$x = 20(Speed\ can\ never\ be\ negative)$$

The speed of Raj's car=20 km/h

(iv) d

Sol: $Time\ taken\ by\ Ajay's\ car = \frac{400}{x+5} = \frac{400}{20+5} = \frac{400}{25} = 16\ hours$

(v) c

Sol: Speed of Ajay's car= $(x+5) = (20+5) = 25\ km/h$

2) **The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.**

(i) **Let speed of the stream be x km/hr. then speed of the motorboat in upstream will be**

a) 20 km/hr b) (20 + x) km/hr c) (20 − x) km/hr d) 2 km/hr

(ii) **What is the relation between speed ,distance and time?**

a) speed = (distance)/time b) distance = (speed)/time c) time = speed x distance d) speed = distance x time

(iii) **Which is the correct quadratic equation for the speed of the current ?**

a) $x^2 + 30x - 200 = 0$ b) $x^2 + 20x - 400 = 0$ c) $x^2\ 30x - 400 = 0$ d) $x^2 - 20x - 400 = 0$

(iv) **What is the speed of current ?**

a) 20 km/hour b) 10 km/hour c) 15 km/hour d) 25 km/hour

(v) **How much time boat took in downstream?**

a) 90 minute b) 15 minute c) 30 minute d) 45 minute

Sol: The speed of a motor boat = 20 km/hr

Speed of the stream= x km/hr

Speed of the motorboat in upstream=$(20-x)$ km/h

Speed of the motorboat in downstream=$(20+x)$ km/h

Distance=15 km

Time taken for upstream$(T_1) = \frac{\text{Distance}}{\text{Speed}} = \frac{15}{20-x}\ h$

Time taken for downstream$(T_2) = \frac{\text{Distance}}{\text{Speed}} = \frac{15}{20+x}\ h$

According to problem : $T_1 - T_2 = 1\ h$

$$\frac{15}{20-x} - \frac{15}{20+x} = 1$$

$$15\left(\frac{1}{20-x} - \frac{1}{20+x}\right) = 1$$

$$\frac{20+x-20+x}{(20-x)(20+x)} = \frac{1}{15}$$

$$\frac{2x}{400 - x^2} = \frac{1}{15}$$

$30x = 400 - x^2$

$x^2 + 30x - 400 = 0$

$x^2 + 40x - 10x - 400 = 0$

$x(x + 40) - 10(x + 40) = 0$

$(x + 40)(x - 10) = 0$

$x + 40 = 0 \ or \ x - 10 = 0$

$x = -40 \ or \ 10$

$x = 10(Speed \ can \ never \ be \ negative)$

Speed of the stream= $x = 10$ km/hr

Time taken for downstream$(T_2) = \frac{15}{20 + x} = \frac{15}{20 + 10} = \frac{15}{30} = \frac{1}{2} h = 30 \ minutes$

(i) **c**) $(20 - x) km/hr$

(ii) **b**) $distance = (speed)/ \ time$

(iii) **c**) $x2 + 30x - 400 = 0$

(iv) **b**) $10 \ km/hour$

(v) **c**) $30 \ minutes$

3) A rectangular floor area can be completely tiled with 200 square tiles . If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

(ii) Write the corresponding quadratic equation in standard form.

(iii) (a) Find the value of x , the length of side of a tile by factorisation. (OR)

(b) Solve the quadratic equation for x using quadratic formula.

Sol: Let the original length of each side of tile $= x \ units$

Area of each tile $= x \times x = x^2 \ sq \ units$

Area of floor $= Area \ of \ 200 \ tiles = 200x^2 \ sq \ units \rightarrow (1)$

The length of tile if the side is increased by 1 unit $= (x + 1) \ units$

Area of each new tile $= (x + 1) \times (x + 1) = (x + 1)^2 \ sq \ units$

Area of floor $= Area \ of \ 128 \ new \ tiles = 128(x + 1)^2 \ sq \ units \rightarrow (2)$

$From\ (1) and\ (2): 200x^2 = 128(x+1)^2$

$200x^2 = 128(x^2 + 2x + 1)$

$200x^2 - 128x^2 - 256x - 128 = 0$

$200x^2 - 256x - 128 = 0$

$9x^2 - 32x - 16 = 0$

(i) The required quadratic equation: $200x^2 = 128(x+1)^2$

(ii) The required quadratic equation in standard form: $9x^2 - 32x - 16 = 0$

(iii)a) $9x^2 - 32x - 16 = 0$

$9x^2 - 36x + 4x - 16 = 0$

$9x(x-4) + 4(x-4) = 0$

$(x-4)(9x+4) = 0$

$(x-4) = 0\ or\ (9x+4) = 0$

$x = 4\ or\ x = \frac{-4}{9}$

$x = 4$

b)Using quadratic formula

$a = 9, b = -32, c = -16$

$b^2 - 4ac = (-32)^2 - 4 \times 9 \times (-16) = 1024 + 576 = 1600$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{32 \pm \sqrt{1600}}{2 \times 9} = \frac{32 \pm 40}{18}$

$x = \frac{32+40}{18}\ or\ \frac{32-40}{18}$

$x = \frac{72}{18}\ or\ \frac{-8}{18}$

$x = 4\ or\ \frac{-4}{9}$

$Hence\ x = 4$

Some more problems for brain boosting:

1. Does $(x - 1)^2 + 2(x + 1) = 0$ have a real root? Justify your answer.
2. Find the roots of the quadratic equation $x^2 - 2\sqrt{2}x - 6 = 0$ using the quadratic formula.
3. Find the roots of the quadratic equation $2x^2 - \sqrt{5}x - 2 = 0$ using the quadratic formula
4. Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ by the factorisation of the corresponding quadratic polynomial.
5. Find the roots of $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ by the factorisation of the corresponding quadratic polynomial.

6. Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?
7. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.
8. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
9. A train travels at a certain average speed for a distance 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than the original speed. If it takes 3 hours to complete total journey, what is its original average speed?(CBSE-2023)
10. two water taps together can fill a tank in 15/ 8 hours .the tap with longer diameter takes 2 hours less than the smaller one to fill the tank separately .find the time in which each tap can fill the tank separately?(CBSE-2023).
11. Find the sum and product of the roots of the equation $2x^2 - 9x + 4$.(CBSE-Delhi-2023)
12. Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. .(CBSE-Delhi-2023)
13. Solve the quadratic equation:$x^2 - 2ax + (a^2 - b^2) = 0$ $for\ x$ (CBSE-2022)
14. The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers?(CBSE-2022)
15. The hypotenuse (in cm) of a right angled triangle is 6 cm more than twice the length of the length of the shortest side. If the length of third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle.(CBSE-2022)
16. If $\alpha\ and\ \beta$ are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$
17. Solve the quadratic equation $(x-1)^2 - 5(x-1) - 6 = 0$
18. If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that 2a=b+c
19. Find the value of k, if the quadratic equation $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots.
20. If the equation $(1+m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1+m^2)$
21.

Answers:

1) The discriminant of the equation is less than zero. Therefore, the equation has no real roots.
2) The roots of the equation are √2 and -3√2
3) the roots are (√5 + √21)/4 and (√5 - √21)/4.
4) the roots of the equation are -√2/3 and √2/2
5) the roots of the equation are √2 and -1/3√2.
6) Ajita scored 15 marks in the examination

7) the natural number is x = 8

8) the present age of Zeba is 14 years.

9) 42km/h

10) 5 hours and 3 hours.

11) 9/2 and 2

12) Discriminant=70 and roots are real distinct

13) x= a+b or a-b

14) 16,18 or 23,11

15) 26cm,24cm and 10cm

16) 15/4

17) x=0 or 7

18)

19) k=-3,5

20)

MCQ

1. Which of the following is not a quadratic equation?

(A) $2(x-1)^2 = 4x^2 - 2x + 1$ (B) $2x - x^2 = x^2 + 5$ (C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

2. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$ (C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(A) 2 (B) – 2 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

4. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(A) 0 only (B) 4 (C) 8 only (D) 0, 8

5. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) Two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

6. $(x^2 + 1)^2 - x^2 = 0\ has(11)$

(A) four real roots (B) two real roots (C) no real roots (D) one real root.

7. The quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has

(A) two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

8. The quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0\ has$

(A) two distinct real roots (B) two equal real roots (C) no real roots (D) more than 2 real roots

9. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is

(A) x^2-4x+1 (B) x^2+4x+1 (C) $4x^2-3$ (D) x^2-1

10. The root of the equation $x^2-3x-m(m+3)=0$, where m is constant are

(A) m,m+3 (B) −m,m+3 (C) m,-(m+3) (D) -m,-(m+3)

11. If 1 is a root of the equations $my^2+my+3=0$ and $x^2+x+n=0$ then $mn=$

A) 3 B) $-\frac{7}{2}$ C) 6 D) -3

12. If the roots of equation $ax^2+bx+c=0, a\neq 0$ are real and equal, then which of the following relation is true(CBSE-2024)

A) $a=\frac{b^2}{c}$ B) $b^2=ac$ C) $ac=\frac{b^2}{4}$ D) $c=\frac{b^2}{a}$

1)C	2)C	3)A	4)C	5)C	6)C	7)C	8)B	9)A	10)B	11)D	12)C

1. Assertion: If one root of the quadratic equation $6x^2-x-k=0$ is 2/3, then the value of k is 2.

   Reason: The quadratic equation $ax^2+bx+c=0$, $a\neq 0$ has almost two roots.

2. Assertion: $(2x-1)^2-4x^2+5=0$ is not a quadratic equation.

   Reason: An equation of the form $ax^2+bx+c=0$, $a\neq 0$, where a, b, c $\in$ R is called a quadratic equation.

3. Assertion: The roots of the quadratic equation $x^2+2x+2=0$ are imaginary

   Reason: If discriminant $D=b^2-4ac<0$ then the roots of quadratic equation $ax^2+bx+c=0$ are imaginary.

4. Assertion: $3x^2-6x+3=0$ has equal roots.

   Reason: The quadratic equation $ax^2+bx+c=0$ have equal roots if discriminant $D>0$.

5. Assertion: The quadratic equation $4x^2+6x+3$ has no real roots.

   Reason: The value of the discriminant is -12

6. Assertion : The values of x are $-\frac{a}{2}$, a for a quadratic equation $2x^2-ax-a^2=0$

   Reason : For quadratic equation $ax^2+bx+c=0$; $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

7. Assertion : $4x^2-12x+9=0$ has repeated roots.

Reason: The quadratic equation $ax^2+bx+c=0$ have repeated roots if discriminant $D>0$.

1)B	2)A	3)A	4)C	5)A	6) A	7)C

CHAPTER 5

X-MATHEMATICS-NCERT-2024-25

ARITHMETIC PROGRESSIONS

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https://sureshmathsmaterial.com

Arithmetic Progressions

1. An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
2. This fixed number is called the common difference of the AP it is denoted by d
3. The first term is denoted by a_1, $second\ term = a_2$, $third\ term = a_3, \dots \dots ..$
4. **General form of AP**: $\boldsymbol{a, a+d, a+2d, a+3d} \dots \dots$
5. $\boldsymbol{a_1} = a$; $\boldsymbol{a_2} = a + d$; $\boldsymbol{a_3} = a + 2d$; $\boldsymbol{a_7} = a + 6d$; $\boldsymbol{a_{13}} = a + 12d$; $\boldsymbol{a_{20}} = a + 19d$; ...
6. $\boldsymbol{n^{th}}$ **term of AP** : $a_n = a + (n-1)d$
7. $If\ \boldsymbol{a, b, c\ are\ in\ AP}\ then\ b - a = c - b \Rightarrow 2b = a + c \Rightarrow b = \frac{a+c}{2}$
8. In the list of numbers $a_1, a_2, a_3, a_4, ..$ if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, \dots .$

give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.

Example1: For the AP $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \dots \dots$ write the first term a and the common difference d.

Sol: $a_1 = \frac{3}{2}, a_2 = \frac{1}{2}, a_3 = \frac{-1}{2}, a_4 = \frac{-3}{2}, \dots$

First term $= a = a_1 = \frac{3}{2}$

Common difference $= d = a_2 - a_1 = \frac{1}{2} - \frac{3}{2} = \frac{1-3}{2} = \frac{-2}{2} = -1$

Example 2 : Which of the following list of numbers form an AP? If they form an AP, write the next two terms:

(i) 4, 10, 16, 22, . . .

Sol: $a_1 = 4, a_2 = 10, a_3 = 16, a_4 = 22$

$a_2 - a_1 = 10 - 4 = 6$

$a_3 - a_2 = 16 - 10 = 6$

$a_4 - a_3 = 22 - 16 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 6$

The next two terms are: $22 + 6 = 28$ and $28 + 6 = 34$

(ii) 1, – 1, – 3, – 5, . . .

Sol: $a_1 = 1, a_2 = -1, a_3 = -3, a_4 = -5, ..$

$a_2 - a_1 = -1 - 1 = -2$

$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$

$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next two terms are: $-5 - 2 = -7$ *and* $-7 - 2 = -9$

(iii) – 2, 2, – 2, 2, – 2, . . .

Sol: $a_1 = -2, a_2 = 2, a_3 = -2, a_4 = 2, a_5 = -2$

$a_2 - a_1 = 2 - (-2) = 2 + 2 = 4$

$a_3 - a_2 = -2 - 2 = -4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

(iv) 1, 1, 1, 2, 2, 2, 3, 3, 3, . . .

Sol: $a_1 = 1, a_2 = 1, a_3 1, = a_4 = 2,$

$a_2 - a_1 = 1 - 1 = 0$

$a_3 - a_2 = 1 - 1 = 0$

$a_4 - a_3 = 2 - 1 = 1$

$a_3 - a_2 \neq a_4 - a_3$

So, the given list of numbers does not form an AP.

EXERCISE 5.1

1. **In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?**

(i) The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.

Sol: Taxi fare for first km= ₹15

Taxi fare for second km= ₹15+₹8=₹23

Taxi fare for third km=₹23+₹8=₹31

Taxi fare for fourth km=₹31+₹8=₹39

∴ The taxi fares are ₹15, ₹23, ₹31, ₹39,……

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = 8$

It is an arithmetic progression with common difference=8

(ii) The amount of air present in a cylinder when a vacuum pump removes 1/ 4 of the air remaining in the cylinder at a time.

Sol: let the amount of air present in cylinder=x

(If a vacuum pump removes $\frac{1}{4}$ of the air then the remaining air is $\frac{3}{4}$ of it)

When vacuum pump use first time remaining air $= \frac{3}{4} \times x = \frac{3x}{4}$

Vacuum pump use second time remaining air $= \frac{3}{4} \times \frac{3x}{4} = \frac{9x}{16}$

Vacuum pump use third time remaining air $= \frac{3}{4} \times \frac{9x}{16} = \frac{27x}{64}$

List of air present in cylinder is $x, \frac{3x}{4}, \frac{9x}{16}, \frac{27x}{64}, \ldots$

$a_2 - a_1 = \frac{3x}{4} - x = \frac{3x - 4x}{4} = \frac{-x}{4}$

$a_3 - a_2 = \frac{9x}{16} - \frac{3x}{4} = \frac{9x-12x}{16} = \frac{-3x}{16}$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP.

(iii) The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.

Sol: The cost of digging for one metre=₹150

The cost of digging for two metres=₹150+₹50=₹200

The cost of digging for three metres=₹200+₹50=₹250

The cost of digging for four metres=₹250+₹50=₹300

The costs are ₹150, ₹200, ₹250, ₹300,….

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots =$ ₹50

It is an arithmetic progression with common difference=₹50

(iv) The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8 % per annum.

Sol: P = ₹10000 , $R = 8$ %,

$A = P\left(1 + \frac{R}{100}\right)^n$

$First\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} = ₹10800$

$Second\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^2 = 10000 \times \frac{108}{100} \times \frac{108}{100} = ₹11664$

$Third\ year\ ammount = 10000\left(1 + \frac{8}{100}\right)^1 = 10000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100} = ₹12597.12$

The amounts are ₹10000, ₹10800, ₹11664, ₹12597.12, …

$a_2 - a_1 = ₹10800 - ₹10000 = ₹800$

$a_3 - a_2 = ₹11664 - ₹10800 = ₹864$

$a_2 - a_1 \neq a_3 - a_2$

The given situations does not form an AP

2. **Write first four terms of the AP, when the first term a and the common difference d are given as follows:**

(i) $\boldsymbol{a = 10, d = 10}$

Sol: $a_1 = a = 10$

$a_2 = a + d = 10 + 10 = 20$

$a_3 = a + 2d = 10 + 2 \times 10 = 10 + 20 = 30$

$a_4 = a + 3d = 10 + 3 \times 10 = 10 + 30 = 40$

The first four terms of AP are 10,20,30,40

(ii) $\boldsymbol{a = -2, d = 0}$

Sol: $a_1 = a = -2$

$a_2 = a + d = -2 + 0 = -2$

$a_3 = a + 2d = -2 + 2 \times 0 = -2 + 0 = -2$

$a_4 = a + 3d = -2 + 3 \times 0 = -2 + 0 = -2$

The first four terms of AP are $-2, -2, -2, -2, ..$

(iii) **a = 4, d = – 3**

Sol: $a_1 = a = 4$

$a_2 = a + d = 4 + (-3) = 4 - 3 = 1$

$a_3 = a + 2d = 4 + 2 \times (-3) = 4 - 6 = -2$

$a_4 = a + 3d = 4 + 3 \times (-3) = 4 - 9 = -5$

The first four terms of AP are $4, 1, -2, -5$

(iv) $\boldsymbol{a = -1, d = \frac{1}{2}}$

Sol: $a_1 = a = -1$

$$a_2 = a + d = -1 + \frac{1}{2} = \frac{-2+1}{2} = \frac{-1}{2}$$

$$a_3 = a + 2d = -1 + 2 \times \left(\frac{1}{2}\right) = -1 + 1 = 0$$

$$a_4 = a + 3d = -1 + 3 \times \left(\frac{1}{2}\right) = -1 + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$$

$$The\ first\ four\ terms\ of\ AP\ are\ -1, \frac{-1}{2}, 0, \frac{1}{2}$$

(v) $a = -1.25, d = -0.25$

Sol: $a_1 = a = -1.25$

$a_2 = a + d = -1.25 + (-0.25) = -1.25 - 0.25 = -1.5$

$a_3 = a + 2d = -1.25 + 2 \times (-0.25) = -1.25 - 0.50 = -1.75$

$a_4 = a + 3d = -1.25 + 3 \times (-0.25) = -1.25 - 0.75 = -2$

The first four terms of AP are $-1.25, -1.5, -1.75, -2$

3. For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . .

Sol: First term=a=3

Common difference=$d = a_2 - a_1 = 1 - 3 = -2$

(ii) – 5, – 1, 3, 7, . . .

Sol: First term=a=-5

Common difference=$d = a_2 - a_1 = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Sol: First term=a=$\frac{1}{3}$

Common difference $= d = a_2 - a_1 = \frac{5}{3} - \frac{1}{3} = \frac{5-1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9, . . .

Sol: First term=a=0.6

Common difference=$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

4. Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16, . . .

Sol: $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(ii) $\mathbf{2, \frac{5}{2}, 3, \frac{7}{2} \ldots.}$

Sol: $a_1 = 2, a_2 = \frac{5}{2} a_3 = 3, a_4 = \frac{7}{2}$

$$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$$

$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$

$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{3} = \frac{1}{2}$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \frac{1}{2}$

The next three terms are:$\frac{7}{2} + \frac{1}{2} = \frac{8}{2}, \frac{8}{2} + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = \frac{10}{2} \Rightarrow 4, \frac{9}{2}, 5$

(iii) $\mathbf{-1.2, -3.2, -5.2, -7.2, \ldots}$

Sol: $a_1 = -1.2, a_2 = -3.2, a_3 = -5.2, a_4 = -7.2$

$$a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$$

$$a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$$

$$a_4 - a_3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -2$

The next three terms are:$-7.2 - 2 = -9.2, -9.2 - 2 = -11.2, -11.2 - 2 = -13.2$

$\Rightarrow -9.2, -11.2, -13.2$

(iv) $\mathbf{-10, -6, -2, 2, \ldots}$

Sol: $a_1 = -10, a_2 = -6, a_3 = -2, a_4 = 2$

$$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$$

$$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$$

$$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 4$

The next three terms are: 2+4=6,6+4=10,10+4=14

$\Rightarrow$ 6,10,14

(v) $\mathbf{3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots \ldots}$

Sol: $a_1 = 3, a_2 = 3 + \sqrt{2}, a_3 = 3 + 2\sqrt{2}, a_4 = 3 + 3\sqrt{2}$

$$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$$

$a_3 - a_2 = 3 + 2\sqrt{2} - (3 + \sqrt{2}) = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 3 + 3\sqrt{2} - (3 + 2\sqrt{2}) = 3 + 3\sqrt{2} - 3 + 2\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are:−

$(3 + 3\sqrt{2}) + \sqrt{2} = 3 + 4\sqrt{2}; (3 + 4\sqrt{2}) + \sqrt{2} = 3 + 5\sqrt{2}; (3 + 5\sqrt{2}) + \sqrt{2} = 3 + 6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222, . . .

Sol: $a_1 = 0.2, a_2 = 0.22, a_3 = 0.222, a_4 = 0.2222$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(vii) $0, -4, -8, -12, \ldots.$

Sol: $a_1 = 0, a_2 = -4, a_3 = -8, a_4 = -12$

$a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = -4$

The next three terms are:−

$-12 - 4 = -16; \quad -16 - 4 = -20; \quad -20 - 4 = -24$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots.$

Sol: $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 0$

The next three terms are: $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$

(ix) 1,3,9,27,…….

Sol: $a_2 - a_1 = 3 - 1 = 2$

$a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(x) $\boldsymbol{a, 2a, 3a, 4a, \ldots \ldots}$

Sol: $a_2 - a_1 = 2a - a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_4 - a_3 = 4a - 3a = a$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = a$

The next three terms are: $5a, 6a, 7a$

(xi) $\boldsymbol{a, a^2, a^3, a^4, \ldots}$

Sol: $a_2 - a_1 = a^2 - a = a(a-1)$

$a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xii) $\boldsymbol{\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots \ldots}$

Sol: $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots \ldots$

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$a_2 - a_1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$

The next three terms are: $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

$\Rightarrow \sqrt{50}, \sqrt{72}, \sqrt{98}$

(xiii) $\boldsymbol{\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots \ldots}$

Sol: $a_2 - a_1 = \sqrt{6} - \sqrt{3}$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xiv) $\boldsymbol{1^2, 3^2, 5^2, 7^2, \ldots}$

Sol: $a_2 - a_1 = 3^2 - 1^2 = 9 - 1 = 8$

$a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

$a_2 - a_1 \neq a_3 - a_2$

So, the given list of numbers does not form an AP

(xv) $1^2, 5^2, 7^2, 73, \ldots$

Sol: $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$

$a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$

$a_3 - a_2 = 73 - 7^2 = 73 - 49 = 24$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 24$

The next three terms are: 73+24, 97+24,121+24

$\Rightarrow$ 97,121,145

n^{th} Term of an AP(general term of the AP)

The nth term a_n of the AP with first term a and common difference d is given by

$$a_n = a + (n - 1)d.$$

If there are **m** terms in the AP, then **m** represents the last term which is sometimes also denoted by l.

n^{th} Term of an AP from the end

If d be the common difference and l be the last term of an AP, then n^{th} term from the end $= l - (n - 1)d$

Example 3 : Find the 10^{th} term of the AP : 2, 7, 12, . . .

Sol: Given AP is 2, 7, 12,. . .

$a = 2;\ d = a_2 - a_1 = 7 - 2 = 5$

The 10th term $= a_{10} = a + 9d$

$= 2 + 9 \times (5)$

$= 2 + 45 = 47$

Example 4 : Which term of the AP : 21, 18, 15, . . . is – 81? Also, is any term 0? Give reason for your answer.

Sol: First term=a=21

Common difference=$d = a_2 - a_1 = 18 - 21 = -3$

$Let\ a_n = -81$

$\Rightarrow a + (n - 1)d = -81$

$\Rightarrow 21 + (n - 1) \times (-3) = -81$

$\Rightarrow (n-1) \times (-3) = -81 - 21 = -102$

$\Rightarrow n - 1 = \dfrac{-102}{-3} = 34$

$\Rightarrow n = 34 + 1 = 35$

$\therefore$ -81 is the 35^{th} term of the given AP.

$Let\ a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 21 + (n-1) \times (-3) = 0$

$\Rightarrow (n-1) \times (-3) = -21$

$\Rightarrow n - 1 = \dfrac{-21}{-3} = 7$

$\Rightarrow n = 7 + 1 = 8$

$\therefore$ The 8^{th} term of the given AP is 0.

Example 5 : Determine the AP whose 3rd term is 5 and the 7th term is 9.

Sol: 3^{rd} term of AP=5$\Rightarrow a + 2d = 5 \rightarrow (1)$

7^{th} term of AP=9 $\Rightarrow a + 6d = 9 \rightarrow (2)$

$(2) - (1) \Rightarrow\ a + 6d = 9$

$$\begin{array}{r} a + 2d = 5 \\ \underline{(-)\ (-)\ \ (-)} \\ \underline{4d = 4} \end{array}$$

$d = 1$

Substitute d=1 value in (1)

$a + 2 \times 1 = 5$

$a = 5 - 2$

$a = 3$

Hence, the required AP is 3,4,5,6,......

Example 6 : Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .

Sol: Given list of numbers 5, 11, 17, 23, ...

$a_2 - a_1 = 11 - 5 = 6$

$a_3 - a_2 = 17 - 11 = 6$

$a_4 - a_3 = 23 - 17 = 6$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP with the common difference $d = 6$ and $a = 5$

$Let\ a_n = 301$

$\Rightarrow a + (n-1)d = 301$

$\Rightarrow 5 + (n-1) \times (6) = 301$

$\Rightarrow (n-1) \times (6) = 301 - 5 = 296$

$\Rightarrow n - 1 = \frac{296}{6} = \frac{148}{3}$

$\Rightarrow n = \frac{148}{3} + 1 = \frac{151}{3}$ it is not a positive integer

So, 301 is not a term of the given list of numbers.

Example 7 : How many two-digit numbers are divisible by 3?

Sol: The list of two-digit numbers divisible by 3 is : 12, 15, 18, . . . , 99

Clearly it is an AP. $a = 12$ and $d = 15 - 12 = 3$

$Let\ a_n = 99 \Rightarrow a + (n-1)d = 99$

$12 + (n-1) \times 3 = 99$

$(n-1) \times 3 = 99 - 12 = 87$

$n - 1 = \frac{87}{3} = 29$

$n = 29 + 1 = 30$

So, there are 30 two-digit numbers divisible by 3.

Example 8 : Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Sol: Given AP is 10,7,4,...

$a = 10, d = 7 - 10 = -3$

$Let\ a_n = -62 \Rightarrow a + (n-1)d = -62$

$10 + (n-1) \times (-3) = -62$

$(n-1) \times (-3) = -62 - 10 = -72$

$n - 1 = \frac{-72}{-3} = 24$

$n = 24 + 1 = 25$

So, there are 25 terms in the given AP.

The 11^{th} term from the last $= (25 - 10)^{th}\ term$

$= 15^{th}\ term = a + 14d$

$= 10 + 14 \times (-3)$

$= 10 - 42 = -32$

The 11^{th} term from the last of the AP is -32.

Alternative Solution 1:

If we write the given AP in the reverse order then

$a = -62 \text{ and } d = 3$

$11\,th\ term = a + 10d = -62 + 10 \times 3 = -62 + 30 = -32$

Alternative Solution 2:

$l = -62\,; d = -3$

n^{th} term from the last of the AP series$= l - (n-1)d$

$11^{th}\ term\ from\ the\ last\ of\ the\ AP\ series = l - 10d$

$= -62 - 10(-3) = -62 + 30 = -32$

Example 9 : A sum of ₹ 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.

Sol: simple interest$(I) = \dfrac{P \times T \times R}{100}$

Here P=1000, R= 8%

The interest at the end of 1^{st} year $= \dfrac{1000 \times 1 \times 8}{100} =$ ₹ 80

The interest at the end of 2^{nd} year $= \dfrac{1000 \times 2 \times 8}{100} =$ ₹ 160

The interest at the end of 3^{rd} year $= \dfrac{1000 \times 3 \times 8}{100} =$ ₹ 240

The interests are 80,160,240,.....

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = 80$

The interests form an AP with $a = 80, d = 80$

The interest at the end of 30 years$=a_{30} = a + 29d$

$= 80 + 29 \times 80 =$ ₹2400

Example 10 : In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

Sol: The number of rose plants in the 1^{st},2^{nd},3^{rd},....rows are

23, 21, 19,....., 5 Clearly it is an AP

$a = 23, d = 21 - 23 = -2$

$Let\ a_n = 5 \Rightarrow a + (n-1)d = 5$

$23 + (n-1) \times (-2) = 5$

$(n-1) \times (-2) = 5 - 23 = -18$

$n - 1 = \dfrac{-18}{-2} = 9$

$n = 9 + 1 = 10$

So, there are 10 rows in the flower bed.

EXERCISE 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and an the nth term of the AP:

(i) $a = 7, d = 3, n = 8, a_n = ?$

Sol: $a_n = a + (n-1)d$

$= 7 + (8-1) \times 3$

$= 7 + 7 \times 3$

$= 7 + 21 = 28$

(ii) $a = -18, d = ?, n = 10, a_n = 0$

Sol: $a_n = 0$

$a + (n-1)d = 0$

$-18 + (10-1)d = 0$

$9d = 18$

$d = \frac{18}{9} = 2$

(iii) $a = ?, d = -3, n = 18, a_n = -5$

Sol: $a_n = -5$

$a + (n-1)d = -5$

$a + (18-1) \times (-3) = -5$

$a + 17 \times (-3) = -5$

$a - 51 = -5$

$a = -5 + 51 = 46$

(iv) $a = -18.9, d = 2.5, n = ?, a_n = 3.6$

Sol: $a_n = 3.6$

$a + (n-1)d = 3.6$

$-18.9 + (n-1) \times (2.5) = 3.6$

$(n-1) \times (2.5) = 3.6 + 18.9$

$n - 1 = \frac{22.5}{2.5} = 9$

$n = 9 + 1 = 10$

(v) $a = 3.5, d = 0, n = 105, a_n = ?$

Sol: $a_n = a + (n-1)d$

$= 3.5 + (105-1) \times 0 = 3.5$

2. Choose the correct choice in the following and justify :

(i) 30th term of the AP: 10, 7, 4, . . . , is

(A) 97 (B) 77 (C) −77 (D) − 87 [C]

Sol: Given A.P is 10,7,4,........

$a = 10, d = 7 - 10 = -3$

30^{th} term of the A.P $= a + 29d$

$= 10 + 29 \times (-3)$

$= 10 - 87 = -77$

(ii) 11th term of the AP: – 3, $\frac{-1}{2}$, 2 ..., is

(A) 28 (B) 22 (C) –38 (D) – 48 1/ 2 [B]

Sol: Given A.P is $-3, \frac{-1}{2}, 2, \ldots..$

$a = -3, \quad d = a_2 - a_1 = \frac{-1}{2} - (-3) = \frac{-1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$

11^{th} term of the A.P$= a + 10d$

$= -3 + 10 \times \left(\frac{5}{2}\right)$

$= -3 + 25 = 22$

3. In the following APs, find the missing terms in the boxes :

(i) 2, ☐ , 26

Sol: $a_1 = a = 2$

$a_3 = a + 2d = 26$

$\Rightarrow 2 + 2d = 26$

$\Rightarrow 2d = 26 - 2$

$\Rightarrow d = \frac{24}{2} = 12$

Now $a_2 = a + d = 2 + 12 = 14$

(ii) ☐ , 13, ☐ , 3

Sol: $a_2 = a + d = 13 \rightarrow (1)$

$a_4 = a + 3d = 3 \rightarrow (2)$

(2)-(1)$\Rightarrow$ $a + 3d = 3$

$a + d = 13$

$(-)\ (-)\ (-)$

$2d = -10$

$d = \frac{-10}{2} = -5$

Substitute $d = -5$ in (1)

$a - 5 = 13$

$a = 13 + 5 = 18$

Now $a_1 = a = 18$

$a_3 = a + 2d = 13 + 2(-5) = 18 - 10 = 8$

(iii) 5, ☐, ☐, $9\frac{1}{2}$

Sol: $a_1 = a = 5$

$a_4 = a + 3d = \frac{19}{2}$

$5 + 3d = \frac{19}{2}$

$3d = \frac{19}{2} - 5 = \frac{19-10}{2} = \frac{9}{2}$

$d = \frac{9}{2 \times 3} = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2 \times \frac{3}{2} = 5 + 3 = 8$

(iv) − 4, ☐, ☐, ☐, ☐, 6

Sol: $a_1 = -4 \Rightarrow a = -4$

$a_6 = 6 \Rightarrow a + 5d = 6$

$-4 + 5d = 6$

$5d = 6 + 4 = 10$

$d = \frac{10}{5} = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2 \times 2 = -4 + 4 = 0$

$a_4 = a + 3d = -4 + 3 \times 2 = -4 + 6 = 2$

$a_5 = a + 4d = -4 + 4 \times 2 = -4 + 8 = 4$

(v) ☐, 38, ☐, ☐, ☐, −22

Sol: $a_2 = 38 \Rightarrow a + d = 38 \rightarrow (1)$

$a_6 = -22 \Rightarrow a + 5d = -22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 5d = -22$

$$\begin{array}{r} a + d \;=\; 38 \\ (-)\;(-)\quad(-) \\ \hline 4d = -60 \\ \hline \end{array}$$

$d = \frac{-60}{4} = -15$

Substitute $d = -15$ in (1)

$a - 15 = 38$

$a = 38 + 15 = 53$

$a_1 = a = 53$

$a_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$a_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$

$a_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$

4. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?

Sol: given A.P: 3, 8, 13, 18, . .

$a = 3; d = 8 - 3 = 5$

$let\ a_n = 78$

$a + (n-1)d = 78$

$3 + (n-1) \times 5 = 78$

$(n-1) \times 5 = 78 - 3 = 75$

$n - 1 = \frac{75}{5} = 15$

$n = 15 + 1 = 16$

∴ 78 is the 16^{th} term of A.P

5. Find the number of terms in each of the following APs :
(i) 7, 13, 19, . . . , 205

Sol: $a = 7, d = 13 - 7 = 6$

$let\ a_n = 205$

$a + (n-1)d = 205$

$7 + (n-1) \times 6 = 205$

$(n-1) \times 6 = 205 - 7 = 198$

$n - 1 = \frac{198}{6} = 33$

$n = 33 + 1 = 34$

The number of terms in given A.P are 34.

(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Sol: $a = 18,$

$d = \frac{31}{2} - 18 = \frac{31 - 36}{2} = \frac{-5}{2}$

$let\ a_n = -47$

$a + (n-1)d = -47$

$18 + (n-1) \times \left(\frac{-5}{2}\right) = -47$

$(n-1) \times \left(\frac{-5}{2}\right) = -47 - 18$

$(n-1) \times \left(\frac{-5}{2}\right) = -65$

$n - 1 = -65 \times \frac{-2}{5} = 26$

$n = 26 + 1 = 27$

The number of terms in given A.P are 27.

6. Check whether – 150 is a term of the AP : 11, 8, 5, 2 . . .

Sol: $a = 11, d = 8 - 11 = -3$

$let\ a_n = -150$

$a + (n-1)d = -150$

$11 + (n-1) \times (-3) = -150$

$(n-1) \times (-3) = -150 - 11 = -161$

$n - 1 = \frac{-161}{-3} = \frac{161}{3}$ *it is not a natural number*

$\therefore -150$ is not a term of given AP

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol: 11^{th} term is 38 $\Rightarrow a + 10d = 38 \rightarrow (1)$

16^{th} term is 73 $\Rightarrow a + 15d = 73 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 15d = 73$

$$\begin{array}{r} a + 10d = 38 \\ (-)\ (-)\quad (-) \\ \hline 5d = 35 \\ \hline \end{array}$$

$\Rightarrow d = \frac{35}{5} = 7$

Substitute d=7 in (1)

$a + 10 \times 7 = 38$

$a = 38 - 70 = -32$

31^{st} term $= a + 30d$

$= -32 + 30 \times 7$

$= -32 + 210$

$= 178$

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol: In AP 3rd term = 12

$a + 2d = 12 \rightarrow (1)$

Last term=50^{th} term=106

$a + 49d = 106 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 49d - a - 2d = 106 - 12$

$47d = 94 \Rightarrow d = 2$

Substitute d=1 in (1) we get

$a + 2 \times 2 = 12$

$a = 12 - 4 = 8$

$a_{29} = a + 28d = 8 + 28 \times 2 = 8 + 56 = 64$

9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?

Sol: 3^{rd} term of an A.P=4$\Rightarrow a + 2d = 4 \rightarrow (1)$

9^{th} term of an A.P$= -8 \Rightarrow a + 8d = -8 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 8d = -8$

$$\begin{array}{r} a + 2d = \ \ 4 \\ (-)\ (-)\quad (-) \\ \hline 6d = -12 \\ \hline \end{array}$$

$$d = \frac{-12}{6} = -2$$

Substitute d=-2 in (1) we get

$a + 2 \times (-2) = 4$

$a - 4 = 4$

$a = 4 + 4 = 8$

$let\ a_n = 0$

$a + (n-1)d = 0$

$8 + (n-1) \times (-2) = 0$

$(n-1) \times (-2) = 0 - 8$

$$n - 1 = \frac{-8}{-2} = 4$$

$n = 4 + 1 = 5$

∴ The 5^{th} term of A.P is '0'

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol: 17^{th} term of an AP=10^{th} term+7

$a + 16d = a + 9d + 7$

$a + 16d - a - 9d = 7$

$7d = 7 \Rightarrow d = 1$

The common difference=1

11. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?

Sol: $a = 3$; $d = 15 - 3 = 12$

$Let\ a_n = a_{54} + 132$

$a + (n-1)d = a + 53d + 132$

$(n-1)d = 53d + 132$

$(n-1) \times 12 = 53 \times 12 + 132$

$(n-1) \times 12 = 768$

$$n - 1 = \frac{768}{12} = 64$$

$n = 65$

Therefore, 65^{th} term will be 132 more than 54^{th} term.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol: Let the first A.P is $a, a+d, a+2d, a+3d, \ldots\ldots$

The second A.P is $b, b+d, b+2d, b+3d, \ldots \ldots$

The difference between their 100^{th} terms = 100

$a_{100} - b_{100} = 100$

$(a + 99d) - (b + 99d) = 100$

$a + 99d - b - 99d = 100$

$a - b = 100 \rightarrow (1)$

The difference between their 1000th terms $= a_{1000} - b_{1000}$

$= (a + 999d) - (b + 999d)$

$= a + 999d - b - 999d$

$= a - b$

$= 100 \quad (from\ (1))$

The difference between their 1000^{th} terms = 100.

13. How many three-digit numbers are divisible by 7?

Sol: The three-digit numbers are divisible by 7 are

105,112, 119,……., 994

$a = 105, d = 7$

$let\ a_n = 994$

$a + (n-1)d = 994$

$105 + (n-1) \times 7 = 994$

$(n-1) \times 7 = 994 - 105 = 889$

$n - 1 = \frac{889}{7} = 127$

$n = 127 + 1 = 128$

∴ 128 three digit numbers are divisible by 7

14. How many multiples of 4 lie between 10 and 250?

Sol: Multiples of 4 lie between 10 and 250 are

12,16,20,.......,248

$a = 12, d = 4$

$let\ a_n = 248$

$a + (n-1)d = 248$

$12 + (n-1) \times 4 = 248$

$(n-1) \times 4 = 248 - 12 = 236$

$n - 1 = \frac{236}{4} = 59$

$n = 59 + 1 = 60$

∴ 60 multiples of 4 lie between 10 and 250.

15. For what value of n, are the nth terms of two APs: 63, 65, 67, . . . and 3, 10, 17, . . . equal?

Sol: First A.P : 63,65,67,....

$a = 63, d = 2$

$a_n = a + (n-1)d$

$= 63 + (n-1) \times 2$

$= 63 + 2n - 2$

$= 2n + 61$

Second A.P: 3,10,17,.....

$a = 3, d = 7$

$a_n = a + (n-1)d$

$= 3 + (n-1) \times 7$

$= 3 + 7n - 7 = 7n - 4$

If n^{th} terms of two A.Ps are equal then

$7n - 4 = 2n + 61$

$7n - 2n = 61 + 4$

$5n = 65$

$n = \frac{65}{5} = 13$

∴ 13^{th} terms of the two A.Ps are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol: Third term of AP=16⇒ $a + 2d = 16 \rightarrow (1)$

7^{th} term=5^{th} term + 12

$a + 6d = a + 4d + 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Substitute $d = 6$ in (1) we get

$a + 2 \times 6 = 16$

$a = 16 - 12 = 4$

The required AP is $a, a + d, a + 2d, a + 3d, \ldots.$

⇒ 4,10,16,22, … ….

17. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.

Sol: $a = 3, d = 8 - 3 = 5$

$let \quad a_n = l = 253$

$a + (n - 1)d = 253$

$3 + (n - 1) \times 5 = 253$

$(n - 1) \times 5 = 253 - 3 = 250$

$n - 1 = \frac{250}{5} = 50$

$n = 50 + 1 = 51$

The 20^{th} term from the end of the AP= (51-20)+1=32^{th} term from first

$= a + 31d = 3 + 31 \times 5 = 3 + 155$=158

(OR)

$a = 3, \qquad d = 8 - 3 = 5$

$a_n = l = 253$

n^{th}term from the end of the AP $= l - (n - 1)d$

20^{th}term from the end of the AP $= 253 - 19 \times 5 = 253 - 95 = 158$

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Sol: 4^{th} term +8^{th} term of an AP =24

$\Rightarrow a + 3d + a + 7d = 24$

$\Rightarrow 2a + 10d = 24$

$\Rightarrow a + 5d = 12 \rightarrow (1)$

6th term +10th term of an AP =44

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 14d = 44$

$\Rightarrow a + 7d = 22 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 7d = 22$

$$\begin{array}{r} a + 5d = 12 \\ (-)\ (-)\quad (-) \\ \hline 2d = 10 \\ \hline \end{array}$$

$d = 5$

Substitute d=5 in (1) we get

$a + 5 \times 5 = 12$

$a = 12 - 25$

$a = -13$

$\therefore$ The first three terms of AP are $a, a + d, a + 2d$

$\Rightarrow -13, -13 + 5, -13 + 10$

$\Rightarrow -13, -8, -3$

19. Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?

Sol: subbarao salary in 1995=₹5000, Increment =₹200

Salary in 1996=5000+200=₹5200

Salary in 1997=5200+200=₹5400

Salary in 1998=5400+200=₹5600

The salaries are ₹5000, ₹5200, ₹5400, ₹5600,…….forms an AP

$a = 5000, d = 200$

Let $a_n = 7000$

$a + (n - 1) \times 200 = 7000$

$5000 + (n - 1) \times 200 = 7000$

$(n - 1) \times 200 = 7000 - 5000 = 2000$

$n - 1 = \frac{2000}{200} = 10$

$n = 10 + 1$

$n = 11$

$\therefore$ In 11th year subbarao income reached 7000

20. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.

Sol: Ramkali savings:

First week=₹ 5

Second week=₹ 5+₹ 1.75=₹ 6.75

Third week=₹ 6.75+₹ 1.75=₹ 8.50

Fourth week=₹ 8.50+₹ 1.75=₹ 10.25

.....

Ramkali's savings in the consecutive weeks are ₹ 5, ₹ 6.75, ₹ 8.50, ₹ 10.25,...

These are in AP with a=5 and d=1.75

The nth week savings = ₹ 20.75

$a + (n-1)d = 20.75$

$5 + (n-1) \times 1.75 = 20.75$

$(n-1) \times 1.75 = 20.75 - 5$

$(n-1) \times 1.75 = 15.75$

$n - 1 = \frac{15.75}{1.75} = \frac{1575}{175} = 9$

$n = 9 + 1$

$n = 10$

5.4 Sum of First n Terms of an AP

Carl Fredrich Gauss (1777-1855) is a great German Mathematician

i. If first term of an AP is $\boldsymbol{a}$ and common difference is $\boldsymbol{d}$ then

Sum of first n terms $= S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + a_n)$

ii. If first term is $\boldsymbol{a}$, last term is $\boldsymbol{l}$ and number of terms is $\boldsymbol{n}$ then

$S_n = \frac{n}{2}(a + l)$

iii. $a_n = S_n - S_{n-1}$

Example 11 : Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

$S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_{22} = \frac{22}{2}[2 \times 8 + (22-1)(-5)] = 11[16-105] = 11 \times (-89) = -979$$

Example-12. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term

Sol: $a = 10, n = 14,$

$S_{14} = 1050$

$\frac{n}{2}[2a + (n-1)d] = 1050$

$\frac{14}{2}[2 \times 10 + (14-1)d] = 1050$

$7[20 + 13d] = 1050$

$20 + 13d = \frac{1050}{7} = 150$

$13d = 150 - 20$

$13d = 130$

$d = 10$

20th term$= a + 19d$

$= 10 + 19 \times 10 = 10 + 190 = 200$

Example-13. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

Sol: $a = 24, d = 21 - 24 = -3$

$Let\ S_n = 78$

$\frac{n}{2}[2a + (n-1)d] = 78$

$n[2 \times 24 + (n-1)(-3)] = 2 \times 78$

$n[48 - 3n + 3] = 156$

$n[-3n + 51] = 156$

$-3n^2 + 51n - 156 = 0$

$3n^2 - 51n + 156 = 0$

$n^2 - 17n + 52 = 0$

$(n-4)(n-13) = 0$

$n - 4 = 0 \quad or \quad n - 13 = 0$

$n = 4\ or\ 13$

So, the number of terms is either 4 or 13.

Remark: Two answers are possible because the sum of the terms from 5th to 13th will be zero.

Example-14. (i) Find the sum of the first 1000 positive integers.

Sol: The first 1000 positive integers are 1,2,3,4,.....,1000

$a = 1, d = 1, n = 1000$

$S_n = \frac{n}{2}[a + l]$

$$S_{1000} = \frac{1000}{2}[1 + 1000] = 500 \times 1001 = 500500$$

(ii) Find the sum of the first n positive integers

Sol: $a = 1, d = 1, n = n$

$$S_n = \frac{n}{2}[a + l] = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

$$The\ sum\ of\ the\ first\ n\ positive\ integers = \frac{n(n+1)}{2}$$

Example-15. Find the sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$

Sol: $a_n = 3 + 2n$

$a_1 = 3 + 2 \times 1 = 3 + 2 = 5$

$a_2 = 3 + 2 \times 2 = 3 + 4 = 7$

$a_3 = 3 + 2 \times 3 = 3 + 6 = 9$

List of numbers are 5,7,9,...... clearly it is an AP

$a = 5, d = 7 - 5 = 2, n = 24$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{24} = \frac{24}{2}[10 + (24 - 1) \times 2]$$

$= 12[10 + 23 \times 2]$

$= 12 \times 56$

$= 672$

Example-16. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years.

Sol: $a_3 = 600, a_7 = 700$

$a_7 = 700 \Rightarrow a + 6d = 700 \rightarrow (1)$

$a_3 = 600 \Rightarrow a + 2d = 600 \rightarrow (2)$

$4d = 100$

$$d = \frac{100}{4} = 25$$

Substitute d=25 in (2)

$a + 2 \times 25 = 600$

$a + 50 = 600$

$a = 600 - 50 = 550$

(i) The production in the 1st year=550

(ii) The production in the 10th year$=a+9d$

$= 550 + 9 \times 25$

$= 550 + 225$

$= 775$

(iii) The total production in first 7 years$=S_7$

$= \frac{7}{2}[2 \times 550 + (7-1) \times 25]$

$= \frac{7}{2}[1100 + 6 \times 25]$

$= \frac{7}{2}[1250] = 7 \times 625 = 4375$

$S_n = \frac{n}{2}[2a + (n-1)d]$

EXERCISE 5.3

1. Find the sum of the following APs:

(i) 2, 7, 12, . . ., to 10 terms.

Sol: $a = 2, d = 7 - 2 = 5, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$= 5[4 + 45]$

$= 5 \times 49$

$= 245$

(ii) –37, –33, –29, . . ., to 12 terms.

Sol: $a = -37, d = -33 + 37 = 4, n = 12$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{12} = \frac{12}{2}[2 \times (-37) + (12-1) \times 4]$

$= 6[-74 + 44]$

$= 6 \times (-30)$

$= -180$

(iii) 0.6, 1.7, 2.8, . . ., to 100 terms

Sol: $a = 0.6, d = 1.7 - 0.6 = 1.1, n = 100$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2 \times 0.6 + (100-1) \times 1.1]$

$= 50[1.2 + 99 \times 1.1]$

$= 50[1.2 + 108.9]$

$= 50 \times 110.1$

$= 5505$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots, \text{to } 11 \text{ terms}$

Sol: $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}, n = 11$

$S_n = \frac{n}{2}[2n + (n-1)d]$

$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\left(\frac{1}{60}\right)\right]$

$= \frac{11}{2}\left[\frac{2}{15} + 10 \times \frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$= \frac{11}{2}\left[\frac{4+5}{30}\right]$

$= \frac{11}{2} \times \frac{9}{30} = \frac{11}{2} \times \frac{3}{10}$

$= \frac{33}{20} = 1\frac{13}{20}$

2.Find the sums given below

(i) $7 + 10\frac{1}{2} + 14 + \cdots + 84$

Sol: $a = 7, d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}, l = 84$

$l = a_n = 84$

$a + (n-1)d = 84$

$7 + (n-1)\left(\frac{7}{2}\right) = 84$

$(n-1)\left(\frac{7}{2}\right) = 84 - 7$

$n - 1 = 77 \times \frac{2}{7} = 22$

$n = 22 + 1 = 23$

$S_n = \frac{n}{2}(a + l)$

$S_{23} = \frac{23}{2}(7 + 84)$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2} = 1046\frac{1}{2}$

(ii) 34 + 32 + 30 + . . . + 10

Sol: $a = 34, d = 32 - 34 = -2$

$l = a_n = a + (n-1)d = 10$

$34 + (n-1)(-2) = 10$

$(n-1)(-2) = 10 - 34$

$(n-1)(-2) = -24$

$n - 1 = \frac{-24}{-2} = 12$

$n = 12 + 1 = 13$

$S_n = \frac{n}{2}(a + l)$

$S_{13} = \frac{13}{2}(34 + 10)$

$= \frac{13}{2} \times 44$

$= 13 \times 22$

$= 286$

(iii) –5 + (–8) + (–11) + . . . + (–230)

Sol: $a = -5, d = -8 + 5 = -3$

$l = a_n = a + (n-1)d = -230$

$-5 + (n-1)(-3) = -230$

$(n-1)(-3) = -230 + 5$

$(n-1)(-3) = -225$

$n - 1 = \frac{-225}{-3} = 75$

$n = 75 + 1 = 76$

$S_n = \frac{n}{2}(a + l)$

$S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38 \times (-235)$

$= -8930$

3. **In an AP:**

(i) Given $a = 5, d = 3, a_n = 50$, find n and S_n.

Sol: $a_n = 50$

$a + (n-1)d = 50$

$5 + (n-1) \times 3 = 50$

$(n-1) \times 3 = 50 - 5$

$(n-1)\times 3 = 45$

$n-1 = \frac{45}{3} = 15$

$n = 15+1$

$n = 16$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{16} = \frac{16}{2}[2\times 5+(16-1)\times 3]$

$= 8[10+15\times 3]$

$= 8[10+45]$

$= 8\times 55 = 440$

(ii) ***Given*** $a = 7, a_{13} = 35$, ***find*** d ***and*** S_{13}.

Sol: $a_{13} = 35$

$a+12d = 35$

$7+12d = 35$

$12d = 35-7$

$12d = 28$

$d = \frac{28}{12} = \frac{7}{3}$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{13} = \frac{13}{2}\left[2\times 7+(13-1)\times\frac{7}{3}\right]$

$= \frac{13}{2}\left[14+12\times\frac{7}{3}\right]$

$= \frac{13}{2}[14+28]$

$= \frac{13}{2}\times 42 = 13\times 21 = 273$

(iii) ***Given*** $a_{12} = 37, d = 3$, ***find*** a ***and*** S_{12}.

Sol: $a_{12} = 37$

$a+11d = 37$

$a+11\times 3 = 37$

$a+33 = 37$

$a = 37-33 = 4$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{12} = \frac{12}{2}[2\times 4+(12-1)\times 3]$

$= 6[8 + 33]$

$= 6 \times 41 = 246$

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}

Sol: $a_3 = 15$

$a + 2d = 15 \Rightarrow a = 15 - 2d \rightarrow (1)$

$S_{10} = 125$

$\frac{10}{2}[2a + (10 - 1)d] = 125$

$[2(15 - 2d) + 9d] = \frac{125}{5}$

$30 - 4d + 9d = 25$

$5d = 25 - 30$

$d = \frac{-5}{5} = -1$

Substitute $d = -1$ in (1)

$a = 15 - 2 \times (-1) = 15 + 2 = 17$

$a_n = a + 9d$

$= 17 + 9 \times (-1)$

$= 17 - 9 = 8$

(v) given $d = 5, S_9 = 75$, find a and a_9.

Sol: $S_9 = 75$

$\frac{9}{2}[2a + (9 - 1) \times 5] = 75$

$\frac{9}{2}[2a + (9 - 1) \times 5] = 75$

$\frac{9}{2}[2a + 40] = 75$

$18a + 360 = 150$

$18a = 150 - 360$

$18a = -210$

$a = \frac{-210}{18} = \frac{-35}{3}$

$a_9 = a + 8d = \frac{-35}{3} + 8 \times 5 = \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$

(vi) Given $a = 2, d = 8, S_n = 90$, find n and a_n.

Sol: $S_n = 90$

$\frac{n}{2}[2a + (n - 1)d] = 90$

$\frac{n}{2}[2 \times 2 + (n-1) \times 8] = 90$

$n[4 + 8n - 8] = 90 \times 2$

$4n + 8n^2 - 8n - 180 = 0$

$8n^2 - 4n - 180 = 0$

$2n^2 - n - 45 = 0$

$2n^2 - 10n + 9n - 45 = 0$

$2n(n-5) + 9(n-5) = 0$

$(n-5)(2n+9) = 0$

$n - 5 = 0 \; or \; 2n + 9 = 0$

$n = 5 \; or \; n = \frac{-9}{2}$

$\therefore n = 5 \; (\, n \; is \; a \; natural \; number)$

$a_n = a_5 = a + 4d$

$= 2 + 4 \times 8 = 2 + 32 = 34$

(vii) Given a $= 8, a_n = 62, S_n = 210$, find n and d.

Sol: $S_n = 210$

$\frac{n}{2}(a + a_n) = 210$

$\frac{n}{2}(8 + 62) = 210$

$n = \frac{210 \times 2}{70} = 6$

$a_n = 62$

$a + (n-1)d = 62$

$8 + (6-1)d = 62$

$5d = 62 - 8 = 54$

$d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Sol: $a_n = 4$

$a + (n-1)d = 4$

$a + (n-1) \times 2 = 4$

$a + 2n - 2 = 4$

$a = 4 - 2n + 2$

$a = 6 - 2n \rightarrow (1)$

$S_n = -14$

$\frac{n}{2}[a + a_n] = -14$

$n[6 - 2n + 4] = -14 \times 2$

$n[10 - 2n] = -28$

$10n - 2n^2 + 28 = 0$

$-2n^2 + 10n + 28 = 0$

$n^2 - 5n - 14 = 0$

$(n - 7)(n + 2) = 0$

$n - 7 = 0 \ or \ n + 2 = 0$

$n = 7 \ or \ n = -2$

$\therefore n = 7$ (n is a natural number)

From (1)

$a = 6 - 2 \times 7 = 6 - 14 = -8$

(ix) given a = 3, n = 8, S = 192, find d

Sol: $S = 192$

$\frac{8}{2}[2 \times 3 + (8 - 1)d] = 192$

$4[6 + 7d] = 192$

$24 + 28d = 192$

$28d = 192 - 24$

$28d = 168$

$d = \frac{168}{28} = 6$

(x) Given $l = 28, S = 144$, and there are total 9 terms. Find a.

Sol: $l = a_n = 28, S = 144, n = 9$

$S = 144$

$\frac{n}{2}[a + l] = 144$

$\frac{9}{2}[a + 28] = 144$

$a + 28 = \frac{144 \times 2}{9}$

$a = 32 - 28 = 4$

4. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

Sol: $a = 9; d = 17 - 9 = 8$

$S_n = 636$

$\frac{n}{2}[2a + (n - 1)d] = 636$

$\frac{n}{2}[2 \times 9 + (n-1) \times 8] = 636$

$n[18 + 8n - 8] = 636 \times 2$

$18n + 8n^2 - 8n - 1272 = 0$

$8n^2 + 10n - 1272 = 0$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n + 53) - 12(4n + 53) = 0$

$(4n + 53)(n - 12) = 0$

$n = \frac{-53}{4} \text{ or } n = 12$

$n = 12 (\, n \text{ is natural number})$

5. **The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

Sol: $a = 5; l = a_n = 45;\ S_n = 400$

$S_n = 400$

$\frac{n}{2}(a + l) = 400$

$\frac{n}{2}(5 + 45) = 400$

$n = \frac{400 \times 2}{50} = 16$

$a_n = 45$

$a + (n-1)d = 45$

$5 + (16 - 1) \times d = 45$

$15d = 45 - 5$

$d = \frac{40}{15} = \frac{8}{3}$

6. **The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?**

Sol: $a = 17\ , d = 9 \text{ and } l = a_n = 350$

$a_n = 350$

$a + (n-1)d = 350$

$17 + (n-1) \times 9 = 350$

$(n-1)\times 9 = 350 - 17$

$n - 1 = \frac{333}{9} = 37$

$n = 37 + 1 = 38$

$S_n = \frac{n}{2}(a + l)$

$= \frac{38}{2}(17 + 350)$

$= 19 \times 367 = 6973$

There are 38 terms and their sum is 6973.

Given A.P. contains 38 terms and the sum of the terms is 6973.

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Sol: $a_{22} = 149$

$a + 21d = 149$

$a + 21 \times 7 = 149$

$a + 147 = 149$

$a = 2$

$S_n = \frac{n}{2}[a + l]$

$S_{22} = \frac{22}{2}[2 + 149] = 11 \times 151 = 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol: $a_2 = 14 \Rightarrow a + d = 14 \rightarrow (1)$

$a_3 = 18 \Rightarrow a + 2d = 18 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 2d = 18$

$a + d = 14$

$(-)\ (-)\ \ (-)$

$d = 4$

Substitute d=4 in (1)

$a + 4 = 14$

$a = 14 - 4 = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times 4]$

$= \frac{51}{2}[20 + 50 \times 4]$

$= \frac{51}{2} \times 220$

$= 51 \times 110$

$= 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol: $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = 49 \Rightarrow \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow [2a + 6d] = \frac{2 \times 49}{7}$

$\Rightarrow 2a + 6d = 14$

$\Rightarrow a + 3d = 7 \rightarrow (1)$

$S_{17} = 289 \Rightarrow \frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow [2a + 16d] = \frac{2 \times 289}{17}$

$\Rightarrow 2a + 16d = 34$

$\Rightarrow a + 8d = 17 \rightarrow (2)$

$(2) - (1) \Rightarrow a + 8d = 17$

$a + 3d = 7$

$(-)\ (-)\quad (-)$

$5d = 10$

$d = 2$

Substitute d=2 in (1)

$a + 3 \times 2 = 7 \Rightarrow a + 6 = 7 \Rightarrow a = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2} \times 2n = n^2$

10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below. Also find the sum of the first 15 terms in each case

(i) $a_n = 3 + 4n$

Sol: $a_n = 3 + 4n$

$a_1 = 3 + 4 \times 1 = 3 + 4 = 7$

$a_2 = 3 + 4 \times 2 = 3 + 8 = 11$

$a_3 = 3 + 4 \times 3 = 3 + 12 = 15$

$a_4 = 3 + 4 \times 4 = 3 + 16 = 19$

The list of terms are 7,11, 15,19,.....

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 7, d = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 7 + (15-1) \times 4]$

$= \frac{15}{2}[14 + 56]$

$= \frac{15}{2} \times 70 = 15 \times 35 = 525$

(ii) $a_n = 9 - 5n$

Sol: $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

The list of terms is $4, -1, -6, -11, \ldots \ldots$

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 + 1 = -5$

$a_4 - a_3 = -11 + 6 = -5$

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots$

i.e., $a_{k+1} - a_k$ is same every time

So, the given list of numbers forms an AP. $a = 4, d = -5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 4 + (15-1) \times (-5)]$

$= \frac{15}{2}[8 - 70]$

$= \frac{15}{2} \times (-62) = 15 \times (-31) = -465$

11. **If the sum of the first n terms of an AP is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms**

Sol: $S_n = 4n - n^2$

$S_1 = 4 \times 1 - 1^2 = 4 - 1 = 3$

$S_2 = 4 \times 2 - 2^2 = 8 - 4 = 4$

$S_3 = 4 \times 3 - 3^2 = 12 - 9 = 3$

$S_4 = 4 \times 4 - 4^2 = 16 - 16 = 0$

$a_1 = S_1 = 3$

$a_2 = S_2 - S_1 = 4 - 3 = 1$

$a_3 = S_3 - S_2 = 3 - 4 = -1$

$\therefore a = 3, d = a_2 - a_1 = 1 - 3 = -2$

$a_{10} = a + 9d = 3 + 9 \times (-2) = 3 - 18 = -15$

$a_n = a + (n-1)d = 3 + (n-1) \times (-2) = 3 - 2n + 2 = 5 - 2n$

12. **Find the sum of the first 40 positive integers divisible by 6.**

Sol: The first 40 positive integers divisible by 6 are

$6 \times 1, 6 \times 2, 6 \times 3, \dots\dots\dots, 6 \times 40$

$\Rightarrow 6, 12, 18, \dots\dots, 240$

$a = 6, d = 6, n = 40, l = 240$

$S_n = \frac{n}{2}[a + l]$

$S_{40} = \frac{40}{2}[6 + 240]$

$= 20 \times 246 = 4920$

Shortcut:

$S_{40} = 6 + 12 + 18 + \cdots .. + 240$

$= 6(1 + 2 + 3 + 4 + \cdots + 40)$

$= 6 \times \frac{40 \times 41}{2} = 3 \times 1640 = 4920$

13. **Find the sum of the first 15 multiples of 8.**

Sol: The multiples of 8 are 8, 16, 24, 32, ...

These numbers are in an A.P.

$a = 8 \quad , d = 8 \quad , n = 15$

The sum of first n terms $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)8]$

$= \frac{15}{2}[16 + 14 \times 8]$

$= \frac{15}{2}[16 + 112]$

$= \frac{15}{2} \times 128$

$= 15 \times 64 = 960$

14. Find the sum of the odd numbers between 0 and 50.

Sol: The odd numbers lying between 0 and 50 are 1, 3, 5, 7, 9 ... 49

These odd numbers are in an A.P.

$a = 1; \quad d = 2; \quad l = 49$

We know that nth term of AP, $a_n = l = a + (n - 1)d$

49 = 1 + (n - 1) 2

48 = 2(n - 1)

n - 1 = 24

n = 25

$S_n = \frac{n}{2}[a + l]$

$S_{25} = \frac{25}{2}(1 + 49)$

$= \frac{25}{2} \times 50$

$= 25 \times 25$

$= 625$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol: Penalty for 1st day = Rs. 200

Penalty for 2nd day = Rs. 250

Penalty for 3rd day = Rs. 300

These penalties are in A.P. a = 200, d = 50, n = 30

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{30} = \frac{30}{2}[2 \times 200 + (30 - 1) 50]$$

$$= 15[400 + 1450]$$

$$= 15 \times 1850$$

$$= 27750$$

Therefore, the contractor has to pay Rs. 27750 as a penalty.

16. **A sum of 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is 20 less than its preceding prize, find the value of each of the prizes.**

Sol: Let the prizes be $x, x-20, x-40, x-60, x-80, x-100, x-120$

$a = x, d = -20, l = x - 120$

$S_7 = 700$

$\frac{n}{2}(a + l) = 700$

$\frac{7}{2}[x + x - 120] = 700$

$2x - 120 = \frac{700 \times 2}{7} = 200$

$2x = 200 + 120 = 320$

$x = 160$

The prizes are ₹160, ₹140, ₹120, ₹100, ₹80, ₹60, ₹40.

17. **In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?**

Sol: Trees planted by each class are

$3 \times 1, 3 \times 2, 3 \times 3, \dots\dots\dots\dots, 3 \times 12$

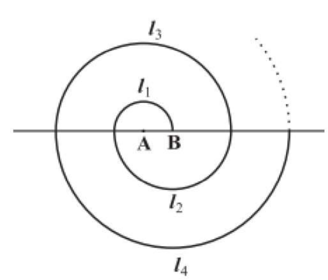
$\Rightarrow 3, 6, 9, \dots\dots\dots, 36$ it is an AP

$a = 3, d = 3, n = 12, l = 36$

$S_n = \frac{n}{2}[a + l]$

$S_{12} = \frac{12}{2}[3 + 36] = 6 \times 39 = 234$

Total plants=234



18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)

Sol: The radii are 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,……. these terms are in AP

$a = 0.5, d = 0.5, n = 13$

$l_1 = \pi \times r = \pi \times 0.5 = \pi \times \frac{1}{2} = \frac{\pi}{2}$

$l_2 = \pi \times 1 = \pi,\qquad l_3 = \pi \times 1.5 = \pi \times \frac{3}{2} = \frac{3\pi}{2}, \dots \dots$

Total length of spiral$= l_1 + l_2 + l_3 + \cdots . + l_{13}$

$= \frac{\pi}{2} + \pi + \frac{3\pi}{2} + \cdots \dots .13\ terms$

$= \frac{\pi}{2}[1 + 2 + 3 + \cdots 13\ terms]$

$$= \frac{\pi}{2}\left[\frac{13(13+1)}{2}\right] = \frac{\pi}{2}\left(\frac{13 \times 14}{2}\right) = \frac{\pi}{2} \times 91 = \frac{22}{7} \times \frac{1}{2} \times 91 = 11 \times 13 = 143\ cm$$

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?

Sol: The logs in rows are 20,19,18,……is an AP

$a = 20, d = -1$

$S_n = 200$

$\frac{n}{2}[2a + (n-1)d] = 200$

$\frac{n}{2}[2 \times 20 + (n-1) \times (-1)] = 200$

$n[40 - n + 1] = 200 \times 2$

$41n - n^2 - 400 = 0$

$-n^2 + 41n - 400 = 0$

$n^2 - 41n + 400 = 0$

$(n-16)(n-25) = 0$

$n - 16 = 0\ \ or\ \ n - 25 = 0$

$n = 16\ or\ n = 25$

$\therefore n = 16\ (\ n\ cannot\ be\ 25)$

$a_{16} = a + 15d = 20 + 15(-1) = 20 - 15 = 5$

The number of logs in the top row=5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Sol: The distance of first ball (from bucket)=5m

The distance of second ball=5+3=8m

The distance of third ball= 8+3=11 m

The distance of fourth ball= 11+3=14 m

……….

The distance of fourth ball= 11+3=14 m

The distance covered the competitor for 1st ,2nd ,3rd, …. Balls are

$2 \times 5m, 2 \times 8m, 2 \times 11m, \dots \dots (10\ terms)$

$10m, 16m, 22m, \dots \dots \dots .(10\ terms)\ \ clearly\ these\ terms\ are\ in\ AP$

$a = 10, d = 6, n = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2 \times 10 + (10-1) \times 6]$

$= 5[20 + 54] = 5 \times 74 = 370m$

CASE STUDY BASED QUESTINS

1) India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

Based on the above information, answer the following questions:

(i) Find the production during first year.

(ii) Find the production during 8th year.

(iii) Find the production during first 3 years.

(iv) In which year, the production is Rs 29,200.

(v) Find the difference of the production during 7th year and 4th year

Sol: The production of TV sets in a factory increases uniformly by a fixed number every year.

Number of TV sets produced by the factory in the 6th year=16000 and in the 9th year=22600

$a + 5d = 16000 \rightarrow (1) and\ a + 8d = 22600 \rightarrow (2)$

From $(2) - (1)$: $a + 8d - a - 5d = 22600 - 16000$

$3d = 6600$

$d = 2200$

$From\ (1): a + 5 \times 2200 = 16000$

$a = 16000 - 1100 = 5000$

(i) The production during the first year=a=₹5000

(ii) The production during the 8th year=a+7d=5000+7×2200=5000+15400=20400

(iii)The production during first 3 years $= \frac{n}{2}[2a + (n-1)d] = \frac{3}{2}[2 \times 5000 + 2 \times 2200]$

$= \frac{3}{2}[10000 + 11000] = \frac{3}{2} \times 21000 = 3 \times 10500 = 31500$

$(iv)\ a_n = 29200 \Rightarrow a + (n-1)d = 29200$

$5000 + (n-1) \times 2200 = 29200$

$(n-1) \times 2200 = 29200 - 5000$

$(n-1) \times 2200 = 24200$

$n - 1 = \frac{24200}{2200} = 11$

$n = 11 + 1 = 12$

In 12th year the production is 29200

(v) The difference of the production during 7th year and 4th year=$a_7 - a_4$

$= (a + 6d) - (a + 3d)$

$= a + 6d - a - 3d = 3d = 3 \times 2200 = 6600$

2) Your friend Veer wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds .

(i) Which of the following terms are in AP for the given situation

a) 51,53,55.... b) 51, 49, 47.... c) -51, -53, -55.... d) 51, 55, 59...

(ii) What is the minimum number of days he needs to practice till his goal is achieved

a) 10 b) 12 c) 11 d) 9

(iii) Which of the following term is not in the AP of the above given situation

a) 41 b) 30 c) 37 d) 39

(iv) If n^{th} term of an AP is given by $a_n = 2n + 3$ then common difference of an AP is

a) 2 b) 3 c) 5 d) 1

(v) The value of x, for which 2x, x+ 10, 3x + 2 are three consecutive terms of an AP

a) 6 b) -6 c) 18 d) -18

Sol: (i) b

First day performance=51 sec

Second day performance=51-2=49 sec

Third day performance=49-2=47 sec

Performances in each day are 51, 49,47,.....(seconds)

These are in AP with a=51 and d=-2

(ii) c

$a_n = 31$

$a + (n-1)d = 31$

$51 + (n-1)(-2) = 31$

$(n-1)(-2) = 31 - 51 = -20$

$(n-1) = \frac{-20}{-2} = 10$

$n = 11$

∴11 days he needs to practice till his goal to achieve.

(iii) b

All terms are odd numbers.

(iv)a

$a_n = 2n + 3$

$a_1 = 2 \times 1 + 3 = 2 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 4 + 3 = 7$

$d = a_2 - a_1 = 7 - 5 = 2$

(v) a

$a_1 = 2x, a_2 = x + 10, a_3 = 3x + 2$

$a_2 - a_1 = a_3 - a_2$

$x + 10 - 2x = 3x + 2 - x - 10$

$10 - x = 2x - 8$

$3x = 18$

$x = 6$

3) Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month , answer the following:

(i) The amount paid by him in 30th installment is

a) 3900 b) 3500 c) 3700 d) 3600

(ii) The amount paid by him in the 30 installlments is

a) 37000 b) 73500 c) 75300 d) 75000

(iii) What amount does he still have to pay offer 30th installment?

a) 45500 b) 49000 c) 44500 d) 54000

(iv) If total instalments are 40 then amount paid in the last installment?

a) 4900 b) 3900 c) 5900 d) 9400

(v) The ratio of the 1st installment to the last installment is

a) 1:49 b) 10:49 c) 10:39 d) 39:10

Sol:

(i) a) 3900

First installment=₹1000

Second installment=₹1000+₹100=₹1100

Third installment=₹1000+2×₹100=₹1200

The installments : ₹1000,₹1100,₹1200,...are in AP

$a = 1000 \ and \ d = 100$

Amount paid in 30th installment=a+29d=1000+29×100=1000+2900=₹3900

(ii) b) 73500

$$Amount\ paid\ in\ 30\ installments = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{30}{2}[2 \times 1000 + 29 \times 100] = 1500 \times 4900 = ₹73500$$

(iii) c)44500

Total amount he still have to pay after the 30th instalment=

$$Total\ loan\ amount - Amount\ paid\ in\ 30\ installments = 118000 - 73500 = ₹44500$$

(iv) a)4900

Amount paid in last installement $= a_{40} = a + 39d = 1000 + 39 \times 100 = 4900$

(v) b)10:49

1st installment : last installment=1000:4900=10:49

Some more problems from exemplar and previous papers

1. For the AP: −3, −7, −11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a30 and a20? Give reasons for your answer.
2. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer.
3. Find the value of the middle most term (s) of the AP : −11, −7, −3,..., 49.
4. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP. (Hint: Let the three terms in AP be a − d, a, a + d)
5. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
6. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.
7. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.
8. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.
9. Find the 12th term from the end of the AP: −2, −4, −6,..., −100.
10. Which term of the AP: 53, 48, 43,... is the first negative term?
11. In an A.P., the sum of the first n terms is given by $S_n=6n-n^2$.Find the 30^{th} term(CBSE-2023)

Answers:

1. 40
2. **No**
3. The two middle most terms are 17 and 21
4. 2,11,20
5. $a = -1; b = 15; c = 31$
6. Yes, 17^{th} term
7. k=0
8. 16^{th} term ;-21
9. -78
10. 12^{th} term
11. -53.

MCQ

1. The 10th term of the AP: 5, 8, 11, 14, ... is
   (A) 32 (B) 35 (C) 38 (D) 185
2. In an AP if a = −7.2, d = 3.6, a_n = 7.2, then n is
   (A) 1 (B) 3 (C) 4 (D) 5

3. In an AP, if d = −4, n = 7, an = 4, then a is
(A) 6 (B) 7 (C) 20 (D) 28

4. In an AP, if a = 3.5, d = 0, n = 101, then a_n will be
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

5. The 21st term of the AP whose first two terms are −3 and 4 is
(A) 17 (B) 137 (C) 143 (D) −143

6. Which term of the AP: 21, 42, 63, 84,... is 210?
(A) 9^{th} (B) 10^{th} (C) 11^{th} (D) 12^{th}

7. If the common difference of an AP is 5, then what is a_{18} -a_{13} ?
(A) 5 (B) 20 (C) 25 (D) 30

8. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be
(A) 7 (B) 11 (C) 18 (D) 0

9. In an AP if a = 1, a_n = 20 and S_n = 399, then n is
(A) 19 (B) 21 (C) 38 (D) 42

10. If the numbers n − 2, 4n − 1 and 5n + 2 are in AP, find the value of n.
(A) 2 (B) 1 (C) 3 (D) 4

11. The famous mathematician associated with finding the sum of the first 100 natural numbers is
(a) Pythagoras (b) Newton (c) Gauss (d) Euclid

12. The 13^{th} term from the end of the A.P:20,13,6,-1,...,-148
(A) 57 (B) -57 (C) 64 (D) -64

13. Assertion (A) : If the nth term of an A.P. is 7- 4n, then its common differences is -4.
Reason (R) : Common differences of an A.P .is given by d=$a_{n+1}-a_n$

14. Assertion (A) : 184 is the5^{th}term of the sequence 3, 7,11,.......

Reason (R) : The nth term of A.P .is given by a_n=a+(n−1)d

1)A	2)D	3)D	4)B	5)B	6)B	7)B	8)D	9)C	10)B	11) C	12) d	13)A	14)D	

Some more problems from previous year papers

1. **Find the 13^{th} term from the last term of the AP : 20, 13, 6,-1,. . ., -148.[CBSE-2023]**

Sol: $a = 20,\ d = 13 - 20 = -7$

$a_n = l = -148$

n^{th}term from the end of the AP $= l - (n-1)d$

13^{th}term from the end of the AP $= -148 - 12 \times (-7) = -148 + 84 = -64$

2. **In an AP ., if the first term a=7, nth term a_n=84 andd the sum of first n terms s_n=2093/2, then find n. [CBSE-2024]**

$$Sol: S_n = \frac{n}{2}[a + a_n]$$

$$\frac{2093}{2} = \frac{n}{2}[7 + 84]$$

$$2093 = n \times 91$$

$$n = \frac{2093}{91} = 23$$

3. The sum of first and eight terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

$Sol: First\ term + eight\ term\ of\ A.P = 32$

$a + a + 7d = 32$

$a + 7d = 32 - a \rightarrow (1)$

Product of first and eight terms=60

$a \times (a + 7d) = 60$

$a(32 - a) = 60$

$32a - a^2 = 60$

$a^2 - 32a + 60 = 0$

$(a - 2)(a - 30) = 0$

$a = 2\ or\ 30$

If $a = 2$ then $2 + 7d = 32 - 2 \Rightarrow 7d = 28 \Rightarrow d = 4$

$The\ sum\ of\ its\ first\ 20\ terms = \frac{n}{2}[2a + (n - 1)d] = \frac{20}{2}[2 \times 2 + 19 \times (4)] = 10[4 + 76]$

$= 10 \times 80 = 800$

If $a = 30$ then $30 + 7d = 32 - 30 \Rightarrow 7d = -28 \Rightarrow d = -4$

$The\ sum\ of\ its\ first\ 20\ terms = \frac{n}{2}[2a + (n - 1)d] = \frac{20}{2}[2 \times 30 + 19 \times (-4)] = 10[60 - 76]$

$= 10 \times (-16) = -160$

4. In an A.P of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687, deter main the first term and common difference of A.P. Also , find the sum of all terms of the A.P.[CBSE-2024]

$Sol: Sum\ of\ first\ 9\ terms\ in\ A.P = 153$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\frac{9}{2}[2a + 8d] = 153$

$2a + 8d = \frac{153 \times 2}{9} = 34$

$2a + 8d = 34 \rightarrow (1)$

The sum of last 6 terms = 687

$a_{35} + a_{356} + a_{37} + a_{38} + a_{39} + a_{40} = 687$

$(a + 34d) + (a + 35d) + (a + 36d) + (a + 37d) + (a + 38d) + (a + 39d) = 687$

$6da + 219d = 687$

$2a + 73d = 229 \rightarrow (2)$

$From\ (2) - (1): 2a + 73d - 2a - 8d = 229 - 34$

$65d = 195$

$d = 3$

Substitute d=3 in (1)

$2a + 8 \times 3 = 34$

$2a = 34 - 24 = 10$

$a = 5$

$The\ sum\ of\ all\ terms\ of\ the\ A.P = \frac{40}{2}[2 \times 3 + 39 \times 3] = 20 \times (6 + 117) = 20 \times 223 = 4460$

5. Find the sum of first 20 terms of an A.P whose n^th^ term is given by $a_n = 5 - 2n$ [CBSE-2022]

Sol: $a_n = 5 - 2n$

$a = a_1 = 5 - 2 \times 1 = 5 - 2 = 3$

$a_{20} = 5 - 2 \times 20 = 5 - 40 = -35$

$S_n = \frac{n}{2}[a + a_n]$

$S_{20} = \frac{20}{2}[3 - 35] = 10 \times (-30) = -300$

6. Which term of the A.P $-\frac{11}{2}, -3, -\frac{1}{2}, \ldots is\ \frac{49}{2}$?

Sol: $a = -\frac{11}{2};\ d = -3 + \frac{11}{2} = \frac{-6 + 11}{2} = \frac{5}{2}$

$a_n = \frac{49}{2}$

$a + (n - 1)d = \frac{49}{2}$

$-\frac{11}{2} + (n - 1) \times \frac{5}{2} = \frac{49}{2}$

$(n - 1) \times \frac{5}{2} = \frac{49}{2} + \frac{11}{2} = \frac{60}{2} = 30$

$n - 1 = 30 \times \frac{2}{5} = 12$

$n = 12 + 1 = 13$

7. Find a and b so that the numbers a,7,b,23 are in A.P[CBSE-2022]

Sol: a, 7, b, 23 are in A. P

$a_2 = 7 \Rightarrow a + d = 7 \rightarrow (1)$

$a_4 = 23 \Rightarrow a + 3d = 23 \rightarrow (2)$

$From\ (2) - (1): a + 3d - a - d = 23 - 7$

$2d = 16 \Rightarrow d = 8$

$From(1): a + 8 = 7 \Rightarrow a = 7 - 8 = -1$

$b = a + 2d = -1 + 16 = 15$

CHAPTER 6

X-MATHEMATICS-NCERT-2024-25

Triangles

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1. **congruent figures**: The figures that have the same shape and size are called congruent figures
2. The two triangles are congruent if the **sides** and **angles** of one triangle are **equal** to the **corresponding** sides and angles of the other triangle.
3. If Δ PQR is congruent to Δ ABC, we write Δ PQR ≅ Δ ABC.
4. Congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.
5. **Similar figures:**

   Two figures having the same shape but not necessarily the same size are called **similar figures**.

   Examples: 1) All squares are similar 2) All equilateral triangles are similar 3) All circles are similar

6. All the congruent figures are similar but the similar figures need not be congruent.
7. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion).

EXERCISE 6.1

1. Fill in the blanks with similar / not similar.

(i) All squares are <u>similar</u>

(ii) All equilateral triangles are <u>similar.</u>

(iii) All isosceles triangles are <u>not similar</u>.

(iv) Two polygons with same number of sides are similar. If their corresponding angles are equal and corresponding sides are equal.

(v) Reduced and Enlarged photographs of an object are <u>similar</u>.

(vi) Rhombus and squares are <u>not similar</u> to each other.

2. Give two different examples of pair of

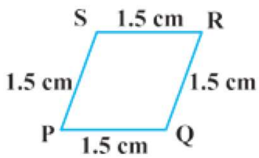
(i) Similar figures:

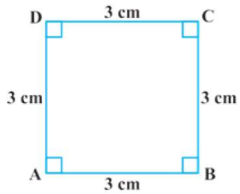
Example: 1. All squares 2. All circles. 3. All equilateral triangles.

(ii) Non-similar figures:

Examples: 1. Square, Rectangle 2. Rectangle, Rhombus

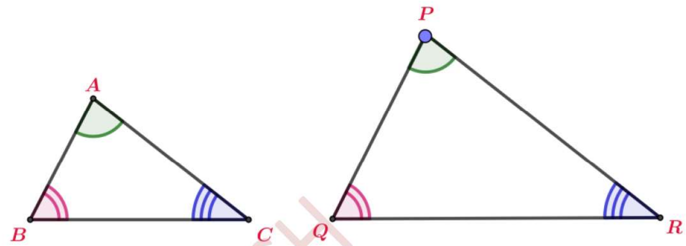
3. State whether the following quadrilaterals are similar or not:





Sol: Corresponding angles are not equal. So, the quadrilateral are not similar.

6.3 Similarity of Triangles



1. Two triangles are similar, if

(i) their corresponding angles are equal and

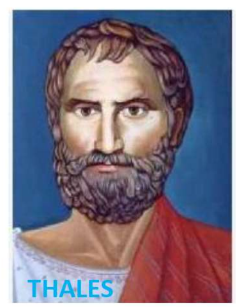
(ii) their corresponding sides are in the same ratio (or proportion).

2. In ΔABC and ΔPQR

(i) $\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Then ΔABC is similar to ΔPQR. It is denoted by $\Delta ABC \sim \Delta PQR$.

(Symbol '$\sim$' is read as "Is similar to")

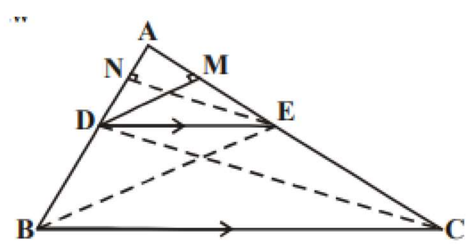


Basic proportionality theorem (Thales theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Given: In ΔABC, $DE \parallel BC$ which intersects sides AB and A C at D and E respectively

RTP: $\frac{AD}{DB} = \frac{AE}{EC}$



Construction: Join B, E and C, D and then draw $DM \perp AC$ and $EN \perp AB$.

$$Proof: Area\ of\ \Delta ADE = \frac{1}{2} \times AD \times EN$$

$$Area\ of\ \Delta BDE = \frac{1}{2} \times BD \times EN$$

$$So, \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times BD \times EN} = \frac{AD}{DB} \rightarrow (1)$$

$$Area\ of\ \Delta ADE = \frac{1}{2} \times AE \times DM$$

$$Area\ of\ \Delta CDE = \frac{1}{2} \times EC \times DM$$

Area of triangle $= \frac{1}{2} \times Base \times Height$

$$\frac{ar(\Delta\, ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \rightarrow (2)$$

But ΔBDE and ΔCDE are on the same base DE and between same parallels BC and DE.

So $ar(\Delta BDE) = ar(\Delta CDE) \rightarrow (3)$

From (1) (2) and (3), we have

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{ar(\Delta ADE)}{ar(\Delta CDE)}$$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved

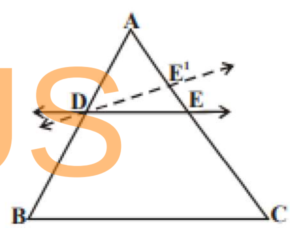
Theorem-6.2 :(Converse of basic proportionality theorem)

If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

Given : In ΔABC, a line DE is drawn such that $\frac{AD}{DB} = \frac{AE}{EC}$

RTP : DE || BC

Proof: Assume that DE is not parallel to BC then draw the line $DE^{I} \parallel BC$



In ΔABC ; $DE^{I} \parallel BC$

$$So, \frac{AD}{DB} = \frac{AE'}{E'C} \quad (\text{From Basic proportionality therem})$$

$$But \frac{AD}{DB} = \frac{AE}{EC} \ (Given)$$

$$\therefore \frac{AE}{EC} = \frac{AE'}{E'C} \Rightarrow \frac{AE}{EC} + 1 = \frac{AE'}{E'C} + 1$$

$$\frac{AE + EC}{EC} = \frac{AE' + E'C}{E'C}$$

$$\frac{AC}{EC} = \frac{AC}{E'C}$$

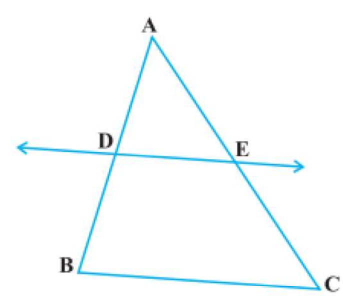
$$\Rightarrow EC = E'C$$

E and E′ must coincide

$\therefore DE \parallel BC$

Example 1 : If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC, prove that

$$\frac{AD}{AB} = \frac{AE}{AC}$$



Solution: In ΔABC, DE ||BC

$$\frac{AD}{DB} = \frac{AE}{EC} \ (from\ basic\ propertionality\ theorem)$$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

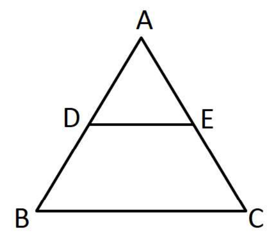
$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

In ΔABC, DE ∥ BC then

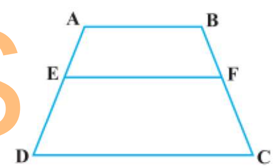
(i) $\frac{AD}{DB} = \frac{AE}{EC}$; $\frac{BD}{DA} = \frac{CE}{EA}$

(ii) $\frac{AB}{AD} = \frac{AC}{AE}$; $\frac{AD}{AB} = \frac{AE}{AC}$

(iii) $\frac{AB}{DB} = \frac{AC}{EC}$; $\frac{BD}{AB} = \frac{EC}{AC}$



Example 2 : ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB (see Fig. 6.14). Show that $\frac{AE}{ED} = \frac{BF}{FC}$



Solution : Let us join AC to intersect EF at G.

AB || DC and EF || AB (given)

⇒ EF || DC (Lines parallel to the same line are parallel to each other)

In ΔADC, EG || DC

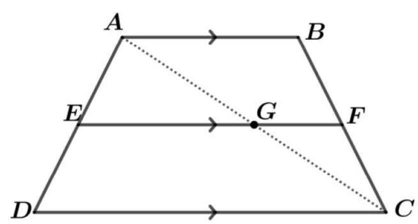
$\Rightarrow \frac{AE}{ED} = \frac{AG}{GC}$ (by BPT) → (1)

Similarly, In ΔCAB, GF || AB

$\Rightarrow \frac{AG}{GC} = \frac{BF}{FC}$ (by BPT) → (2)

From (1) & (2)

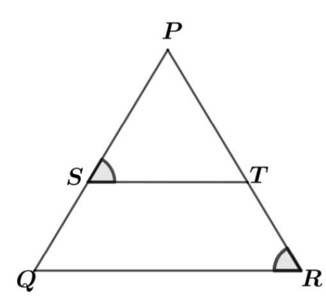
$\frac{AE}{ED} = \frac{BF}{FC}$



Example3: In ΔPQR, ST is a line such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and also ∠PST=∠PRQ . Prove that ΔPQR is an isosceles triangle.

Sol: In ΔPQR, ST is a line such that

$\frac{PS}{SQ} = \frac{PT}{TR}$



$\Rightarrow$ ST ∥ QR (*by converse of* BPT)

$\angle PST = \angle PQR$ (*corresponding angles*) $\rightarrow$ (1)

But $\angle PST = \angle PRQ$ (*given*) $\rightarrow$ (2)

From (1) and (2)

$\angle PQR = \angle PRQ$

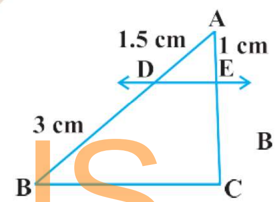
$\Rightarrow$ PR = PQ (*Sides opposite to equal angles are equal*)

Now in ΔPQR , PR = PQ

∴ ΔPQR is an isosceles triangle.

EXERCISE 6.2

1. **In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii)**



Sol: In ΔABC , DE ∥ BC

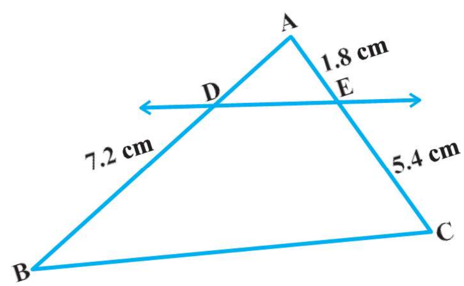
From basic property theorem

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{1\times 3}{1.5} = \frac{3}{1.5} = \frac{30}{15} = 2$$

$\therefore EC = 2\ cm$

(*ii*)



Sol: *In* ΔABC , DE ∥ BC

From basic property theorem

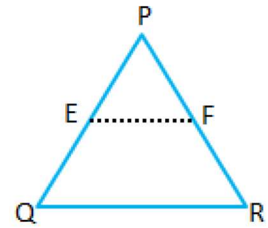
$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} = \frac{18}{54} = \frac{1}{3}$$

$$\Rightarrow AD = \frac{7.2}{3} = 2.4$$

$\therefore AD = 2.4\ cm$

2. **E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :**

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm



Sol: $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$; $\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

$\frac{PE}{EQ} \neq \frac{PF}{FR} \Rightarrow EF \nparallel QR$

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Sol: $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{40}{45} = \frac{8}{9}$; $\frac{PF}{FR} = \frac{8}{9}$

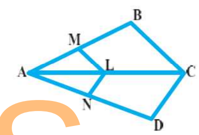
$\frac{PE}{EQ} = \frac{PF}{FR} \Rightarrow EF \parallel QR$

(iii) PQ = 1.28 cm PR = 2.56 cm PE = 1.8 cm and PF = 3.6 cm

Sol: $\frac{PQ}{PE} = \frac{1.28}{1.8} = \frac{128}{180} = \frac{32}{45}$; $\frac{PR}{PF} = \frac{2.56}{3.6} = \frac{256}{360} = \frac{64}{90} = \frac{32}{45}$

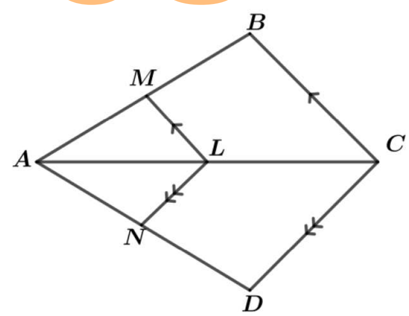
$\frac{PQ}{PE} = \frac{PR}{PF} \Rightarrow EF \parallel QR$

3. In Fig. 6.18, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$



Sol: In ΔACB, LM || CB

$\Rightarrow \frac{AL}{LC} = \frac{AM}{MB}$ (by BPT) → (1)



In ΔACD, LN || CD

$\Rightarrow \frac{AL}{LC} = \frac{AN}{ND}$ (by BPT) → (2)

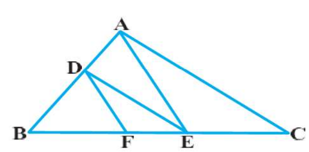
From (1) and (2)

$\frac{AM}{MB} = \frac{AN}{ND}$

$\Rightarrow \frac{AM}{AM + MB} = \frac{AN}{AN + ND}$

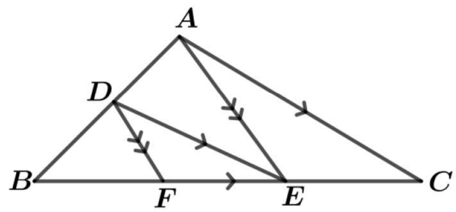
$\Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$

4. In Fig. 6.19, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$



Sol: In ΔABC , DE ∥ AC

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC} (\textit{ by BPT}) \rightarrow (1)$



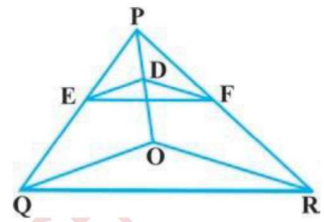
In ΔAEB , $DF \parallel AE$

$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE} (\textit{ by BPT}) \rightarrow (2)$

From (1) and (2)

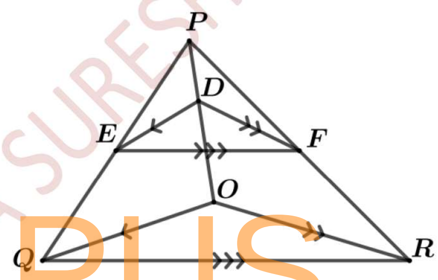
$\frac{BF}{FE} = \frac{BE}{EC}$

5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.



Solution: *In* ΔPQO , $ED \parallel QO$

$\Rightarrow \frac{PD}{DO} = \frac{PE}{EQ} (\textit{ by BPT}) \rightarrow (1)$



In ΔPOR , $DF \parallel OR$

$\Rightarrow \frac{PD}{DO} = \frac{PF}{FR} (\textit{ by BPT}) \rightarrow (2)$

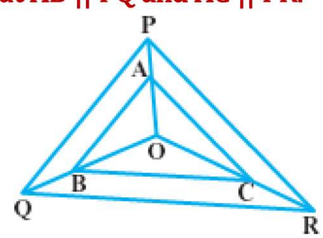
From (1) and (2)

$\frac{PE}{EQ} = \frac{PF}{FR}$

$\Rightarrow EF \parallel QR$ (*by converse of BPT*)

Hence proved

6. In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

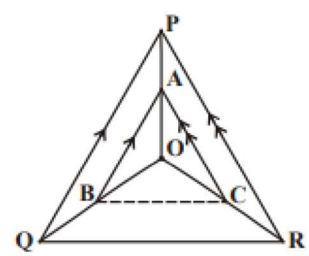


Sol: *In* ΔPQO , $AB \parallel PQ$

$\Rightarrow \frac{OA}{AP} = \frac{OB}{BQ} (\textit{ by BPT}) \rightarrow (1)$

In ΔPRO , $AC \parallel PR$

$\Rightarrow \frac{OA}{AP} = \frac{OC}{CR} (\textit{ by BPT}) \rightarrow (2)$



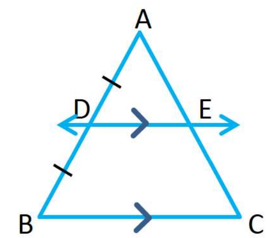
From (1) and (2)

$\frac{OB}{BQ} = \frac{OC}{CR}$

$\Rightarrow BC \parallel QR$ (*by converse of BPT*)

7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Solution: Let $in\ \Delta ABC\ , D\ is\ midpoint\ of\ AB\ and\ DE \parallel BC$



$\Rightarrow \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}\ (by\ BPT)$

$\Rightarrow 1 = \frac{\text{AE}}{\text{EC}}\quad (Since\ D\ is\ midpoint\ of\ AB\ ie, \text{AD} = \text{DB})$

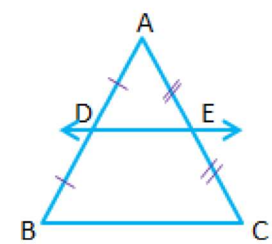
$\Rightarrow \text{AE} = \text{EC}$

$\Rightarrow E\ is\ mid\ point\ of\ AC$

$\Rightarrow \overleftrightarrow{DE}\ bisects\ AC$

8. **Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).**

Sol: $Given:\ In\ \Delta ABC\ , D\ is\ midpoint\ of\ AB\ and\ E\ is\ midpoint\ of\ AC$



RTP: $DE \parallel BC$

Proof:

$\text{AD} = \text{DB}\ \ and\ \ \text{AE} = \text{EC}$

$(D\ is\ midpoint\ of\ AB\ and\ E\ is\ midpoint\ of\ AC)$

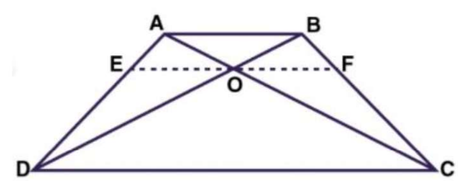
$\Rightarrow \frac{\text{AD}}{\text{DB}} = 1$ and $\frac{\text{AE}}{\text{EC}} = 1$

$\Rightarrow \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$

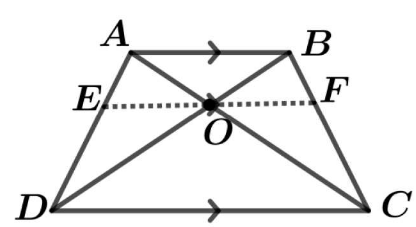
$\Rightarrow \text{DE} \parallel \text{BC}\ (by\ converse\ of\ BPT)$

9. **ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that**

$$\boldsymbol{\frac{AO}{BO} = \frac{CO}{DO}}$$



Given: ABCD is a trapezium in which AB||DC and its diagonals intersect each other at point ‘O’



RTP: $\frac{\text{AO}}{\text{BO}} = \frac{\text{CO}}{\text{DO}}$

Construction: Through ‘O’ draw a line EF ∥ AB ∥ DC .

Proof: $In\ \Delta ADC, EO \parallel DC$

$\frac{\text{AE}}{\text{ED}} = \frac{\text{AO}}{\text{OC}} \rightarrow (1)$

$In\ \Delta ADB, EO \parallel AB$

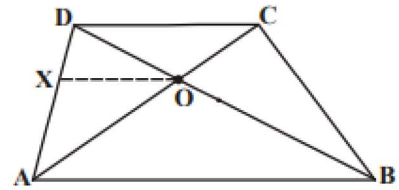
$\frac{\text{AE}}{\text{ED}} = \frac{\text{BO}}{\text{OD}} \rightarrow (2)$

From (1) and (2)

$$\frac{AO}{OC}=\frac{BO}{OD}\Rightarrow\frac{AO}{BO}=\frac{OC}{OD}\Rightarrow\frac{AO}{BO}=\frac{CO}{DO}$$

Hence proved.

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$ Show that ABCD is a trapezium.



Given: In quadrilateral ABCD, $\frac{AO}{BO}=\frac{CO}{DO}$

RTP : ABCD is a trapezium.

Construction: Through 'O' draw a line parallel to AB which meets DA at X.

Proof : In ΔDAB, XO || AB (by construction)

$\Rightarrow\frac{AX}{XD}=\frac{BO}{OD}$ (by B. P. T) → (1)

But $\frac{AO}{BO}=\frac{CO}{DO}$ (given)

$\Rightarrow\frac{AO}{CO}=\frac{BO}{DO}\rightarrow(2)$

From (1) and (2)

$\frac{AX}{XD}=\frac{AO}{CO}$

In ΔADC, XO is a line such that $\frac{AX}{XD}=\frac{AO}{OC}$

⇒ XO ∥ DC (From converse of BPT)

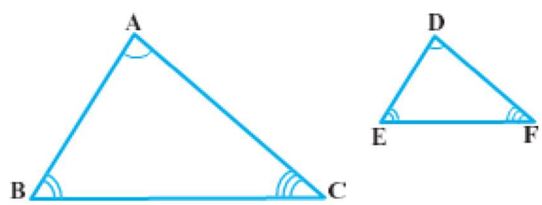
⇒ AB ∥ DC

In quadrilateral ABCD, AB || DC

⇒ ABCD is a trapezium

6.4 Criteria for Similarity of Triangles

Two triangles are similar, if (i) their correspondingangles are equal and (ii) their corresponding sides are in the same ratio (or proportion).



In Δ ABC andΔDEF, if

(i) ∠A = ∠D, ∠B = ∠E, ∠C =∠F and

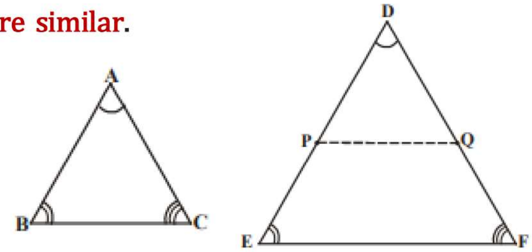
(ii) $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$, then the two triangles are similar .

We write the similarity of these two triangles as'ΔABC ~ Δ DEF' and read it as 'triangle ABC is similar to triangle DEF'.

The symbol '~' stands for 'is similar to'

Theorem 6.3 (AAA criterion for similarity of triangles)

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.



Given: : In triangles ABC and DEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

RTP : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction: Locate points P and Q on DE and DF respectively, such that AB = DP and AC = DQ. Join PQ.

Proof: *In* ΔABC , ΔDPQ

$AB = DP$ (Construction)

$AC = DQ$ (Construction)

$\angle A = \angle D$ (Given)

$\Delta ABC \cong \Delta DPQ$ (SAS congruency)

$\Rightarrow \angle B = \angle P$ (CPCT)

But $\angle B = \angle E$ (given)

$\therefore \angle P = \angle E \Rightarrow PQ \parallel EF$

$\Rightarrow \frac{DP}{PE} = \frac{DQ}{QF}$ (by BPT)

$\Rightarrow \frac{DP}{DP + PE} = \frac{DQ}{DQ + QF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$

Similarly $\frac{AB}{DE} = \frac{BC}{EF}$

So $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

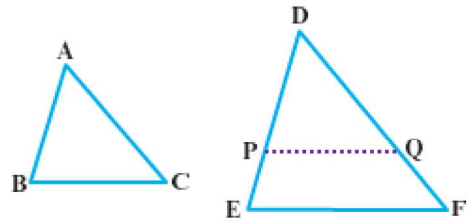
Hence proved.

AA similarity criterion for two triangles:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

SSS (Side–Side–Side) similarity criterion for two triangles.

Theorem 6.4 : If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.



Given: ΔABC and ΔDEF are such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} (< 1)$$

RTP : $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Construction : Locate points P and Q on DE and DF respectively such that $AB = DP$ and $AC = DQ$. Join PQ.

Proof: $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (*from construction*)

$\Rightarrow PQ \parallel EF$ (*By converse of BPT*)

So $\angle P = \angle E$ and $\angle Q = \angle F$ (corresponding angles)

In $\Delta DPQ, \Delta DEF$ corresponding angles are equal

$\Delta DPQ \sim \Delta DEF$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{PQ}{EF}$

$\Rightarrow \frac{BC}{EF} = \frac{PQ}{EF} \Rightarrow BC = PQ$

In $\Delta ABC, \Delta DPQ$

$AB = DP$ and $AC = DQ$ (by construction)

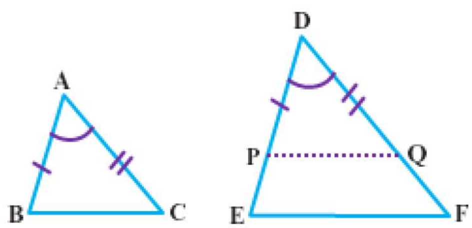
$BC = PQ$ (proved)

$\Rightarrow \Delta ABC \cong \Delta DPQ$

So $\angle A = \angle D$, $\angle B = \angle P = \angle E$ and $\angle C = \angle Q = \angle F$ (CPCT)

SAS (Side–Angle–Side) similarity criterion for two triangles.

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.



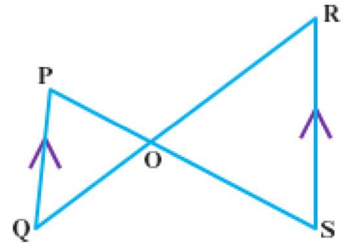
Given : In ΔABC and ΔDEF

$$\frac{AB}{DE} = \frac{AC}{DF} (< 1) \quad and \quad \angle A = \angle D$$

RTP : $\Delta ABC \sim \Delta DEF$

Construction: Locate points P and Q on DE and DF respectively such that $AB = DP$ and $AC = DQ$. Join PQ.

Proof: $\frac{AB}{DE} = \frac{AC}{DF}$ (Given)



$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (*from construction*)

$\Rightarrow$ PQ ∥ EF and ΔABC ≅ ΔDPQ

So ∠A = ∠D , ∠B = ∠P , ∠C = ∠Q

⇒ So ∠A = ∠D , ∠B = ∠E , ∠C = ∠F

⇒ ΔABC ~ ΔDEF (AAA similarity)

Example 4 : In Fig. 6.29, if PQ || RS, prove that Δ POQ ~ Δ SOR.

Solution : PQ || RS (Given)

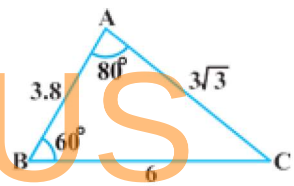
So, ∠ P = ∠ S (Alternate angles)

and ∠ Q = ∠ R (Alternate angles)

∠ POQ = ∠ SOR (Vertically opposite angles)

∴ Δ POQ ~ Δ SOR (AAA similarity criterion)

Example 5 : Observe adjacent figure and then find ∠ P

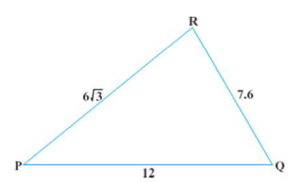


Solution : In ΔABC and ΔPQR,

$$\frac{AB}{RQ} = \frac{3.8}{7.6} = \frac{1}{2}; \quad \frac{BC}{PQ} = \frac{6}{12} = \frac{1}{2}; \frac{CA}{PR} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

$\frac{AB}{RQ} = \frac{BC}{PQ} = \frac{CA}{PR}$

Δ ABC ~ ΔRQP (By SSS similarity)



∠C =∠P (Corresponding angles of similar triangles CAST)

∠ C = 180° – ∠ A – ∠B(Angle sum property)

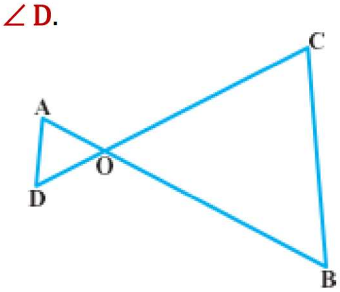
= 180° – 80° – 60° = 40°

∠P = 40°

Example 6 : In Fig. 6.31, OA . OB = OC . OD. Show that ∠ A = ∠ C and ∠ B = ∠ D.

Solution : OA .OB = OC .OD (Given)

$\frac{OA}{OC} = \frac{OD}{OB}$



∠ AOD = ∠ COB (Vertically opposite angles)

Δ AOD ~ Δ COB (SAS similarity criterion)

∠ A = ∠ C and ∠ D = ∠ B (Corresponding angles of similar triangles)

Example 7 : A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Solution :

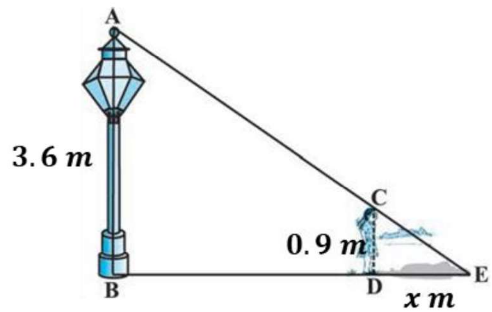
Lamp post (AB)=3.6 m

Height of girl (CD)=90cm=0.9 m

Length of shadow=DE=$x\ m$

Distance from pole to girl(BD)=$speed \times time$

$= 1.2 \times 4 = 4.8\ m$



In ΔABE and ΔCDE

$\angle E = \angle E$ (commom)

$\angle D = \angle B = 90^0$(lamp − post as well as the girl are standing vertical to the ground)

$\Delta ABE \sim \Delta CDE(\ AA\ similarity)$

$$\frac{BE}{DE} = \frac{AB}{CD}$$

$$\frac{BD + DE}{DE} = \frac{AB}{CD}$$

$$\frac{4.8 + x}{x} = \frac{3.6}{0.9} = 4$$

$$4x = 4.8 + x$$

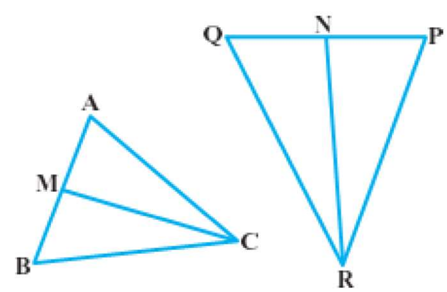
$$3x = 4.8$$

$$x = \frac{4.8}{3} = 1.6$$

The length of shadow of girl after 4 seconds=1.6 m

Example 8 : In Fig. 6.33, CM and RN are respectively the medians of Δ ABC and Δ PQR. If Δ ABC ~ Δ PQR, prove that:

(i) ΔCMB ~ ΔRNQ (ii) $\frac{CM}{RN} = \frac{AB}{PQ}$



Sol: $\Delta ABC \sim \Delta PQR$

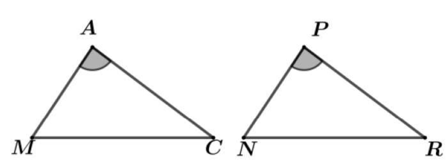
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \rightarrow (1)$$

CM and RN are medians of similar triangles ΔABC and ΔPQR

$$AM = MB = \frac{AB}{2} \Rightarrow AB = 2AM = 2MB \rightarrow (2)$$

$$PN = NQ = \frac{PQ}{2} \Rightarrow PQ = 2PN = 2NQ \rightarrow (3)$$

From (1),(2) and (3)



$\frac{2AM}{2PN} = \frac{AC}{PR} \Rightarrow \frac{AM}{PN} = \frac{AC}{PR} \rightarrow (4)$

In ΔAMC , ΔPNR

$\angle A = \angle P$ (CAST)

$\frac{AM}{PN} = \frac{AC}{PR}$ (*from* (4))

$\therefore \Delta AMC \sim \Delta PNR$ (SAS similarity)-→(i)

$\frac{CM}{RN} = \frac{AC}{PR}$

$But\ \frac{AC}{PR} = \frac{AB}{PQ}\ (from\ (1))$

$\frac{CM}{RN} = \frac{AB}{PQ} \rightarrow (ii)$

$A\ gain\ \frac{AB}{PQ} = \frac{BC}{QR}(from\ (1))$

$\frac{CM}{RN} = \frac{BC}{QR}$

$Also\ \frac{CM}{RN} = \frac{AB}{PQ} = \frac{2BM}{2QN}$

$i.e., \frac{CM}{RN} = \frac{BM}{QN}$

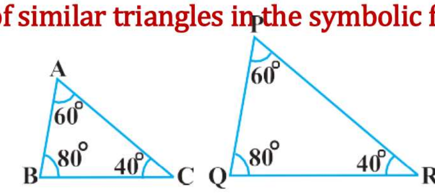
$\frac{CM}{RN} = \frac{BC}{QR} = \frac{BM}{QN}$

$\Delta CMB \sim \Delta RNQ$ (SSS similarity)

EXERCISE 6.3

1. **State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form :**

(i)



(i)

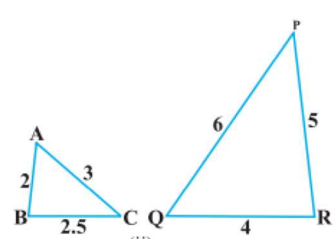
Sol: $\angle A = \angle P = 60^0$

$\angle B = \angle Q = 80^0$

$\angle C = \angle R = 40^0$

$\Delta ABC \sim \Delta PQR(\ AAA\ similarity)$

(ii)



(ii)

Sol: $\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$

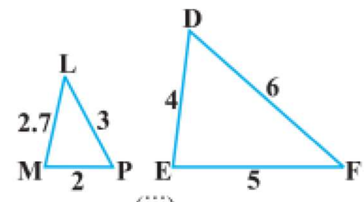
$\frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{CA}{PR} = \frac{3}{6} = \frac{1}{2}$

$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PR}$

$\Delta ABC \sim \Delta QRP(\text{ } SSS \text{ } similarity)$

(iii)



(iii)

Sol: $\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$

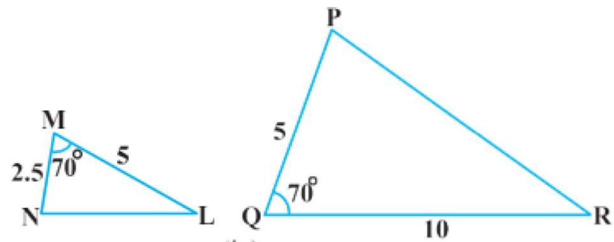
$\frac{PL}{DF} = \frac{3}{6} = \frac{1}{2}$

$\frac{LM}{FE} = \frac{2.7}{5} = \frac{27}{50}$

$\frac{MP}{ED} = \frac{PL}{DF} \neq \frac{LM}{FE}$

$\Delta MPL, \Delta EDF$ *are not similar.*

(iv)



(iv)

Sol: $\frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$

$\angle M = \angle Q = 70^0$

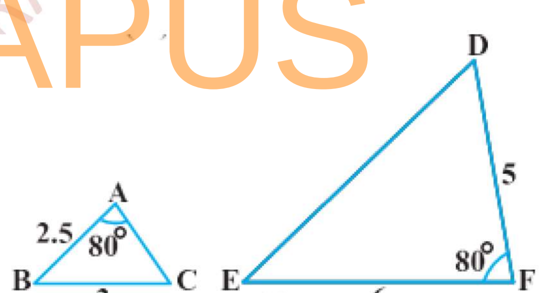
$\Delta MNL \sim \Delta QPR(\text{ } SAS \text{ } similarity)$

(v)

Don't say they are similar or not.

If AC=3 and DE=6 then

ΔABC~ΔFDE (SSS or SAS similarity)



(v)

(vi)

Sol:$\angle D + \angle E + \angle F = 180^0$(angle sum property)

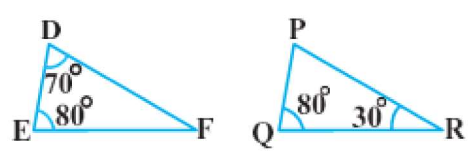
$70^0 + 80^0 + \angle F = 180^0$

$\angle F = 180^0 - 150^0 = 30^0$

$\angle E = \angle Q = 80^0$

$\angle F = \angle R = 30^0$

ΔDEF~ΔPQR(AA similarity)



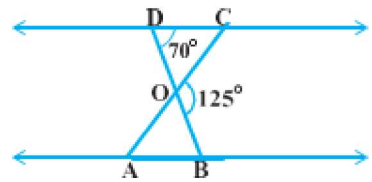
(vi)

2. In adjacent Fig Δ ODC ~ Δ OBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB.

Sol: ∠DOC + ∠ COB = 180° (Linear pair)

⇒ ∠DOC = 180° – 125° (Given, ∠ BOC = 125°)

$\Rightarrow \angle DOC = 55°$



$\angle DCO + \angle CDO + \angle DOC = 180°$ (Angle sum property)

$\Rightarrow \angle DCO + 70° + 55° = 180°$

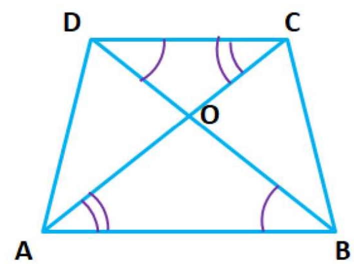
$\Rightarrow \angle DCO = 55°$

It is given that, $\Delta ODC \sim \Delta OBA$

$\angle OAB = \angle OCD$(Corresponding angles of similar triangles are equal)

$\Rightarrow \angle OAB = 55°$

3. **Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$**



Sol: In ΔAOB and ΔCOD

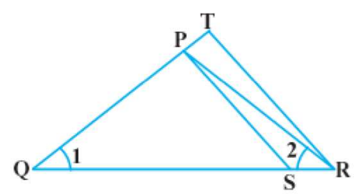
$\angle OAB = \angle OCD$ (AB || DC, alternate interior angles)

$\angle OBA = \angle ODC$ (AB || DC, alternate interior angles)

$\Delta AOB \sim \Delta COD$(AA similarity)

$\frac{OA}{OC} = \frac{OB}{OD}$ (corresponding sides are propertional)

4. **In adjacent Fig. , $\frac{QR}{QS} = \frac{QT}{PR}$ = and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.**



Sol: In ΔPQR, $\angle PQR = \angle PRQ$

$\Rightarrow PQ = PR$(Sides opposite to equal angles are equal)$\rightarrow$(1)

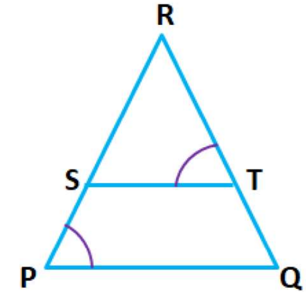
$Given, \frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{QP}$ (from (1))$\rightarrow$(2)

In ΔPQS and ΔTQR

$\frac{QR}{QS} = \frac{QT}{QP}$ and $\angle Q = \angle Q$

$\therefore \Delta PQS \sim \Delta TQR$ [By SAS similarity criterion]

5. **S and T are points on sides PR and QR of Δ PQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.**



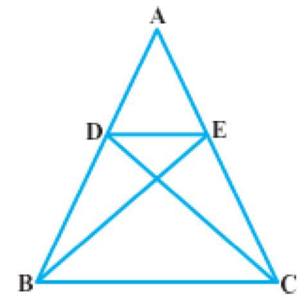
Sol: In ΔRPQ and ΔRTS

$\angle QPR = \angle RTS$ (Given)

$\angle R = \angle R$ (Common angle)

$\therefore \Delta RPQ \sim \Delta RTS$ (AA similarity criterion)

6. In Fig. 6.37, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.



Sol: Given, ΔABE ≅ ΔACD.

AB=AC and AE=AD (By CPCT)→(1)

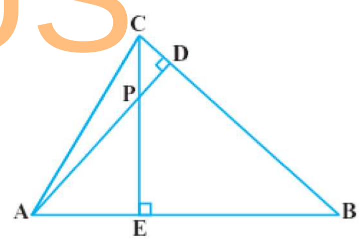
In ΔADE and ΔABC

$\frac{AD}{AB} = \frac{AE}{EC}$ (from (1))

$\angle A = \angle A$ [Common angle]

$\therefore \Delta ADE \sim \Delta ABC$ [SAS similarity criterion]

7. In adjacent Fig., altitudes AD and CE of Δ ABC intersect each other at the point P. Show that: (i) Δ AEP ~ Δ CDP (ii) Δ ABD ~ Δ CBE (iii) Δ AEP ~ Δ ADB (iv) Δ PDC ~ Δ BEC



Sol:(i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ (90° each)

$\angle APE = \angle CPD$ (Vertically opposite angles)

$\Delta AEP \sim \Delta CDP$ (by AA similarity criterion)

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ (90° each)

$\angle ABD = \angle CBE$ (Common Angles)

$\therefore \Delta ABD \sim \Delta CBE$(by AA similarity criterion)

(iii) In ΔAEP and ΔADB,

$\angle AEP = \angle ADB$ (90° each)

$\angle PAE = \angle DAB$ (Common Angles)

$\therefore \Delta AEP \sim \Delta ADB$(by AA similarity criterion)

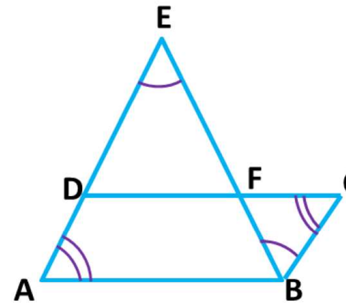
(iv) In ΔPDC and ΔBEC,

$\angle PDC = \angle BEC$ (90° each)

$\angle PCD = \angle BCE$ (Common angles)

$\therefore \Delta PDC \sim \Delta BEC$(by AA similarity criterion)

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta ABE \sim \Delta CFB$



Sol: In ΔABE and ΔCFB

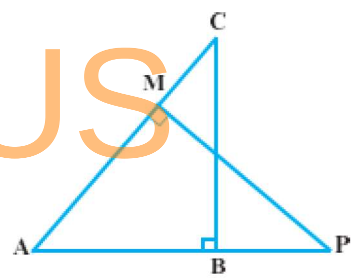
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (AE || BC ,alternate interior angles as)

$\therefore \Delta ABE \sim \Delta CFB$ (AA similarity criterion)

9. In Fig. 6.39, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:

$(i) \Delta ABC \sim \Delta AMP (ii) \frac{CA}{PA} = \frac{BC}{MP}$



Sol: (i) In ΔABC and ΔAMP

$\angle BAC = \angle MAP$ (common angles)

$\angle ABC = \angle AMP = 90°$

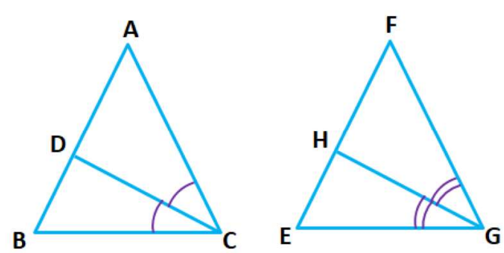
$\therefore \Delta ABC \sim \Delta AMP$ (AA similarity criterion)

(ii) $\Delta ABC \sim \Delta AMP$ (from(i))

$$\frac{CA}{PA} = \frac{BC}{MP} \text{ (Ratio of corresponding sides are equal in similar triangles)}$$

10. CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

$(i) \frac{CD}{GH} = \frac{AC}{FG} (ii) \Delta DCB \sim \Delta HGE (iii) \Delta DCA \sim \Delta HGF$



Sol: (i) Given $\Delta ABC \sim \Delta FEG$.

$\therefore \angle A = \angle F, \angle B = \angle E$, and $\angle ACB = \angle FGE$ (Corresponding angles of similar triangles)$\rightarrow$(1)

$\therefore \angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$ (Angle bisector)$\rightarrow$(2)

In ΔACD and ΔFGH,

$\angle ACD = \angle FGH$ (from(2))

$\angle A = \angle F$ (from(1))

$\therefore \Delta ACD \sim \Delta FGH$ (AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

(ii) In ΔDCB and ΔHGE,

$\angle DCB = \angle HGE$ (from (2))

$\angle B = \angle E$ (from (1))

$\therefore \Delta DCB \sim \Delta HGE$ (AA similarity criterion)

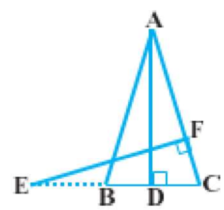
(iii) In ΔDCA and ΔHGF,

$\angle ACD = \angle FGH$ (from (2))

$\angle A = \angle F$ (from (1))

$\therefore \Delta DCA \sim \Delta HGF$ (AA similarity criterion)

11. In adjacent Fig., E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that $\Delta ABD \sim \Delta ECF$.



Sol: Given, ABC is an isosceles triangle.

$\therefore AB = AC$

$\Rightarrow \angle ABD = \angle ECF$ (Angles opposite to equal sides)→(1)

In ΔABD and ΔECF,

$\angle ADB = \angle EFC = 90°$

$\angle ABD = \angle ECF$(from (1))

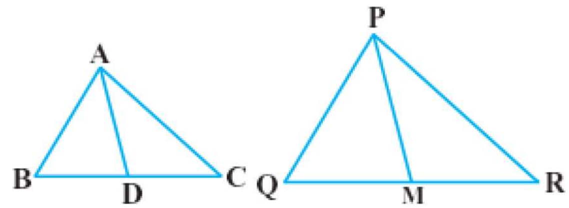
$\therefore \Delta ABD \sim \Delta ECF$ (By AA similarity)

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see Fig. 6.41). Show that $\Delta ABC \sim \Delta PQR$.

Sol: AD and PM are medians of ΔABC and ΔPQR

$BC = 2BD = 2DC$ and $QR = 2QM = 2MR$→(1)

Given $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM} \rightarrow (2)$



$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}\ (from(1))$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

⇒ ΔABD ~ ΔPQM [SSS similarity criterion]

∴ ∠ABD = ∠PQM [Corresponding angles of two similar triangles are equal]

⇒ ∠ABC = ∠PQR→(3)

In ΔABC and ΔPQR

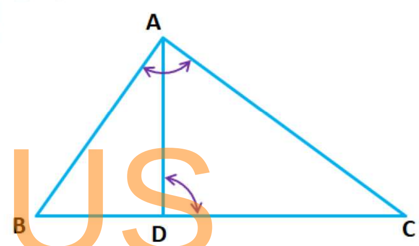
$\frac{AB}{PQ} = \frac{BC}{QR}\ (from(2))$

$\angle ABC = \angle PQR\ (from(3))$

ΔABC ~ ΔPQR [SAS similarity criterion]

13. D is a point on the side BC of a triangleABC such that ∠ADC =∠BAC. Show that CA^2 = CB.CD

Sol: In ΔADC and ΔBAC,



∠ADC = ∠BAC (given)

∠ACD = ∠BCA (Common angles)

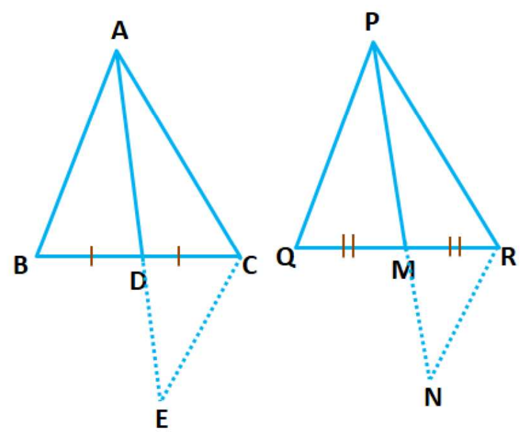
∴ ΔADC ~ ΔBAC (AA similarity criterion)

$\frac{CA}{CB} = \frac{CD}{CA}$ (corresponding sides of similar triangles are in proportion)

$\Rightarrow CA^2 = CB \times CD$

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

Sol: Produce AD to E so that AD = DE. Join CE, Similarly produce PM to N such that PM = MN, also Join RN.



In ΔABD and ΔCDE

AD = DE (By Construction.)

BD = DC (AD is the median)

∠ADB = ∠CDE (Vertically opposite angles)

∴ ΔABD ≅ ΔCDE (SAS criterion of congruence)

⇒ AB = CE (By CPCT)→(1)

In ΔPQM and ΔMNR,

PM = MN (By Construction)

QM = MR (PM is the median)

∠PMQ = ∠NMR (Vertically opposite angles)

∴ ΔPQM = ΔMNR (SAS criterion of congruence)

$\Rightarrow PQ = RN$ [By CPCT]$\rightarrow(2)$

$Given\ \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \rightarrow(3)$

From (1),(2) and (3)

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$

$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$

$\therefore \Delta ACE \sim \Delta PRN$ (SSS similarity)

$\angle CAE = \angle RPN$ (corresponding angles of similar triangles are equal)

$\Rightarrow \angle CAD = \angle PRM$

Similarly $\angle BAD = \angle QRM$

$\Rightarrow \angle CAD + \angle BAD = \angle PRM + \angle QRM$

$\Rightarrow \angle BAC = \angle QPR \rightarrow (4)$

In ΔABC and ΔPQR

$\frac{AB}{PQ} = \frac{AC}{PR}$ $(Given)$

$\angle BAC = \angle QPR$(From(4))

$\therefore \Delta ABC \sim \Delta PQR$ (SAS similarity criterion)

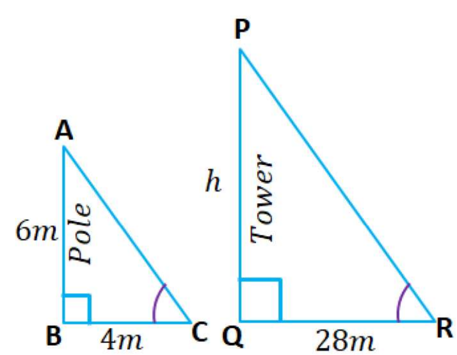
15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Sol: Length of pole (AB)=6 m

Length of shadow of pole (BC)=4m

Let Height of tower (PQ)=h m

Length of shadow of the tower (QR)=28 m



In ΔABC and ΔPQR,

$\angle B = \angle Q = 90°$

$\angle C = \angle R$ (angular elevation of sun)

$\therefore \Delta ABC \sim \Delta PQR$ (AA similarity criterion)

$\frac{AB}{PQ} = \frac{BC}{QR}$(In similar trianglescorresponding sides are proportional)

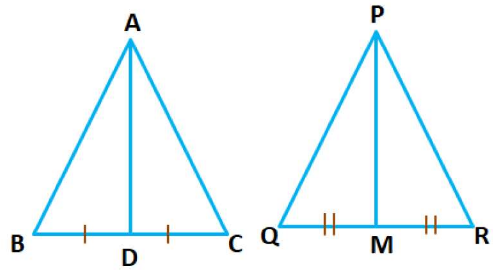
$\frac{6}{h} = \frac{4}{28} \Rightarrow h = \frac{6 \times 28}{4} = 6 \times 7 = 42$

$\therefore$ The height of the tower is 42 m.

16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta\, ABC \sim \Delta\, PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Sol: $\Delta\, ABC \sim \Delta\, PQR$

In similar triangles corresponding angles are equal and corresponding sides are proportional



$\Rightarrow \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ and

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \rightarrow (1)$

Since AD and PM are medians

BC=2BD and QR=2QM

In ΔABD and ΔPQM,

$\angle B = \angle Q$ (from (1))

$\frac{AB}{PQ} = \frac{BC}{QR}$ (from(1))

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$

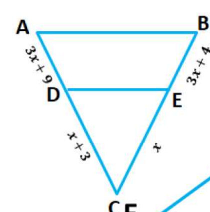
$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore \Delta ABD \sim \Delta PQM$ (SAS similarity criterion)

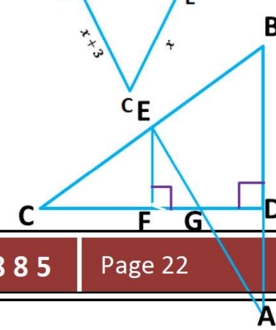
$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM} (CSST)$

Some problems for student brain boosting

1. Legs (sides other than the hypotenuse) of a right triangle are of lengths 16cm and 8 cm. Find the length of the side of the largest square that can be inscribed in the triangle.



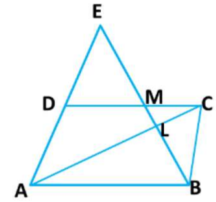
2. Find the value of x for which DE||AB in adjacent Fig.,



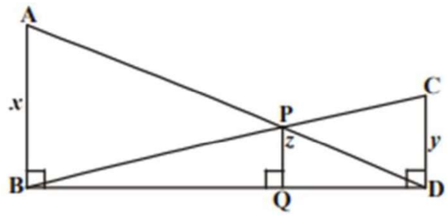
3. If $\Delta\, ABC \sim \Delta\, DEF$, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, find the perimeter of $\Delta\, ABC$

4. In the given figure ,CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\frac{CF}{CD} = \frac{FG}{DG}$ (CBSE-2023)

5. In the given figure, ABCD is a parallelogram.BE bisectsCD at M and intersects Ac at L. Prove that EL=2BL



6. In adjacent fig., PA,QB and RC are each perpendicular to AC. If x=8 cm and z=6 cm, then find y.



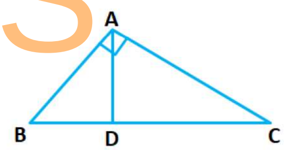
Answers:1)16/3 cm 2)2 3)18 cm 6)24/7 cm

MCQ

1. **D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE BC. Then, length of DE (in cm) is**

(A) 2.5 (B) 3 (C) 5 (D) 6

2. **In Fig. 6.2, ∠BAC = 90° and AD ____________ BC. Then,**



(A) BD.CD = BC^2 (B) AB.AC = BC^2 (C) BD.CD = AD^2 (D) AB.AC = AD^2

3. **If ΔA B C ~ ΔE D F and ΔA B C is not similar to ΔD E F, then which of the following is not true?**

(A) BC.EF = A C.FD (B) AB.EF = AC.DE (C) BC.DE = AB.EF (D) BC.DE = AB.FD

4. **If in two triangles ABC and PQR $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then**

(A) ΔPQR ~ ΔCAB (B) ΔPQR ~ ΔABC(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

5. **If in two triangles DEF and PQR, D = Q and R = E, then which of the following is not true?**

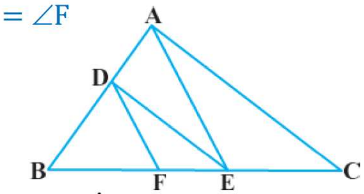
(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$ (C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

6. **It is given that ΔABC ~ ΔDFE, ∠A =30°,∠C = 50°, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true:**

(A) DE = 12 cm, ∠F = 50° (B) DE = 12 cm, ∠F = 100° (C) EF = 12 cm, ∠D = 100° (D) EF = 12 cm, ∠D = 30°

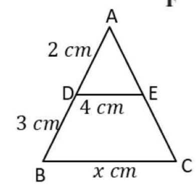
7. **If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when**

(A) ∠B = ∠E (B) ∠A = ∠D (C) ∠B = ∠D (D) ∠A = ∠F



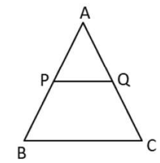
8. In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$. Which of the following makes the two triangles similar?(CBSE-2023)

(A) ∠A=∠D (B) ∠B=∠D (C) ∠B=∠E (D) ∠A=∠F



9. In the given figure, DE||BC. The value of x is

(A) 6 (B) 12.5 (C) 8 (D) 10



10. In ΔABC,PQ||BC. If PB=6 cm, AP=4 cm, AQ=8 cm, find the length of AC.

(A) 12 cm (B) 20 cm (C) 6 cm (D) 14 cm

11. **Assertion (A):** The sides of two similar triangles are in the ratio 2 : 5, then the areas of these triangles are in the ratio 4 : 25.

Reason (R): The ratio of the areas of two similar triangles is equal to the square of the ratio of their sides.

12. **Assertion (A):** If two sides of a right angle are 7 cm and 8 cm, then its third side will be 9 cm.

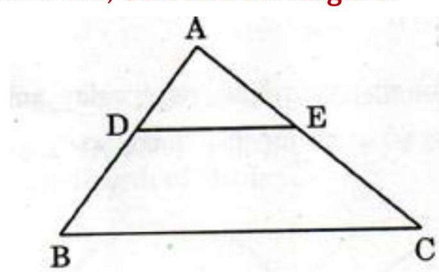
Reason (R): In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

13.

1)B	2)C	3)C	4)A	5)D	6)	7)	8)	8)B	9)D	10)B	11)A	12)D	

Previous year problems:

1. In Δ ABC , $DE \parallel BC$. If $AD = 4\ cm$, $AB = 9\ cm$ and $AC = 13.5\ cm$, then find the length of EC?[CBSE-2024]



Sol: In Δ ABC , $DE \parallel BC$

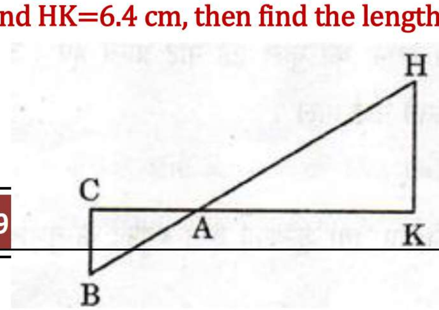
$$\frac{AB}{AD} = \frac{AC}{AE}(By\ basic\ propertionality\ theorem)$$

$$\frac{9}{4} = \frac{13.5}{AE}$$

$$AE = \frac{4}{9} \times 13.5 = 4 \times 1.5 = 6\ cm$$

$$EC = AC - AE = 13.5 - 6 = 7.5\ cm$$

2. In the given figure , $\Delta AHK \sim \Delta ABC$. If AK=8 cm,BC=3.2cm and HK=6.4 cm, then find the length of AC.[CBSE-2024]



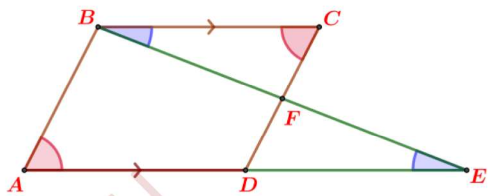
Sol: In the given figure , $\Delta AHK \sim \Delta ABC$

$$\frac{AC}{AK} = \frac{BC}{HK} \ (corresponding\ sides\ are\ proportional)$$

$$\frac{AC}{8} = \frac{3.2}{6.4}$$

$$AC = \frac{32}{64} \times 8 = 4\ cm$$

3. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at EF. Show that $\triangle ABE \sim \Delta CFB$ [CBSE-2024]



Sol: $In \triangle ABE, \Delta CFB$

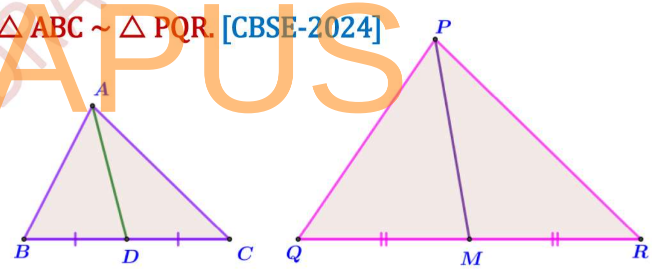
$\angle A = \angle C$ ($opposite\ angles\ of\ a\ parallelogram$)

$\angle AEB = \angle CBF$ ($Alternate\ interior\ angles$)

$\Delta ABE \sim \Delta CFB$ ($AA\ similarity$)

4. Sides AB, BC and the median AD of △ ABC are respectively proportional to sides PQ, QR and the median PM of another △ PQR. Prove that △ ABC ~ △ PQR. [CBSE-2024]

5.



Sol: Given

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM} \ (D, M\ are\ midpoints\ of\ Bc\ and\ QR$$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\Delta ABD \sim \Delta PQM$

$\angle ABD = \angle PQM (\ CAST)$

$\angle ABC = \angle PQR \rightarrow (1)$

$In\ \Delta ABC, \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}(Given)$$

$$\angle ABC = \angle PQR(from(1))$$

$\Delta ABC \sim \Delta PQR(SAS\ similarity\ criterion)$

6.

7.