

1

PATTERNS IN MATHEMATICS



Learning Outcomes:

Learners will be able to

- Identify, describe, and generalize patterns in numbers and shapes. (CG-1)
- Represent patterns algebraically and use them to make predictions. (CG-8)
- Apply logical reasoning to explore mathematical relationships in real-life contexts. (CG-10)
- Appreciate the beauty and structure of Mathematics through pattern recognition. (CG-9)

1.0 Introduction

"Have you ever looked at a honeycomb, the petals of a flower, or the design on your clothes?"



“Do you know what’s common among all of them?”

“They all follow patterns — arrangements that repeat or grow in a regular way. Just like how days follow nights, and seasons follow one another, numbers and shapes in math also follow beautiful patterns.

Now, we’ll become pattern detectives and uncover the hidden rules behind them!”

1.1 What is a Pattern?

A pattern is an arrangement or sequence that repeats in a predictable way. Patterns can be found in numbers, shapes, colours, sounds, or designs. They help us recognize order, make predictions, and understand relationships in mathematics and nature.

Let us see some examples of patterns

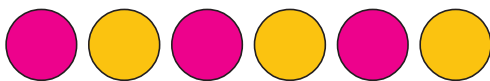
1. Number Patterns

- **Example 1:** 2, 4, 6, 8, 10, ...
 - ★ Each number increases by 2 (an arithmetic pattern).
- **Example 2:** 1, 2, 4, 8, 16, ...
 - ★ Each number is multiplied by 2 (a geometric pattern).

2. Shape Patterns

- Circle, Square, Circle, Square, Circle, Square, ...
 - ★ Shapes repeat in a regular alternating pattern.

3. Colour Patterns



- Colours Pink and Yellow repeat alternately.

4. Patterns in Nature

- The arrangement of petals in a sunflower and the spiral of a shell - both follow natural patterns like the Fibonacci sequence.

A pattern is something that repeats in a regular way and helps us predict what comes next.

1.2 What is Mathematics?

Mathematics is, in large part, the search for patterns, and for the explanations as to why those patterns exist.

Such patterns indeed exist all around us — in nature, in our homes and schools, and in the motion of the sun, moon, and stars. They occur in everything that we do and see, from shopping and cooking, to throwing a ball and playing games, to understand weather patterns and using technology.

The search for patterns and their explanations can be a fun and creative endeavour. It is for this reason that mathematicians think of mathematics both as an art and as a science. This year, we hope that you will get a chance to see the creativity and artistry involved in discovering and understanding mathematical patterns.

It is important to keep in mind that mathematics aims to not just find out what patterns exist, but also the explanations for why they exist. Such explanations can often then be used in applications well beyond the context in which they were discovered, which can then help to propel humanity forward.

For example, the understanding of patterns in the motion of stars, planets, and their satellites led humankind to develop the theory of gravitation, allowing us to launch our own satellites and send rockets to the Moon and to Mars; similarly, understanding patterns in genomes has helped in diagnosing and curing diseases — among thousands of other such examples.



Figure it Out - 1.1

1. Can you think of other examples where mathematics helps us in our everyday lives?
2. How has mathematics helped propel humanity forward? (You might think of examples involving: carrying out scientific experiments; running our economy and democracy; building bridges, houses or other complex structures; making TVs, mobile phones, computers, bicycles, trains, cars, planes, calendars, clocks, etc.)

Math Talk



1.3 Patterns in Numbers

Among the most basic patterns that occur in mathematics are patterns of numbers, particularly patterns of Counting Numbers (Natural Numbers), whole numbers:

1, 2, 3, 4, 5, ...

0, 1, 2, 3, 4, ...

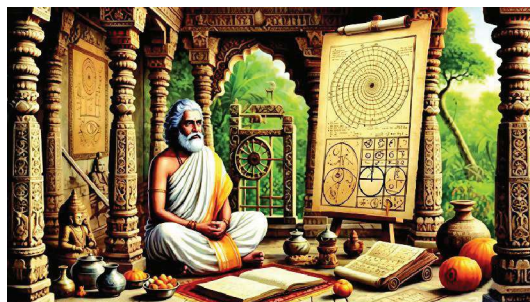
The branch of Mathematics that studies patterns in whole numbers is called **number theory**.

Number sequences are the most basic and among the most fascinating types of patterns that mathematicians study. Table 1 shows some key number sequences that are studied in Mathematics.

Table 1: Examples of number sequences

1, 1, 1, 1, 1, 1, ...	(All 1's)
1, 2, 3, 4, 5, 6, 7, ...	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21, ...	(Virahānka*/Fibonacci numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

- ❁ Virahanka (6th century CE), Hemachandra (1150 CE) basing upon Chanda Shastra of Pingala (2nd century BCE) worked on finding patterns of syllables-laghu, guru, developed a sequence of numbers which is now credited as Fibonacci (1228 CE)



sequence. The sequence starts with 1, and each subsequent number is the sum of the two preceding ones ($V_n = V_{n-1} + V_{n-2}$) giving the series 1, 2, 3, 5, 8, 13, and so on.



Figure it Out - 1.2

1. Can you recognise the pattern in each of the sequences in Table 1?
2. Rewrite each sequence of Table 1 in your notebook, along with the next three numbers in each sequence. After each sequence, write in your own words what is the rule for forming the numbers in the sequence.

Math Talk



1.4 Visualising Number Sequences

Many number sequences can be visualised using pictures. Visualising mathematical objects through pictures or diagrams can be a very fruitful way to understand mathematical patterns and concepts. Let us represent the first seven sequences in Table 1 using the following pictures.

Table 2: Pictorial representation of some number sequences




































					All 1's
1	1	1	1	1	
					Counting numbers
1	2	3	4	5	
					Odd numbers
1	3	5	7	9	
					Even numbers
2	4	6	8	10	
					Triangular numbers
1	3	6	10	15	
					Squares
1	4	9	16	25	
					Cubes
1	8	27	64	125	

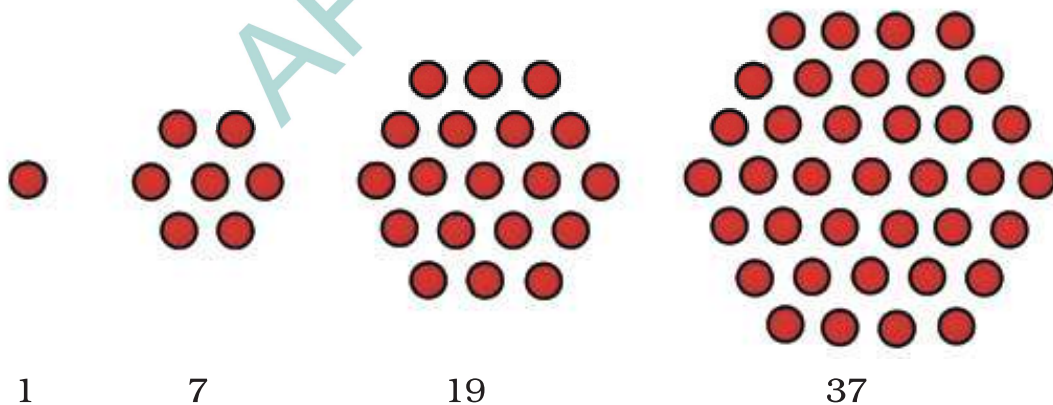


Figure it Out - 1.3

1. Copy the pictorial representations of the number sequences in Table 2 in your notebook, and draw the next picture for each sequence!
2. Why are 1, 3, 6, 10, 15, ... called **triangular numbers**? Why are 1, 4, 9, 16, 25, ... called **square numbers** or squares? Why are 1, 8, 27, 64, 125, ... called **cubes**?



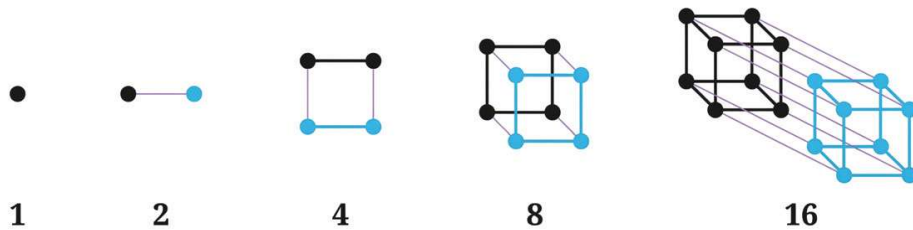
3. You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this! This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!
4. What would you call the following sequence of numbers?



That's right, they are called **hexagonal numbers**! Draw these in your notebook. What is the next number in the sequence?

5. Can you think of pictorial ways to visualise the sequence of Powers of 2? Powers of 3?

Here is one possible way of thinking about Powers of 2:



1.5 Relations among Number Sequences

Sometimes, number sequences can be related to each other in surprising ways.

Example: What happens when we start adding up odd numbers?

$$1 = 1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

$$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2 \dots$$

(We write $a \times a$ as a^2 and read as a square)

This is a really beautiful pattern!

❁ Why does this happen? Do you think it will happen forever?

The answer is that the pattern does happen forever. But why? As mentioned earlier, the reason why the pattern happens is just as important and exciting as the pattern itself.

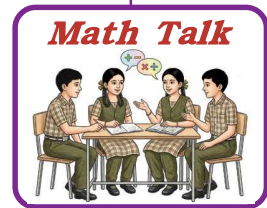
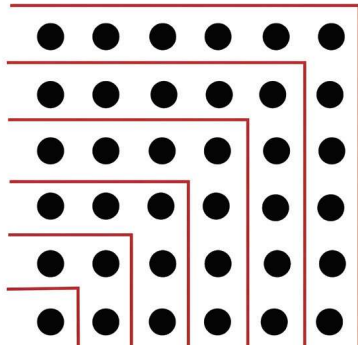
A picture can explain it

Visualising with a picture can help explain the phenomenon. Recall that square numbers are made by counting the number of dots in a square grid.

- ❁ How can we partition the dots in a square grid into odd numbers of dots: 1, 3, 5, 7, ... ?

Think about it for a moment before reading further!

Here is how it can be done:



This picture now makes it evident that

$$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6 \times 6$$

Because such a picture can be made for a square of any size, this explains why adding up odd numbers gives square numbers.

- ❁ By drawing a similar picture, can you say what is the sum of the first 10 odd numbers?
- ❁ Now by imagining a similar picture, or by drawing it partially, as needed, can you say what is the sum of the first 100 odd numbers?

Another example of such a relation between sequences:

Adding up and down

Let us look at the following pattern:

$$1 = 1 = 1 \times 1 = 1^2$$

$$1 + 2 + 1 = 4 = 2 \times 2 = 2^2$$

$$1 + 2 + 3 + 2 + 1 = 9 = 3 \times 3 = 3^2$$

$$1 + 2 + 3 + 4 + 3 + 2 + 1 = 16 = 4 \times 4 = 4^2$$

$$1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5 \times 5 = 5^2$$

$$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36 = 6 \times 6 = 6^2$$

This seems to be giving yet another way of getting the square numbers—by adding the counting numbers up and then down!

❁ Can you find a similar pictorial explanation?

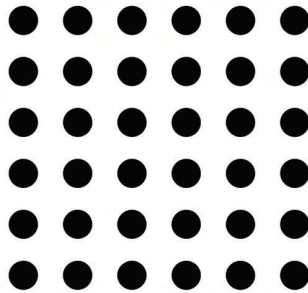
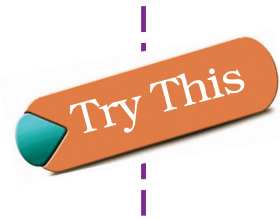
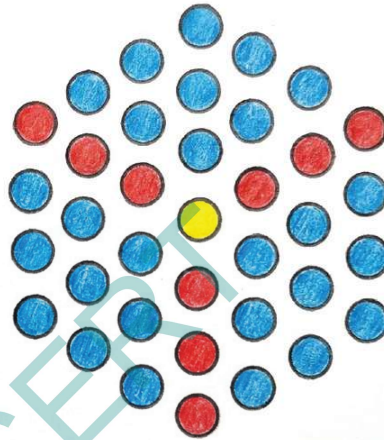
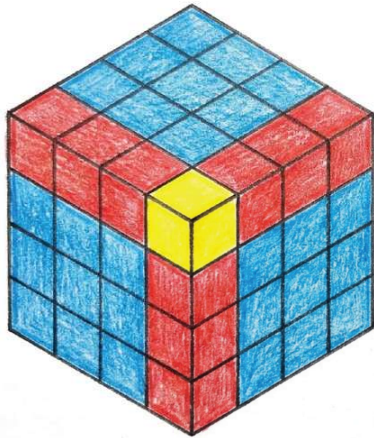


Figure it Out - 1.4

1. Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., $1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, \dots$, gives square numbers?
2. By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be the value of $1 + 2 + 3 + \dots + 99 + 100 + 99 + \dots + 3 + 2 + 1$?
3. Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?
4. Which sequence do you get when you start to add the counting numbers up? Can you give a smaller pictorial explanation?
5. What happens when you add up pairs of consecutive triangular numbers? That is, take $1 + 3, 3 + 6, 6 + 10, 10 + 15, \dots$ Which sequence do you get? Why? Can you explain it with a picture?
6. What happens when you start to add up powers of 2 starting with 1, i.e., take $1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, \dots$? Now add 1 to each of these numbers — what numbers do you get? Why does this happen?



7. What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?
8. What happens when you start to add up hexagonal numbers, i.e., take $1, 1 + 7, 1 + 7 + 19, 1 + 7 + 19 + 37, \dots$? Which sequence do you get? Can you explain it using a picture of a cube?



9. Find your own patterns or relations in and among the sequences in Table 1. Can you explain why they happen with a picture or otherwise?

1.6 Patterns in Shapes

Other important and basic patterns that occur in Mathematics are patterns of shapes. These shapes may be in one, two, or three dimensions (1D, 2D, or 3D) — or in even more dimensions. The branch of Mathematics that studies patterns in shapes is called geometry.

Shape sequences are one important type of shape pattern that mathematicians study. Table 3 shows a few key shape sequences that are studied in Mathematics.

Table 3: Examples of shape sequences

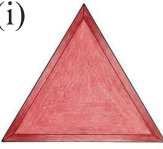

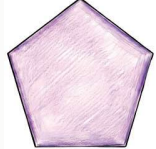
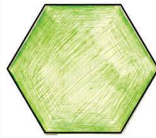


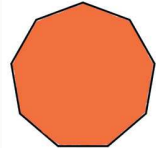


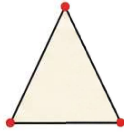
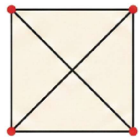

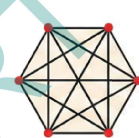


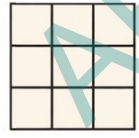
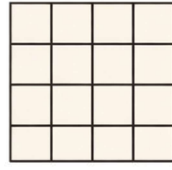
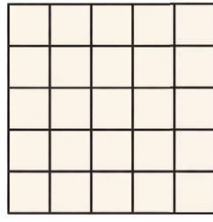


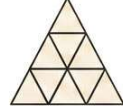
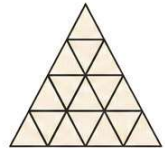
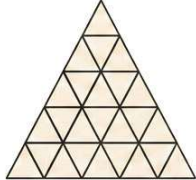
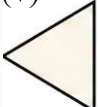

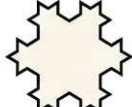
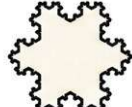
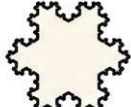
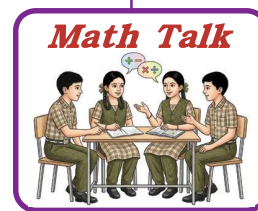
(i)					Regular Polygons	
	Triangle	Quadrilateral	Pentagon	Hexagon		
						
	Heptagon	Octagon	Nonagon	Decagon		
(ii)						Complete Graphs
	K2	K3	K4	K5	K6	
(iii)						Stacked Squares
(iv)						Stacked Triangles
(v)						Koch Snowflake



Figure it Out - 1.5

1. Can you recognise the pattern in each of the sequences in Table 3?
2. Try and redraw each sequence in Table 3 in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence.



1.7 Relation to Number Sequences

Often, shape sequences are related to number sequences in surprising ways. Such relationships can be helpful in studying and understanding both the shape sequence and the related number sequence.

Example: [From table 3(i)] The number of sides in the shape sequence of Regular Polygons is given by the counting numbers starting at 3, i.e., 3, 4, 5, 6, 7, 8, 9, 10, That is why these shapes are called, respectively, **regular triangle, quadrilateral (i.e., square), pentagon, hexagon, heptagon, octagon, nonagon, decagon**, etc.

The word 'regular' refers to the fact that these shapes have equal-length sides and also equal 'angles' (i.e., the sides look the same and the corners also look the same). We will discuss angles in more depth in the next chapter.


The other shape sequences in Table 3 also have beautiful relationships with number sequences.



Figure it Out - 1.6

1. Count the number of sides in each shape in the sequence of Regular Polygons. Which number sequence do you get? What about the number of corners in each shape in the sequence of Regular Polygons? Do you get the same number sequence? Can you explain why this happens?
2. Count the number of lines in each shape in the sequence of Complete Graphs. **[From table 3 (ii)]** Which number sequence do you get? Can you explain why?



3. How many little squares **[From table 3 (iii)]** are there in each shape of the sequence of Stacked Squares? Which number sequence does this give? Can you explain why?
4. How many little triangles **[From table 3 (iv)]** are there in each shape of the sequence of Stacked Triangles? Which number sequence does this give? Can you explain why? (**Hint:** In each shape in the sequence, how many triangles are there in each row?)
5. To get from one shape to the next shape in the Koch Snowflake sequence, one replaces each line segment '—' by a 'speed bump' . As one does this more and more times, the changes become tinier and tinier with very very small line segments. How many total line segments are there in each shape of the Koch Snowflake? What is the corresponding number sequence? (The answer is 3, 12, 48, ..., i.e., 3 times Powers of 4; this sequence is not shown in Table 1.)

1.8 Patterns in Nature

Patterns in Nature - A Mathematical Wonder

Nature is full of beautiful and fascinating patterns - from the smallest leaf to the vastest galaxy. These patterns are not just random; they follow mathematical principles that help us understand how nature organizes itself.

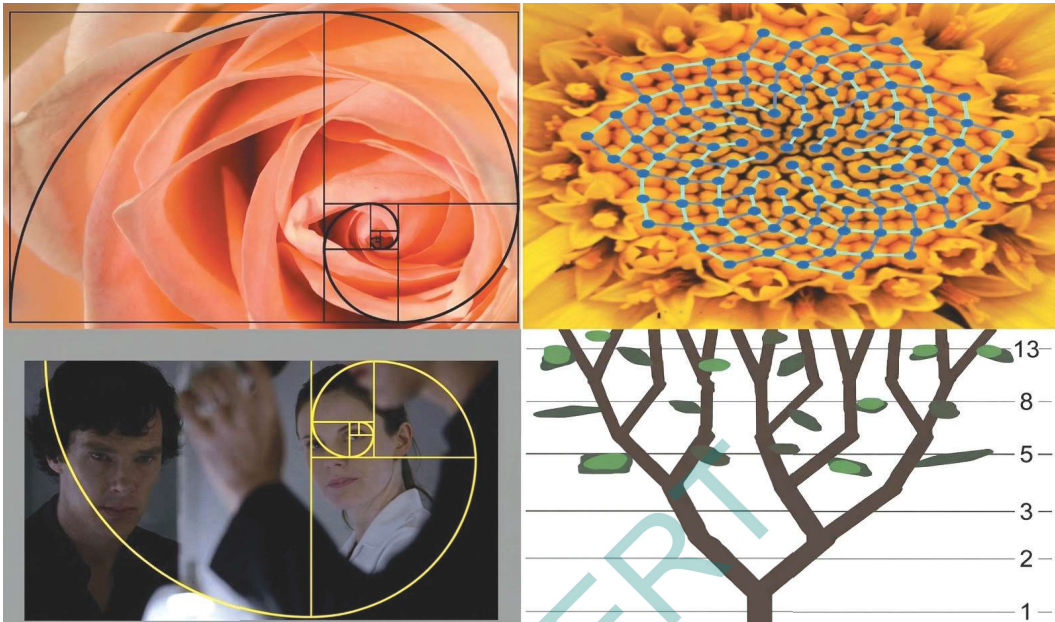
1. Common Patterns Found in Nature

a) Spirals :

Where we find Spirals in Nature?

- Found in snail shells, pinecones, sunflowers, galaxies and also in some art.
- They often follow the Fibonacci sequence, where each number is the sum of the previous two (1, 2, 3, 5, 8, 13...).

- **Example:** The arrangement of seeds in a sunflower follows a spiral pattern for maximum packing.



b) Symmetry

Can you give me some examples for Symmetrical objects in the Nature?

- Seen in butterflies, leaves, flowers, and starfish.



- **Bilateral symmetry** - same on both sides (like a human face).
- **Radial symmetry** - arranged around a center (like a daisy or jellyfish).



Daisy

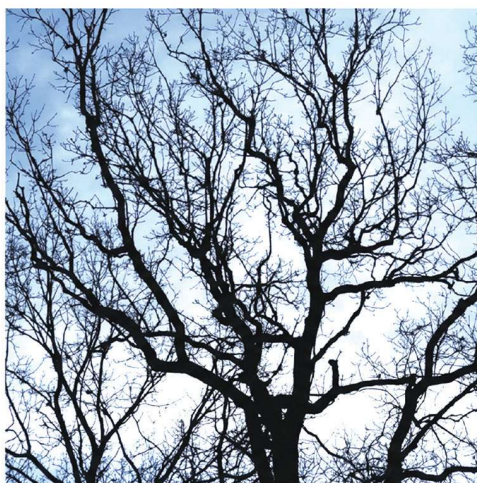


Jellyfish

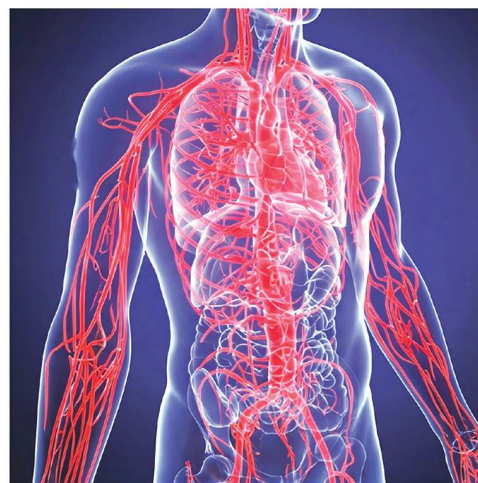
c) Fractals

These are some examples for Fractals.

- Patterns that repeat at different scales - "a pattern within a pattern."
- Found in snowflakes, fern leaves, lightning, and river networks.
- **Example** : The branching of trees or blood vessels.



Branching trees



Blood vessels

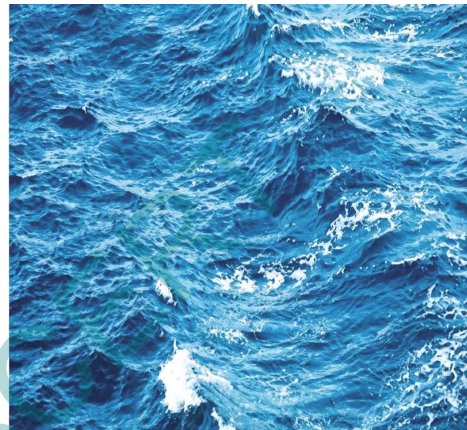
d) Waves and Ripples

Where we observe Waves and Ripples in Nature?

- Seen in sand dunes, ocean waves, and wind-blown patterns on snow or desert sand.
- These form due to the movement of air or water following physical laws.



Sand dunes



Ocean waves

2. Why Do Patterns Occur in Nature?

- Efficiency - Nature uses minimum energy to create maximum order.
- Growth rules - Cells and organisms grow following mathematical ratios.
- Adaptation - Patterns help living things survive (e.g., camouflage patterns in animals).

3. Connection to Mathematics

- Geometry explains shapes and symmetry.
- Algebra describes sequences and patterns like Fibonacci.
- Trigonometry models waves and spirals.
- Fractals connect mathematics with art and computer graphics.



Project Work

- Observe five natural objects like leaf, shell, flower, cloud, fruit, etc. Write your observations (for example kind of patterns viz. spiral, symmetry etc.). Relate them to a mathematical concept (for example, sequence, shape, ratio, etc.) in your project work. Draw neat diagrams and draw lines of symmetry for the objects.

CHAPTER MASTERY

1. Which of the following is Virahanka/Fibonacci sequence?

- A) 1,1,3,5,6, - - - B) 1,2,4,8, - - -
C) 1,2,3,5, - - - D) 1,3,9,27, - - -

2. Match the following:

- (i) square numbers a) 1,3,6,10, - - -
(ii) cube numbers b) 1,4,9,16, - - -
(iii) Triangular numbers c) 1,8,27,64, - - -
A) (i)-a, (ii)-b, (iii)-c B) (i)-b, (ii)-c, (iii)-a
C) (i)-a, (ii)-c, (iii)-b D) (i)-b,(ii)-a, (iii)-c

3. What is the next number in the sequence: 1,7,19,37, - - -

- A) 61 B) 63 C) 59 D) 47

4. Represent the third hexagonal number (19) through a diagram.

5. _____ is an example for both triangular and square number.

6. Assertion(A) : 1,4,9,16, - - - are square numbers

Reason(R) : By adding the counting numbers up and down we get square numbers.

- A) Both (A) and (R) are correct and (R) is the correct explanation of (A)
B) Both (A) and (R) are correct and (R) is not the correct explanation of (A)
C) (A) is correct and (R) is false.
D) (A) is false and (R) is correct.

7. What is a pattern?

8. Which mathematical sequence is often seen in natural patterns like sunflowers and pinecones?

9. How is the Fibonacci sequence connected to spirals in nature?
10. Why do honeycombs have a hexagonal pattern?
11. Explain how symmetry helps in nature.
12. Find the next three terms in the pattern: 2, 4, 8, 16, __, __, __
13. The arrangement of petals in a flower follows Fibonacci numbers. If a flower has 13 petals, predict the number of petals in the next in this pattern.
14. Observe these numbers: 1, 4, 9, 16, 25...
Identify the pattern and explain the rule.
15. Compare a fern leaf and a snowflake. What common pattern do they share?
16. Do you think patterns in nature can help in technological design? Give an example.
17. Explain the efficiency of nature of spirals in space usage?
18. Design a pattern using shapes that could represent growth in nature. Describe your rule.

SUMMARY

- Mathematics may be viewed as the search for patterns and for the explanations as to why those patterns exist.
- Among the most basic patterns that occur in mathematics are number sequences.
- Some important examples of number sequences include the counting numbers, odd numbers, even numbers, square numbers, triangular numbers, cube numbers, Virahānka numbers, and powers of 2.
- Sometimes number sequences can be related to each other in beautiful and remarkable ways. For example, adding up the sequence of odd numbers starting with 1 gives square numbers.
- Visualising number sequences using pictures can help to understand sequences and the relationships between them.
- **Shape sequences** are another basic type of pattern in mathematics. Some important examples of shape sequences include regular polygons, complete graphs, stacked triangles and squares, and Koch snowflake iterations. Shape sequences also exhibit many interesting relationships with number sequences.
- In nature, patterns appear in plants, animals, shells, rivers, clouds, and even weather.

2

LINES AND ANGLES



Learning Outcomes:

Learners will be able to

- Identifies and draws points, line segments, rays, and lines; recognizes intersecting, parallel, and perpendicular lines. (CG-3)
- Classifies angles as acute, right, obtuse, straight, reflex, and complete; measures and draws angles using a protractor. (CG3)
- Identify clockwise and anticlockwise rotations and recognize different types of angles in figures and real-life situations. (CG-3)
- Distinguish between a line and a line segment and understand that exactly one line can pass through two distinct point. (CG-6)
- Verify constructed angles using measuring tools and classify angles as acute, right, obtuse, straight, or reflex based on their measures.. (CG-3)

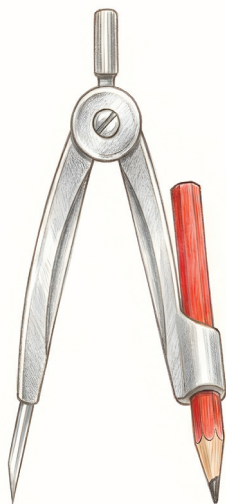
2.0 Introduction

In our daily life we see lines everywhere, like roads, wires, and notebook edges. A line is a straight path that goes on forever in both directions. A line segment has two fixed ends, like a ruler or pencil. A ray starts at one point and goes on in one direction, like sunlight. When two lines or rays meet at a point, they form an angle. Angles can be seen at corners of tables, doors, and walls. The hands of a clock form different angles as they move. Lines and angles are present in nature, like tree branches, in roads and crossings in daily life. These shapes help us understand direction and position. We can learn geometry by observing things around us. Drawing and exploring helps us understand better.

In this chapter, we will explore some of the most basic ideas of geometry including points, line segments, lines, rays, and angles. These ideas form the building blocks of 'plane geometry', and will help us in understanding more advanced topics in geometry such as the construction and analysis of different shapes.

2.1 Point

Mark a dot on the paper with a sharp tip of a pencil. The sharper the tip, the thinner will be the dot. This tiny dot will give you an idea of a point. A point determines a precise location, but it has no length, breadth or height. Some models for a point are given below.



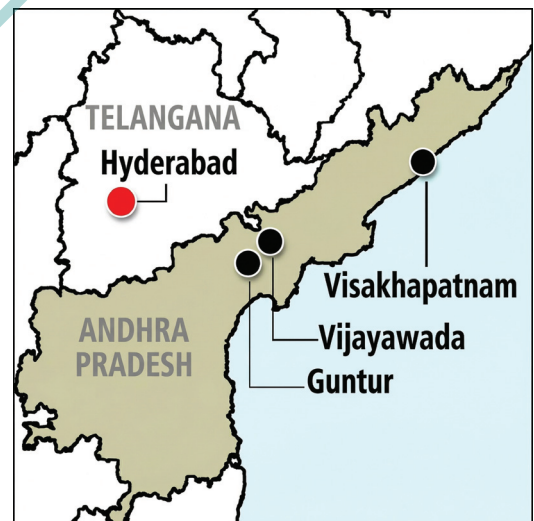
The tip of a compass



The sharpened end of a pencil



The pointed end of a needle



Main cities in Andhra Pradesh Map

If you mark three points on a piece of paper, you may be required to distinguish these three points. For this purpose, each of the three points may be denoted by a single capital letter such as



Z, P and T. These points are read as 'Point Z', 'Point P' and 'Point T'. Of course, the dots represent precise locations and must be imagined to be invisibly thin.

2.2 Line Segment

Fold a piece of paper and unfold it. Do you see a crease? This gives the idea of a line segment. It has two end points, A and B.

Mark any two points A and B on a sheet of paper. Try to connect A to B by various routes (Fig. 2.1).

What is the shortest route from A to B? This shortest path from point A to Point B (including A and B) as shown here is called the line segment from A to B. It is denoted by either \overline{AB} or \overline{BA} . The points A and B are called the end points of the line segment \overline{AB} .

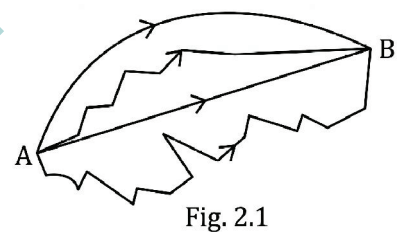
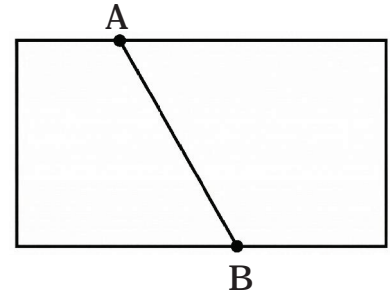


Fig. 2.1

2.3 Line

Imagine that the line segment from A to B (i.e., \overline{AB}) is extended beyond A in one direction and beyond B in the other direction without any end (see Fig. 2.2). This is a model for a line. Do you think you can draw a complete picture of a line? No. Why?

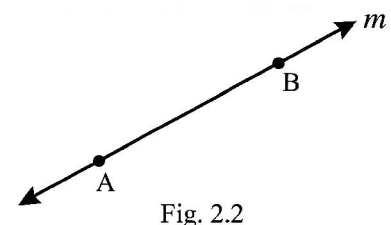


Fig. 2.2

A line through two points A and B is written as \overleftrightarrow{AB} . It extends forever in both directions. Sometimes a line is denoted by a letter like l or m .

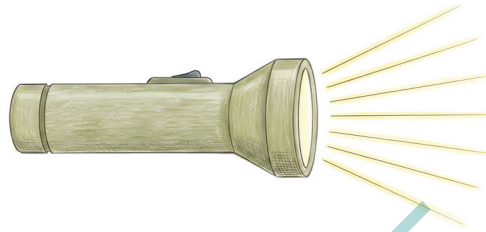
Observe (Fig. 2.1 and Fig.2.2) that any two points determine a unique line that passes through both of them.

2.4 Ray

A ray is a portion of a line that starts at one point (called the starting point or initial point of the ray) and goes on endlessly in a direction. The following are some models for a ray:



Beam of light from a lighthouse



Ray of light from a torch



Sun rays

Look at the diagram (Fig. 2.3) of a ray. Two points are marked on it. One is the starting point A and the other is a point P on the path of the ray. We then denote the ray by \overrightarrow{AP} .

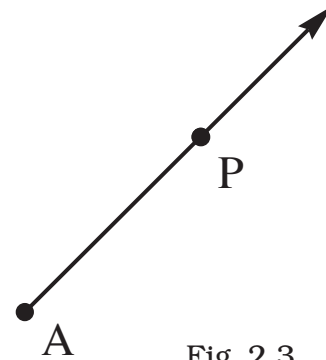


Fig. 2.3



Figure it Out - 2.1

1.

Ramu marked a point on a piece of paper. How many lines can he draw that pass through the point?

Venkat marked two points on a piece of paper. How many different lines can he draw that pass through both of the points?

Can you help Ramu and Venkat find their answers?

2. Name the line segments in Fig. 2.4. Which of the five marked points are on exactly one of the line segments? Which are on two of the line segments?

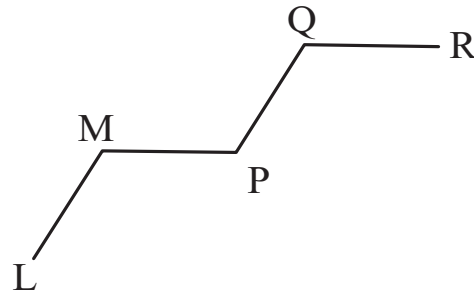


Fig. 2.4.

3. Name the rays shown in Fig. 2.5. Is T the starting point of each of these rays?

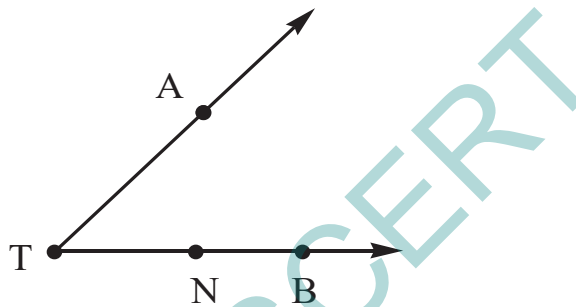


Fig. 2.5

4. Draw a rough figure and write labels appropriately to illustrate each of the following:
- \overleftrightarrow{OP} and \overleftrightarrow{OQ} meet at O.
 - \overleftrightarrow{XY} and \overleftrightarrow{PQ} intersect at point M.
 - Line l contains points E and F but not point D.
 - Point P lies on AB.

5. In Fig. 2.6, name:

- Five points
- A line
- Four rays
- Five line segments

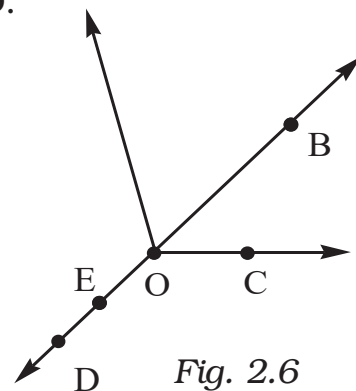


Fig. 2.6

6. Here is a ray \overrightarrow{OA} (Fig. 2.7). It starts at O and passes through the point A. It also passes through the point B.

a. Can you also name it as \overrightarrow{OB} ? Why?

b. Can we write \overrightarrow{OA} as \overrightarrow{AO} ? Why or why not?

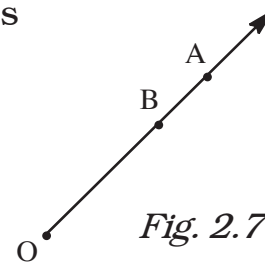


Fig. 2.7

2.5 Angle

An angle is formed by two rays having a common starting point. Here is an angle formed by rays \overrightarrow{BD} and \overrightarrow{BE} where B is the common starting point (Fig. 2.8).

The point B is called the vertex of the angle, and the rays \overrightarrow{BD} and \overrightarrow{BE} are called the arms of the angle. How can

we name this angle? We can simply use the vertex and say that it is the Angle B. To be clearer, we use a point on each of the arms together with the vertex to name the angle. In this case, we name the angle as Angle DBE or Angle EBD. The word angle can be replaced by the symbol ' \angle ', i.e., $\angle DBE$ or $\angle EBD$. Note that in specifying the angle, the vertex is always written as the middle letter.

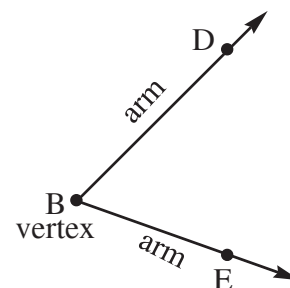


Fig. 2.8

To indicate an angle, we use a small curve at the vertex (refer to Fig. 2.9).

The size of an angle is the amount of rotation or turn that is needed about the vertex to move the first ray to the second ray.

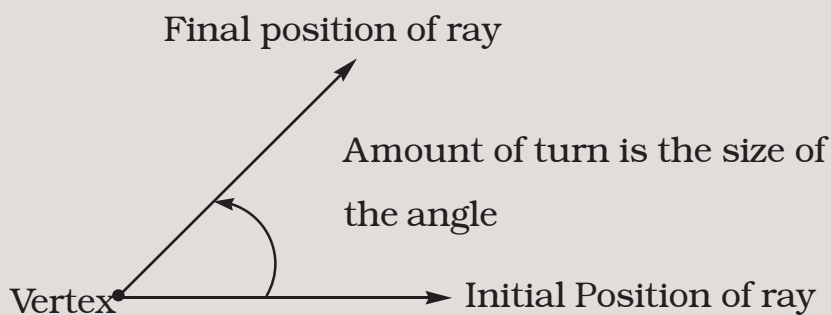
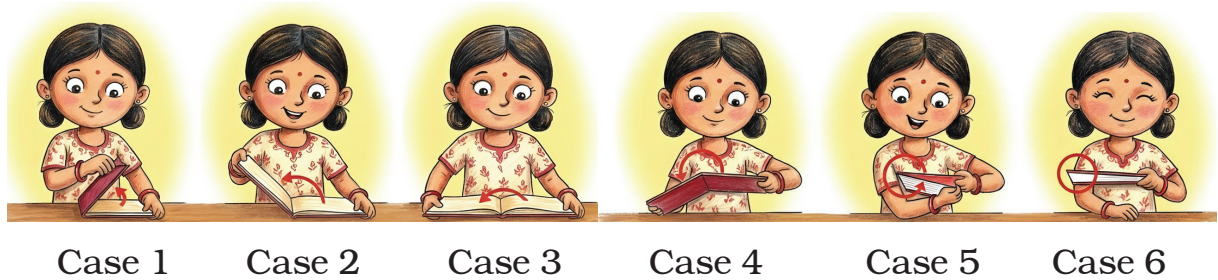


Fig. 2.9

Vidya has just opened her book. Let us observe her opening the cover of the book in different scenarios.



- ❁ Do you see angles being made in each of these cases? Can you mark their arms and vertex?

Which angle is greater—the angle in Case 1 or the angle in Case 2? Just as we talk about the size of a line based on its length, we also talk about the size of an angle based on its amount of rotation.

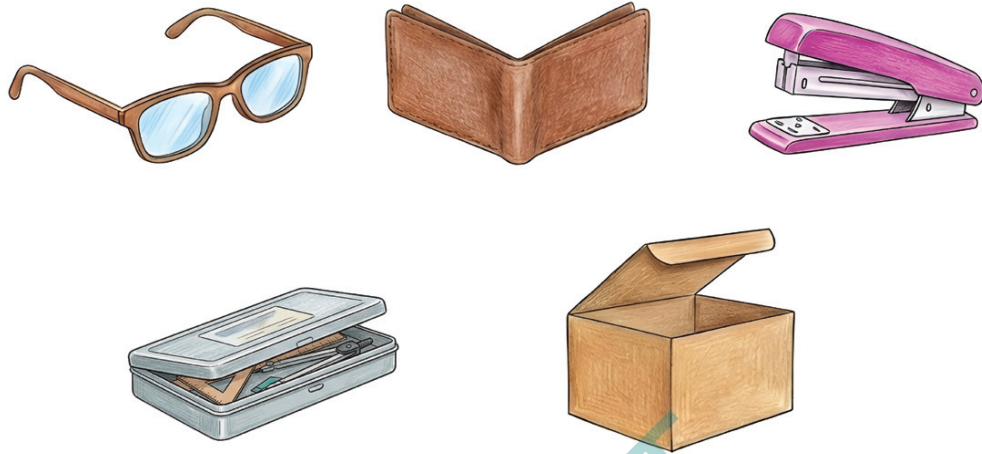
So, the angle in Case 2 is greater as in this case she needs to rotate the cover more. Similarly, the angle in Case 3 is even larger than that of Case 2, because there is even more rotation, and Cases 4, 5, and 6 are successively larger angles with more and more rotation.

Let's look at some other examples where angles arise in real life by rotation or turn:

- In a compass or divider, we turn the arms to form an angle. The vertex is the point where the two arms are joined. Identify the arms and vertex of the angle.
- A pair of scissors has two blades. When we open them (or 'turn them') to cut something, the blades form an angle. Identify the arms and the vertex of the angle.



- Look at the pictures of spectacles, wallet and other common objects. Identify the angles in them by marking out their arms and vertices.



Do you see how these angles are formed by turning one arm with respect to the other?

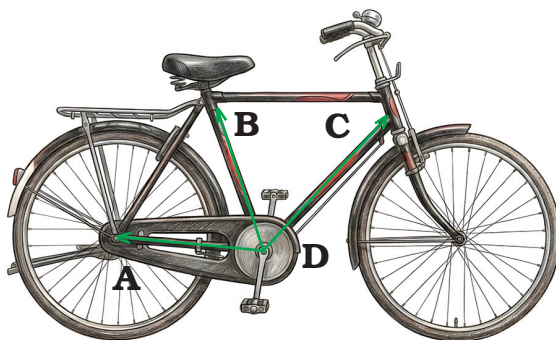
Teacher's Note

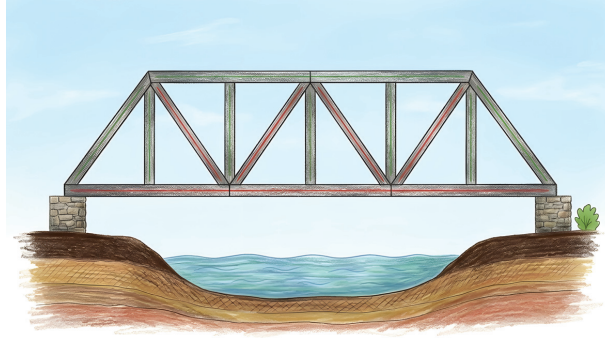
Teacher needs to organise various activities with the students to recognise the size of an angle as a measure of rotation.



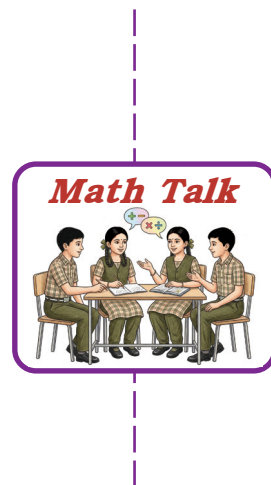
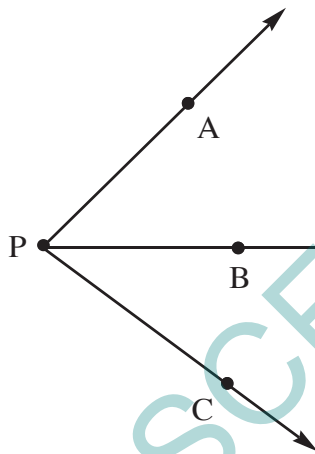
Figure it Out - 2.2

1. Can you find the angles in the given pictures? Draw the rays forming any one of the angles and name the vertex of the angle.

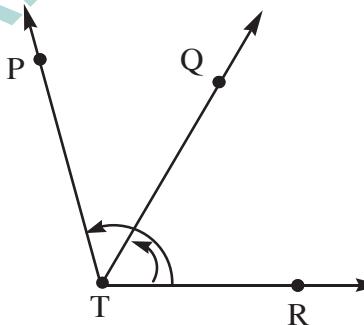




2. Draw and label an angle with arms ST and SR.
3. Explain why $\angle APC$ cannot be labelled as $\angle P$?



4. Name the angles marked in the given figure.

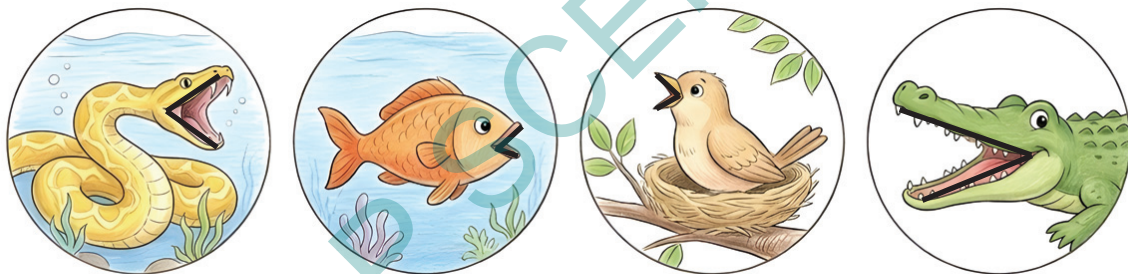


5. Mark any three points on your paper that are not on one line. Label them A, B, C. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C? Write them down, and mark each of them with a curve as in Fig. 2.9.

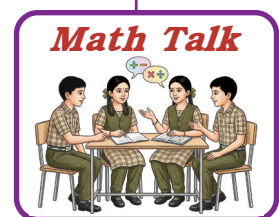
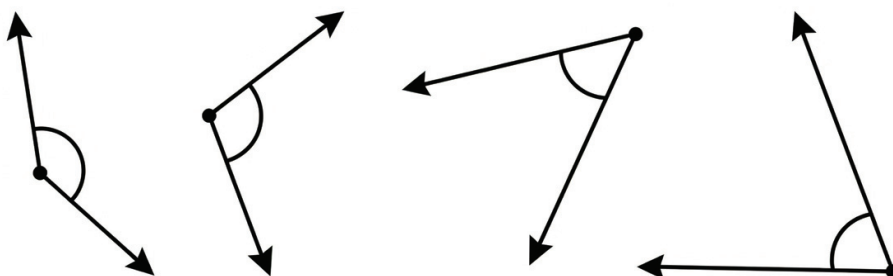
6. Now mark any four points on your paper so that no three of them are on one line. Label them A, B, C, D. Draw all possible lines going through pairs of these points. How many lines do you get? Name them. How many angles can you name using A, B, C, D? Write them all down, and mark each of them with a curve as in Fig. 2.9.

2.6 Comparing Angles

Look at these animals opening their mouths. Do you see any angles here? If yes, mark the arms and vertex of each one. Some mouths are open wider than others; the more the turning of the jaws, the larger the angle! Can you arrange the angles in this picture from smallest to largest?



❁ Is it always easy to compare two angles?

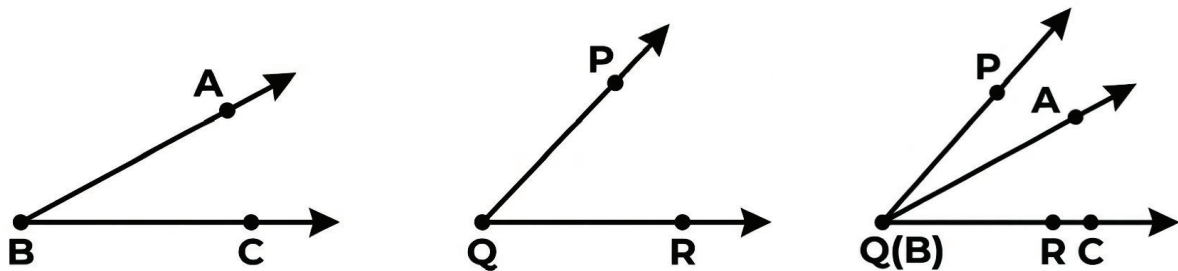


Here are some angles. Label each of the angles. How will you compare them?

Draw a few more angles; label them and compare.

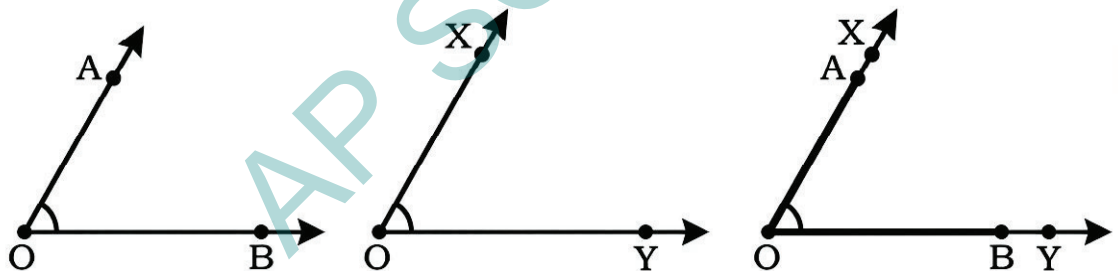
Comparing angles by superimposition

Any two angles can be compared by placing them one over the other, i.e., by superimposition. While superimposing, the vertices of the angles must overlap. After superimposition, it becomes clear which angle is smaller and which is larger.



The picture shows the two angles superimposed. It is now clear that $\angle PQR$ is larger than $\angle ABC$.

Equal angles. Now consider $\angle AOB$ and $\angle XOY$ in the figure. Which is greater?



The corners of both of these angles match and the arms overlap with each other, i.e., $OA \leftrightarrow OX$ and $OB \leftrightarrow OY$. So, the angles are equal in size.

The reason these angles are considered to be equal in size is because when we visualise each of these angles as being formed out of rotation, we can see that there is an equal amount of rotation needed to move \vec{OB} to \vec{OA} and \vec{OY} to \vec{OX} .

From the point of view of superimposition, when two angles are superimposed, and the common vertex and the two rays of both angles lie on top of each other, then the sizes of the angles are equal.

❁ Where else do we use superimposition to compare?

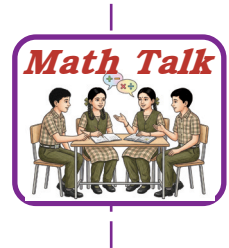
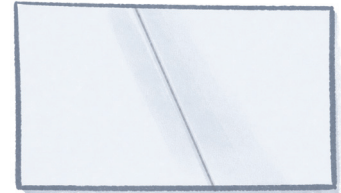


Figure it Out - 2.3

1. Fold a rectangular sheet of paper, then draw a line along the fold created. Name and compare the angles formed between the fold and the sides of the paper. Make different angles by folding a rectangular sheet of paper and compare the angles. Which is the largest and smallest angle you made?



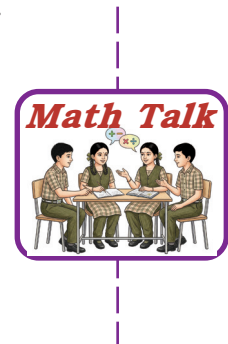
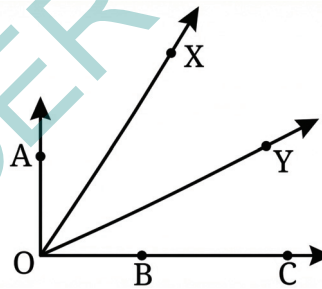
2. In each case, determine which angle is greater and why.

a. $\angle AOB$ or $\angle XOY$

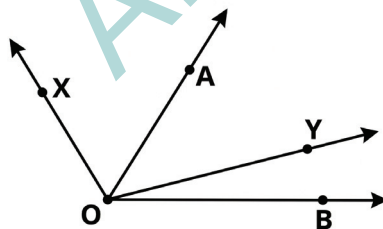
b. $\angle AOB$ or $\angle XOB$

c. $\angle XOB$ or $\angle XOC$

Discuss with your friends on how you decided which one is greater.



3. Which angle is greater: $\angle XOY$ or $\angle AOB$? Give reasons.



Comparing angles without superimposition

Two cranes are arguing about who can open their mouth wider, i.e., who is making a bigger angle.

Let us first draw their angles. How do we know which one is bigger? As seen before,

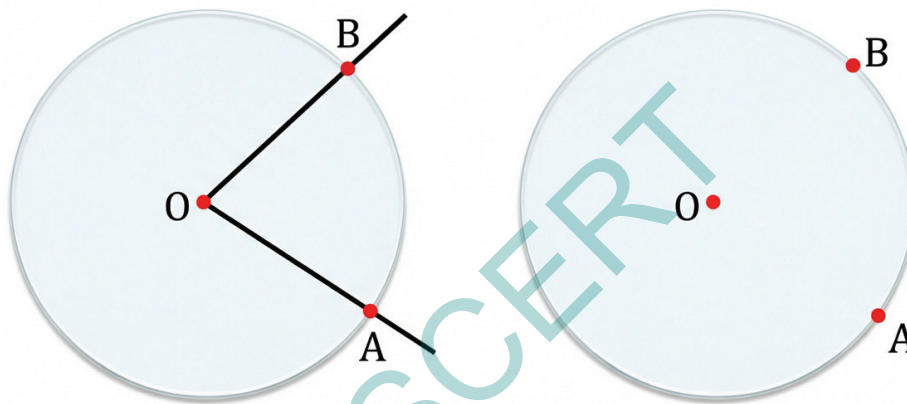


Fig. 2.10

one could trace these angles, superimpose them and then check. But can we do it without superimposition?

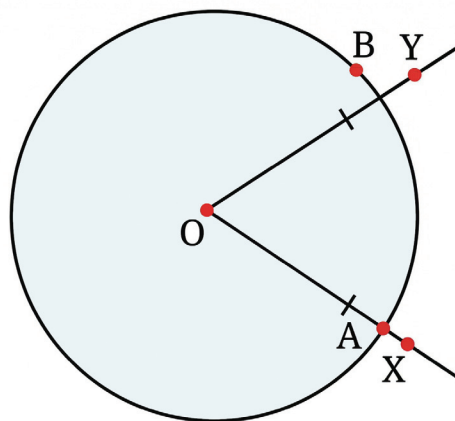
Suppose we have a transparent circle which can be moved and placed on figures. Can we use this for comparison?

Let us place the circular paper on the angle made by the first crane. The circle is placed in such a way that its centre is on the vertex of the angle. Let us mark the points A and B on the edge circle at the points where the arms of the angle pass through the circle.



Can we use this to find out if this angle is greater than, or equal to or smaller than the angle made by the second crane?

Let us place it on the angle made by the second crane so that the vertex coincides with the centre of the circle and one of the arms passes through OA.



Can you now tell which angle is bigger?

Which crane was making the bigger angle?

If you can make a circular piece of transparent paper, try this method to compare the angles in Fig. 2.10 with each other.

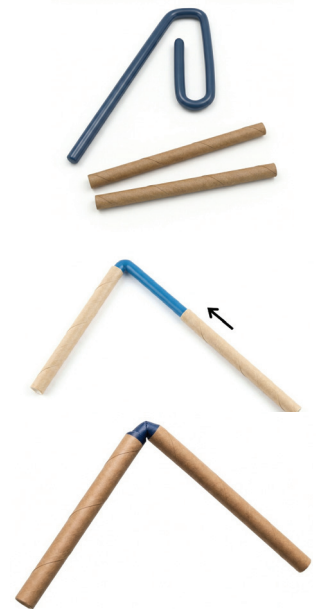
Teacher's Note

A teacher needs to check the understanding of the students around the notion of an angle. Sometimes students might think that increasing the length of the arms of the angle increases the angle. For this, various situations should be posed to the students to check their understanding on the same.

2.7 Making Rotating Arms

Let us make 'rotating arms' using two paper straws and a paper clip by following these steps:

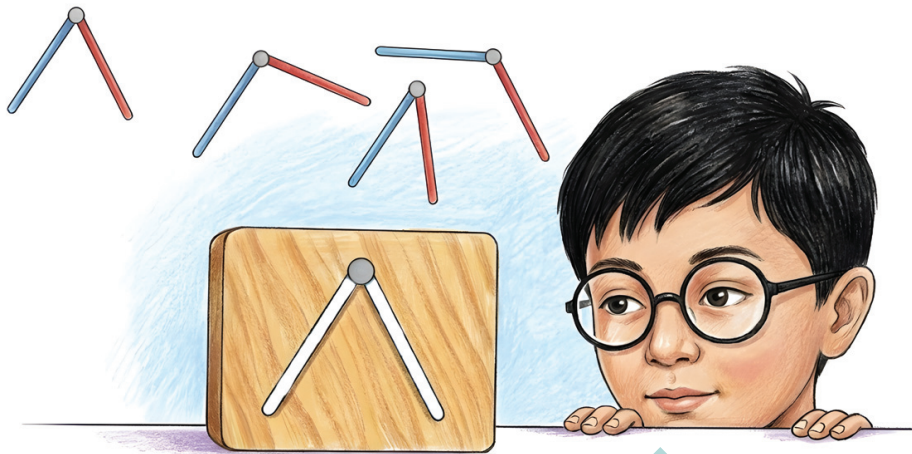
1. Take two paper straws and a paper clip.
2. Insert the straws into the arms of the paper clip.
3. Your rotating arm is ready!



Make several 'rotating arms' with different angles between the arms. Arrange the angles you have made from smallest to largest by comparing and using superimposition.

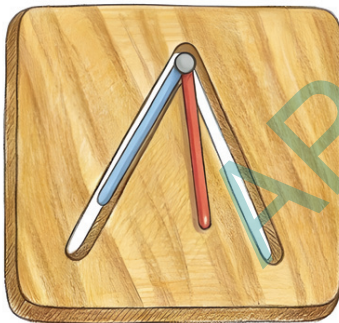
Passing through a slit: Collect a number of rotating arms with different angles; do not rotate any of the rotating arms during this activity.

Take a cardboard and make an angle-shaped slit as shown below by tracing and cutting out the shape of one of the rotating arms.

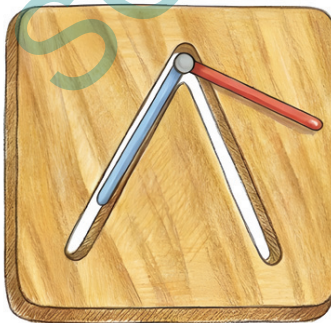


Now, shuffle and mix up all the rotating arms. Can you identify which of the rotating arms will pass through the slit?

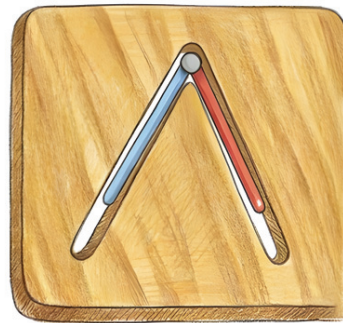
The correct one can be found by placing each of the rotating arms over the slit. Let us do this for some of the rotating arms:



Slit angle is greater than the arms' angle. The arms will not go through the slit.

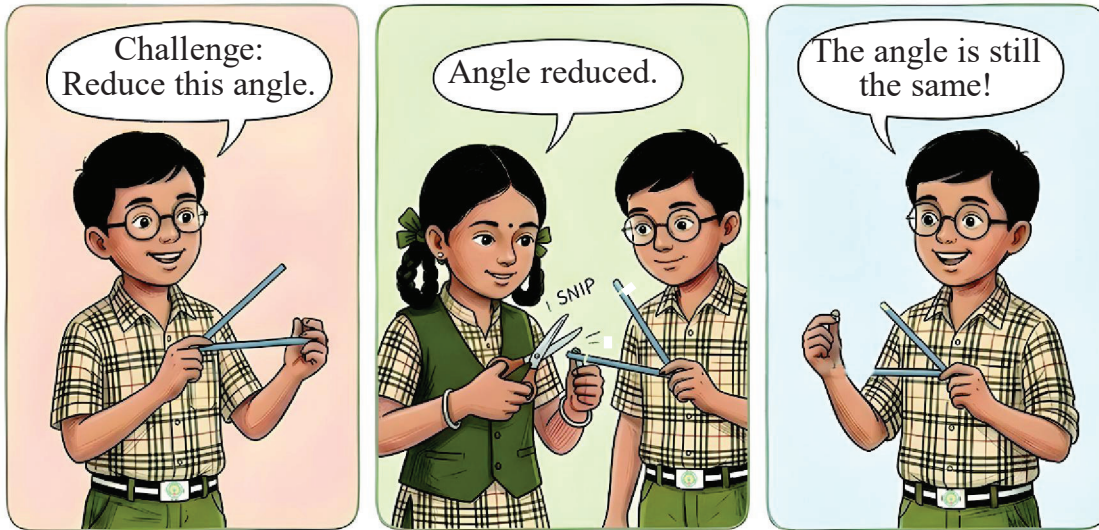


Slit angle is less than the arms' angle. The arms will not go through the slit.



Slit angle is equal to the arms' angle. The arms will go through the slit.

Only the pair of rotating arms where the angle is equal to that of the slit passes through the slit. Note that the possibility of passing through the slit depends only on the angle between the rotating arms and not on their lengths (as long as they are shorter than the length of the slit).



2.8 Special Types of Angles

Let us go back to Vidya's notebook and observe her opening the cover of the book in different scenarios.

She makes a full turn of the cover when she has to write while holding the book in her hand.



She makes a half turn of the cover when she has to open it on her table. In this case, observe the arms of the angle formed. They lie in a straight line. Such an angle is called a straight angle.

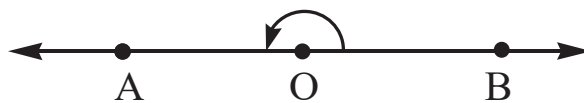
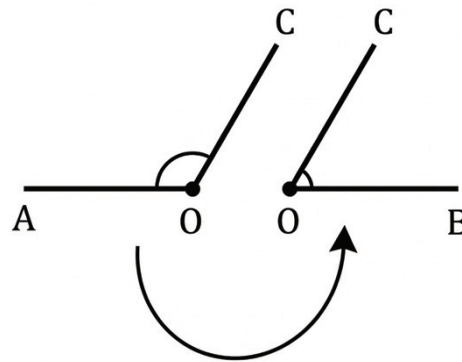
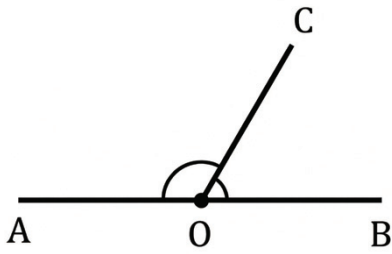
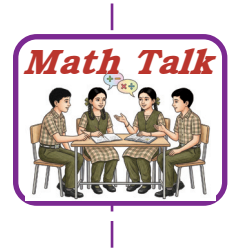


Fig. 2.11

Let us consider a straight angle $\angle AOB$. Observe that any ray \vec{OC} divides it into two angles, $\angle AOC$ and $\angle COB$.



❁ Is it possible to draw \vec{OC} such that the two angles are equal to each other in size?

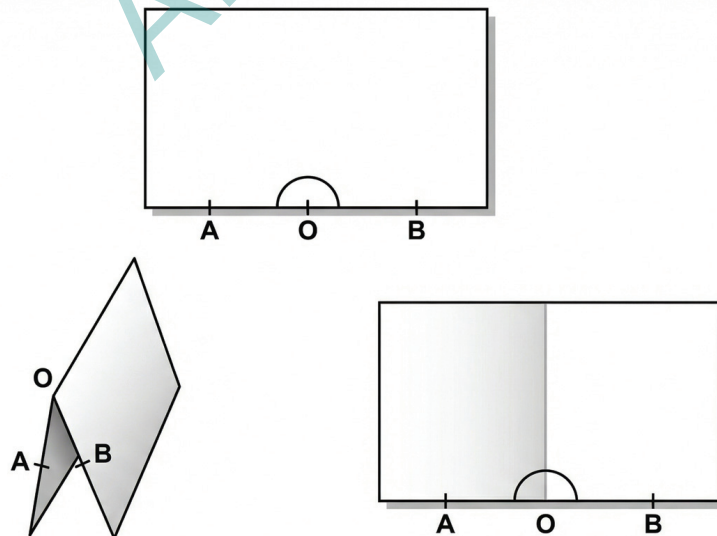


Let's Explore

We can try to solve this problem using a piece of paper. Recall that when a fold is made, it creates a crease which is straight.

Take a rectangular piece of paper and on one of its sides, mark the straight angle AOB. By folding, try to get a line (crease) passing through O that divides $\angle AOB$ into two equal angles.

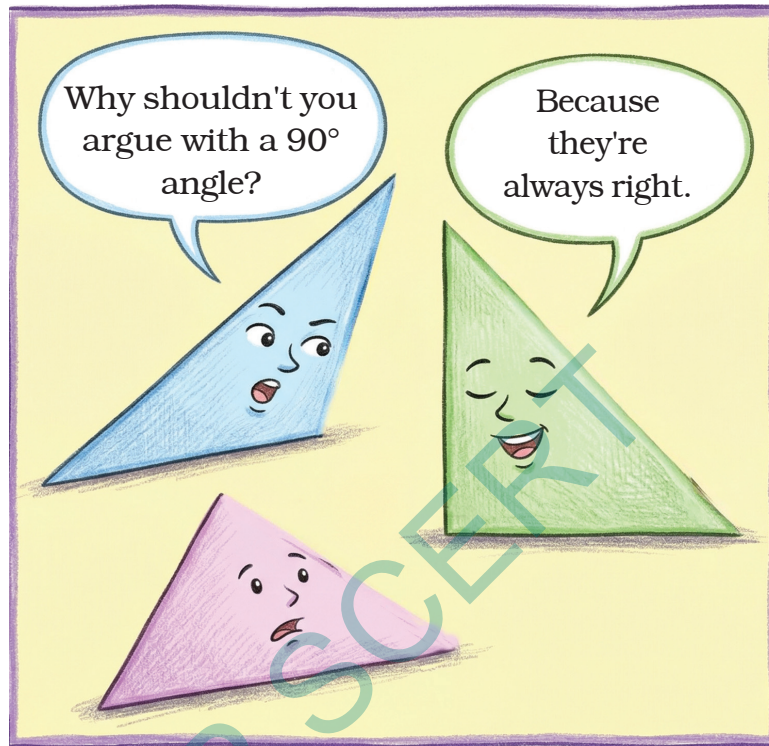
How can it be done?



Fold the paper such that OB overlaps with OA. Observe the crease and the two angles formed.

Justify why the two angles are equal. Is there a way to superimpose and check? Can this superimposition be done by folding?

Each of these equal angles formed are called right angles. So, a straight angle contains two **right angles**.



- ❁ If a straight angle is formed by half of a full turn, how much of a full turn will form a right angle?

Observe that a right angle resembles the shape of an 'L'. An angle is a right angle only if it is exactly half of a straight angle. Two lines that meet at right angles are called **perpendicular lines**. Observe in the figure, the divider lines on the cross roads are perpendicular.

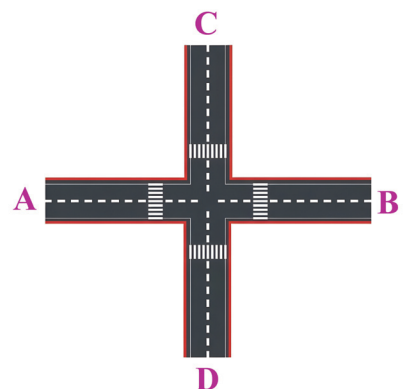
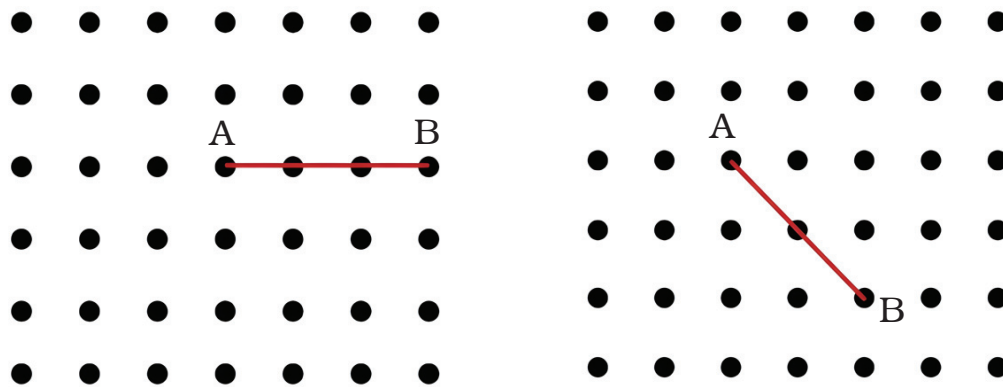


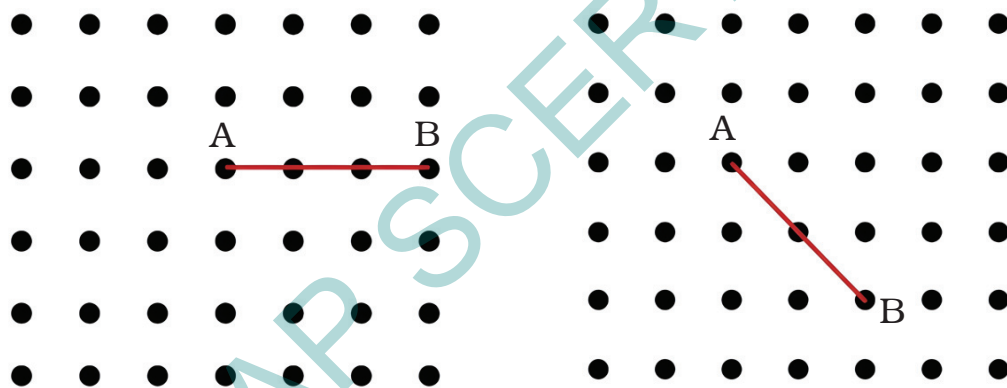
Figure it Out - 2.4

1. How many right angles do the windows of your classroom contain? Do you see other right angles in your classroom?

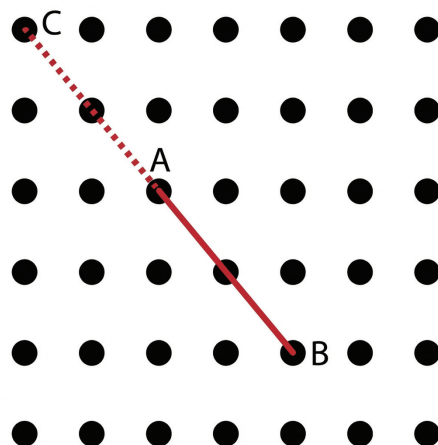
2. Join A to other grid points in the figure by a straight line to get a straight angle. What are all the different ways of doing it?



3. Now join A to other grid points in the figure by a straight line to get a right angle. What are all the different ways of doing it?



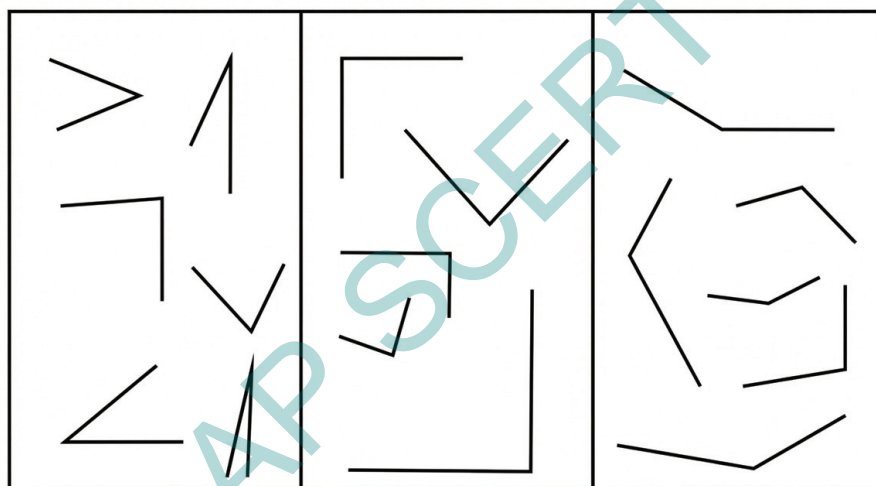
Hint: Extend the line further as shown in the figure below. To get a right angle at A, we need to draw a line through it that divides the straight angle CAB into two equal parts.



4. Get a slanting crease on the paper. Now, try to get another crease that is perpendicular to the slanting crease.
- How many right angles do you have now? Justify why the angles are exact right angles.
 - Describe how you folded the paper so that any other person who doesn't know the process can simply follow your description to get the right angle.

Classifying Angles

Angles are classified in three groups as shown below. Right angles are shown in the second group. What could be the common feature of the other two groups?



In the first group, all angles are less than a right angle or in other words, less than a quarter turn. Such angles are called acute angles.

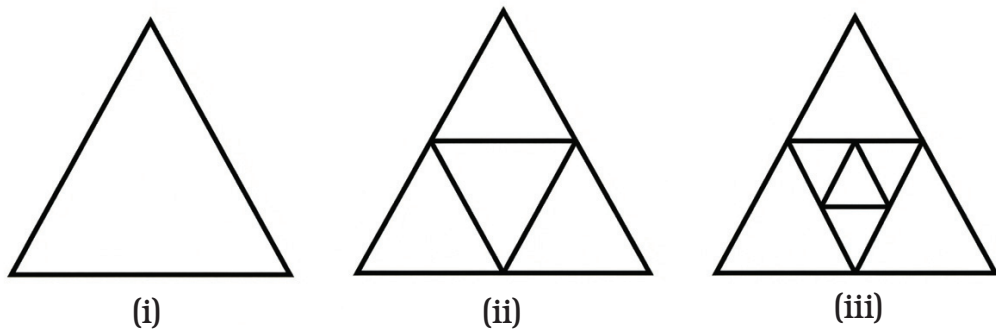
In the third group, all angles are greater than a right angle but less than a straight angle. The turning is more than a quarter turn and less than a half turn. Such angles are called obtuse angles.



Figure it Out - 2.5

- Identify acute, right, obtuse and straight angles in the previous figures.
- Make a few acute angles and a few obtuse angles. Draw them in different orientations.

3. Do you know what the words acute and obtuse mean? Acute means sharp and obtuse means blunt. Why do you think these words have been chosen?
4. Find out the number of acute angles in each of the figures below.



What will be the next figure and how many acute angles will it have? Do you notice any pattern in the numbers?

2.9 Measuring Angles

We have seen how to compare two angles. But can we actually quantify how big an angle is using a number without having to compare it to another angle?

We saw how various angles can be compared using a circle. Perhaps a circle could be used to assign measures for angles?

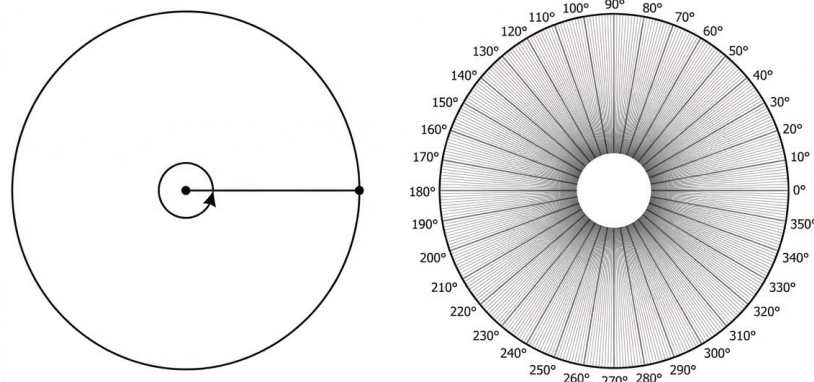
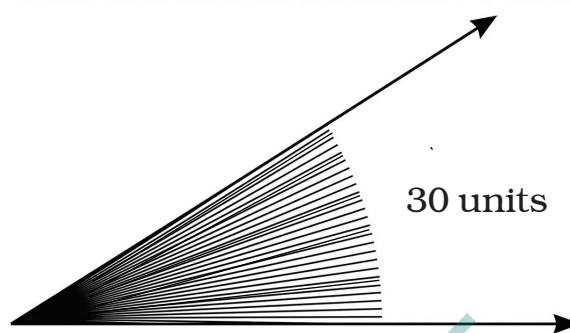


Fig. 2.12

To assign precise measures to angles, mathematicians came up with an idea. They divided the angle in the centre of the circle into

360 equal angles or parts. The angle measure of each of these unit parts is 1 degree, which is written as 1° .

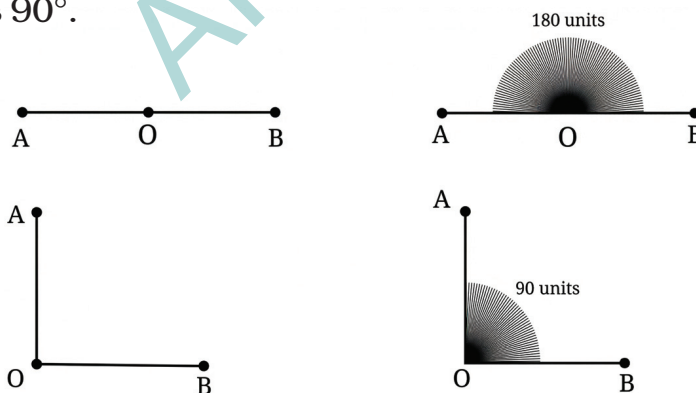
This unit part is used to assign measure to any angle: the measure of an angle is the number of 1° unit parts it contains inside it. For example, see this figure:



It contains 30 units of 1° angle and so we say that its angle measure is 30° .

Measures of different angles: What is the measure of a full turn in degrees? As we have taken it to contain 360 degrees, its measure is 360° .

- What is the measure of a straight angle in degrees? A straight angle is half of a full turn. As a full turn is 360° , a half turn is 180° . What is the measure of a right angle in degrees? Two right angles together form a straight angle. As a straight angle measures 180° , a right angle measures 90° .

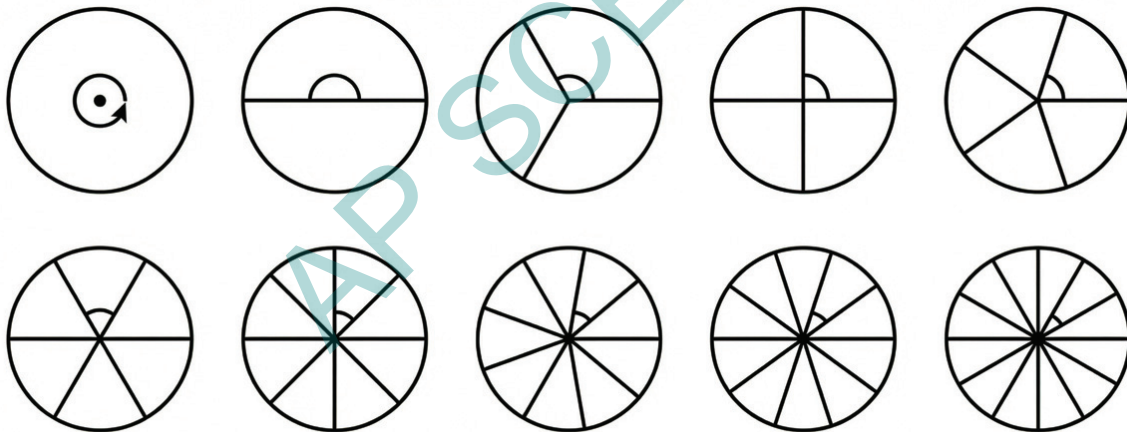


A pinch of history

A full turn or circle has 360 degrees. Why 360° was chosen? The Rigveda the very oldest text speaks of a wheel with 360 spokes. Ancient calendars of India, Persia, Babylonia, and Egypt had 360 days in a year.

Babylonians used the number 60 in their counting, and also 360. Mathematicians continued using 360 because it is the smallest number that can be evenly divided by all numbers up to 10, aside from 7. Note that 360 is also evenly divisible by 12, the number of months in a year, and by 24, the number of hours in a day. This makes it easy to divide a circle into equal parts. Each part is called a degree.

- ❁ The circle has been divided into 1, 2, 3, 4, 5, 6, 8, 9, 10 and 12 parts below. What are the degree measures of the resulting angles? Write the degree measures down near the indicated angles.

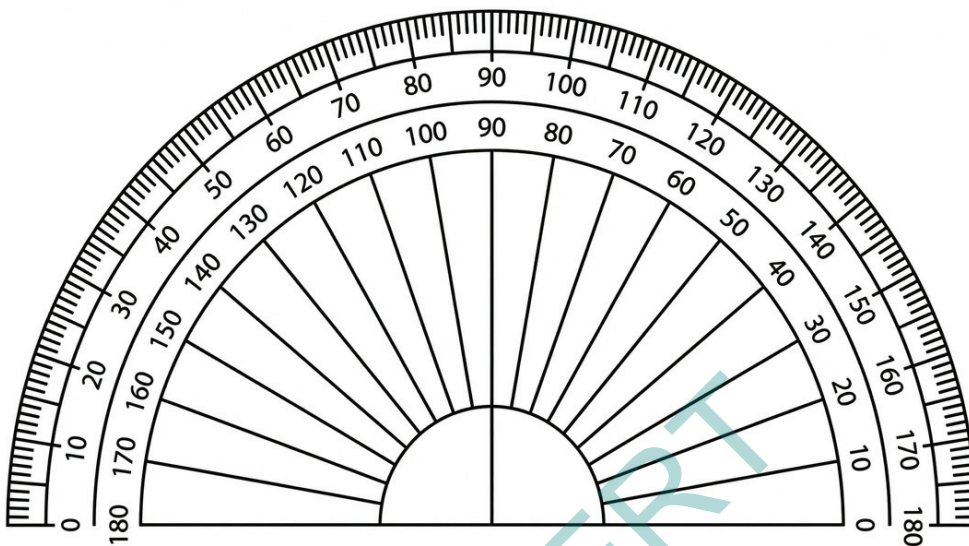


Degree measures of different angles

How can we measure other angles in degrees? It is for this purpose that we have a tool called a protractor that is either a circle divided into 360 equal parts as shown in Fig. 2.12 (on page 32), or a half circle divided into 180 equal parts.

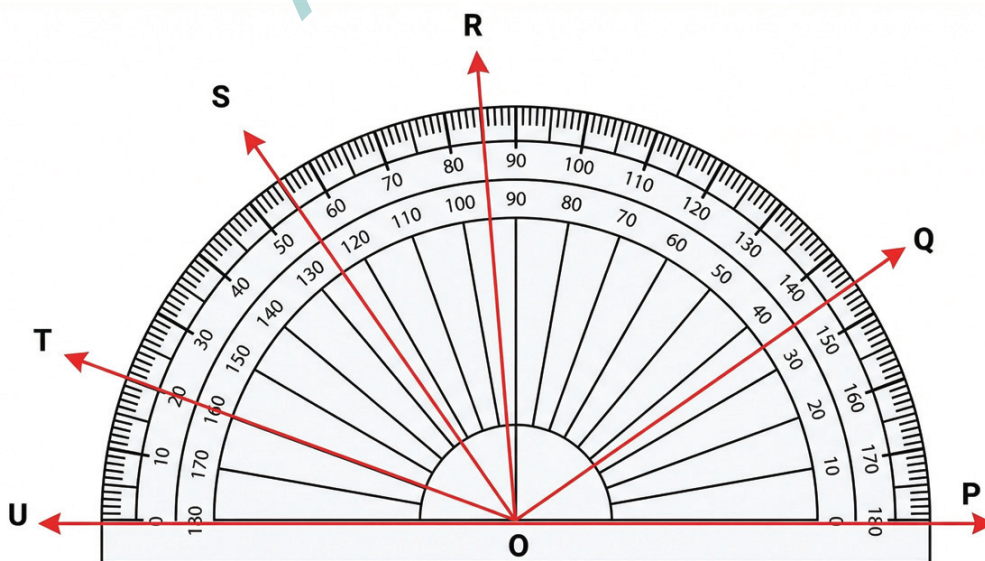
Protractor

This is a protractor that you find in your geometry box. It is used to measure the angles.



There are two sets of numbers on the protractor: one increasing from right to left and the other increasing from left to right. Why does it include two sets of numbers?

- ✿ Name the different angles in the figure and write their measures.



Did you include angles such as $\angle TOQ$?

Which set of markings did you use — inner or outer?

What is the measure of $\angle TOS$?

Can you use the numbers marked to find the angle without counting the number of markings?

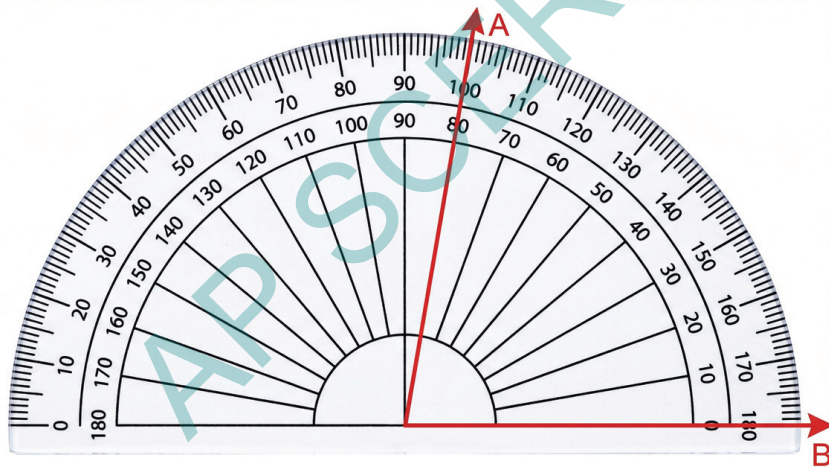
Here, OT and OS pass through the numbers 20 and 55 on the outer scale. How many units of 1 degree are contained between these two arms?

Can subtraction be used here?

How can we measure angles directly without having to subtract?

Place the protractor so the center is on the vertex of the angle.

Align the protractor so that one of the arms passes through the 0° mark as in the picture below.



What is the degree measure of $\angle AOB$?

Make your own Protractor!

You may have wondered how the different equally spaced markings are made on a protractor. We will now see how we can make some of them!

1. Draw a circle of a convenient radius on a sheet of paper. Cut out the circle (Fig. 2.13). A circle or one full turn is 360° .
2. Fold the circle to get two equal halves and cut it through the crease to get a semicircle. Write '0' in the bottom right corner of the semicircle.

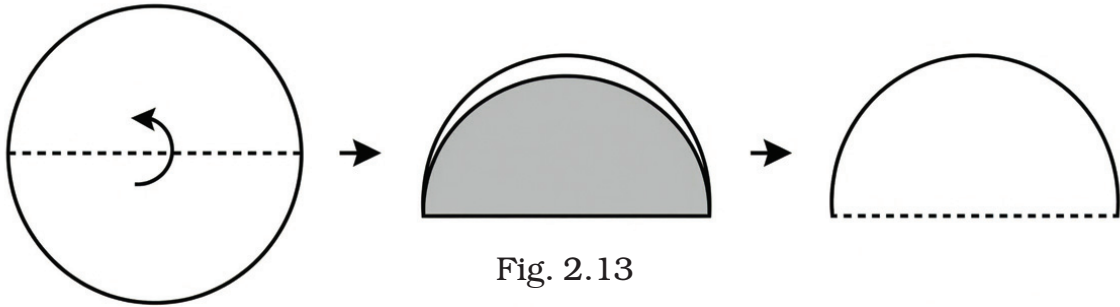


Fig. 2.13

<p>Fig. 2.14</p>	<p>The measure of half a circle is $\frac{1}{2}$ of a full turn. (Fig. 2.14)</p> <p>So, the measure of half a turn = $\frac{1}{2} \times \underline{\hspace{1cm}} = 180^\circ$.</p> <p>Thus, write 180° in the left bottom corner of the semicircle.</p>	
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3. Fold the semi-circular sheet in half as shown in Fig. 2.15 to form a quarter circle.

<p>Fig. 2.15</p>	<p>The measure of a quarter circle is $\frac{1}{4}$ of a full turn.</p> <p>The measure of a $\frac{1}{4}$ turn = $\frac{1}{4}$ of $360^\circ = \underline{\hspace{1cm}}$.</p> <p>Or, the measure of a $\frac{1}{4}$ turn = $\frac{1}{2}$ of a half turn = $\frac{1}{2}$ of $180^\circ = \underline{\hspace{1cm}}$.</p> <p>Thus, mark 90° at the top of the semicircle.</p>	
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4. Fold the sheet again as shown in Figs. 2.16 and 2.17:

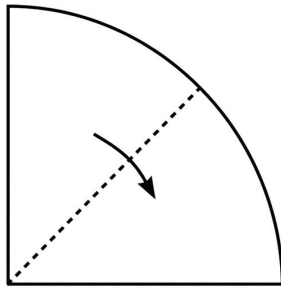


Fig. 2.16

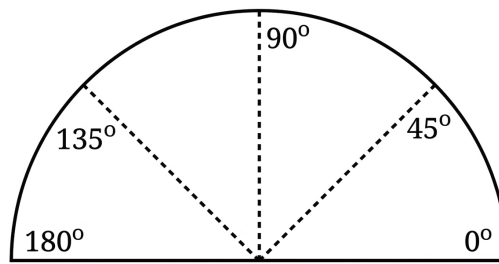


Fig. 2.17

When folded, this is $\frac{1}{8}$ of the circle, or $\frac{1}{8}$ of a turn, or $\frac{1}{8}$ of 360° , or $\frac{1}{4}$ of 180° or $\frac{1}{2}$ of $90^\circ =$ _____.

The new creases formed give us measures of 45° and $180^\circ - 45^\circ = 135^\circ$ as shown. Write 45° and 135° at the correct places on the new creases along the edge of the semicircle.

5. Continuing with another half fold as shown in Fig. 2.18, we get an angle of measure _____.

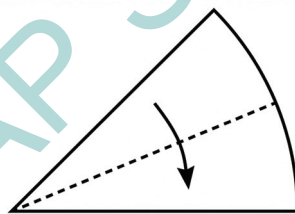


Fig. 2.18

6. Unfold and mark the creases as OB, OC, ..., etc., as shown in Fig. 2.19 and Fig. 2.20.

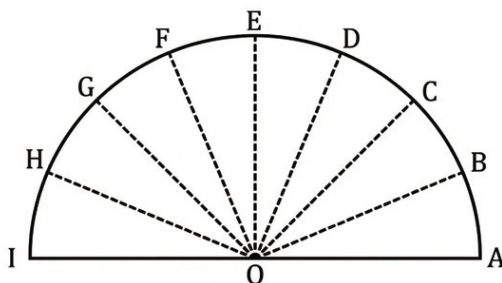


Fig. 2.19

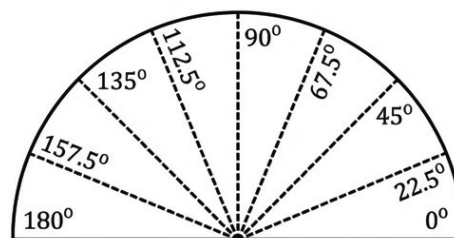


Fig. 2.20

✿ **Think!**

In Fig. 2.19, we have $\angle AOB = \angle BOC = \angle COD = \angle DOE = \angle EOF = \angle FOG = \angle GOH = \angle HOI = \underline{\hspace{2cm}}$. Why?

● **Angle Bisector** ●

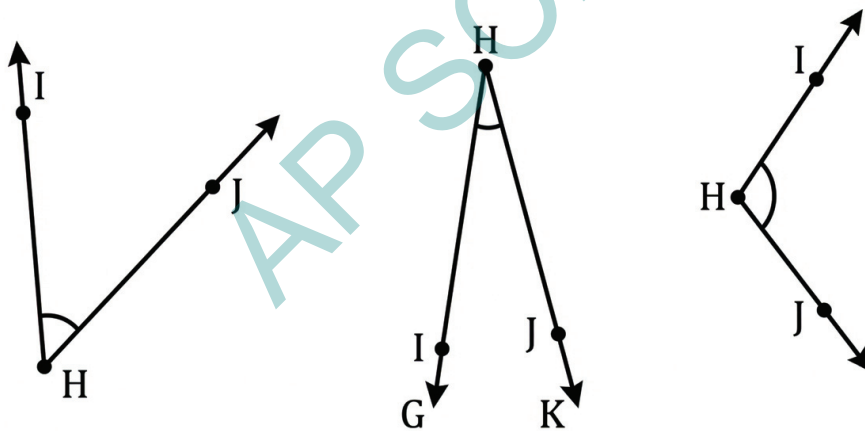
At each step, we folded in halves. This process of getting half of a given angle is called bisecting the angle. The line that bisects a given angle is called the angle bisector of the angle.

Identify the angle bisectors in your handmade protractor. Try to make different angles using the concept of angle bisector through paper folding.



Figure it Out - 2.6

1. Find the degree measures of the following angles using your protractor.

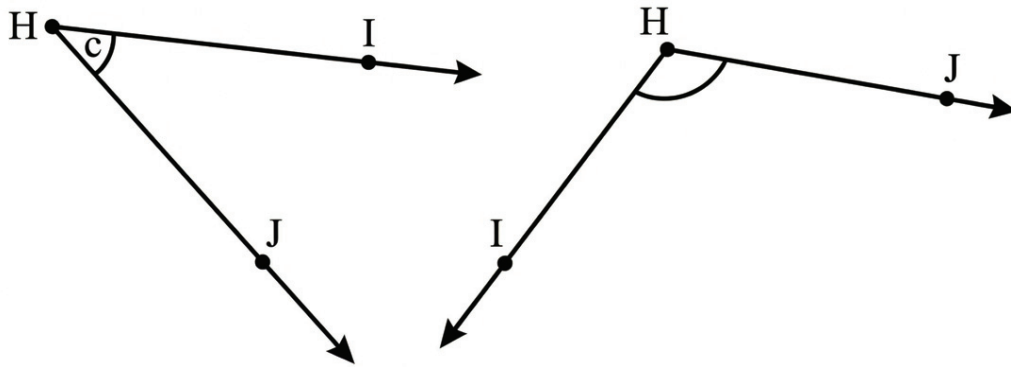


2. Find the degree measures of different angles in your classroom using your protractor.

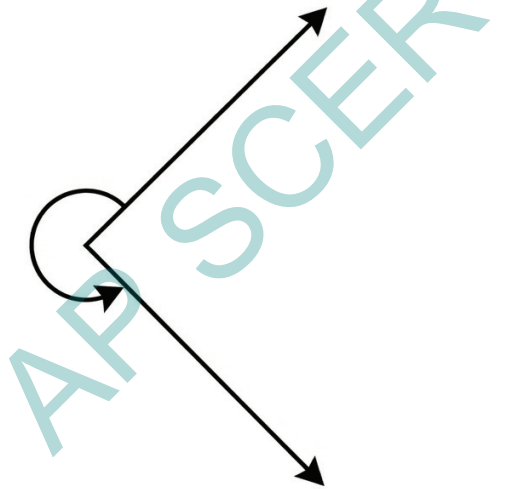
Teacher's Note

It is important that students make their own protractor and use it to measure different angles before using the standard protractor so that they know the concept behind the marking of the standard protractor.

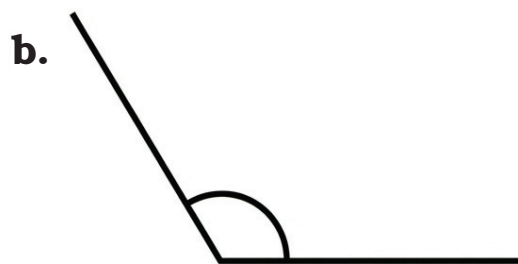
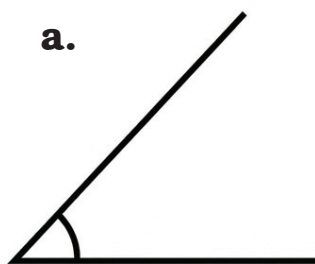
3. Find the degree measures for the $\angle IHJ$ given below. Check if your paper protractor can be used here!

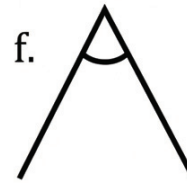
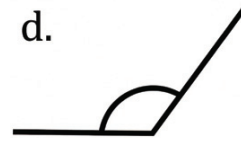
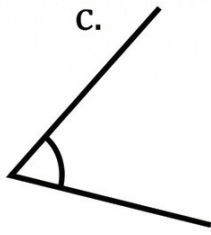


4. How can you find the degree measure of the angle given below using a protractor?

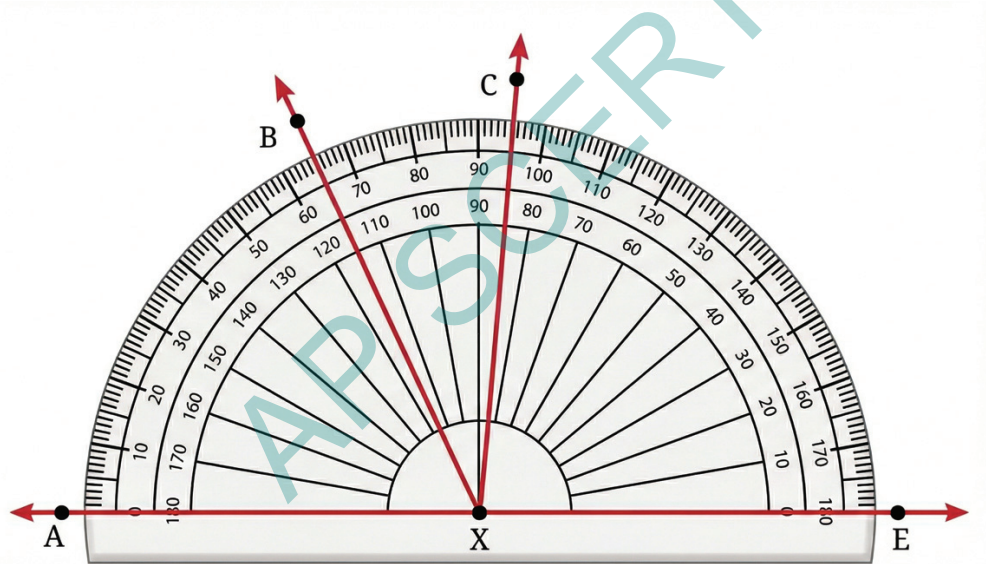


5. Measure and write the degree measures for each of the following angles:

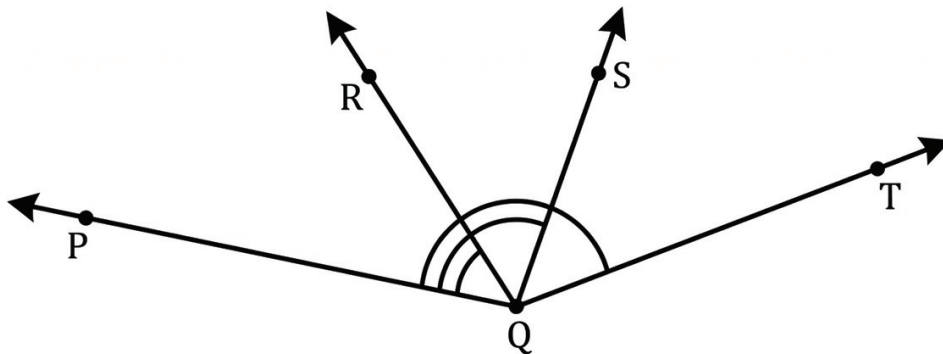




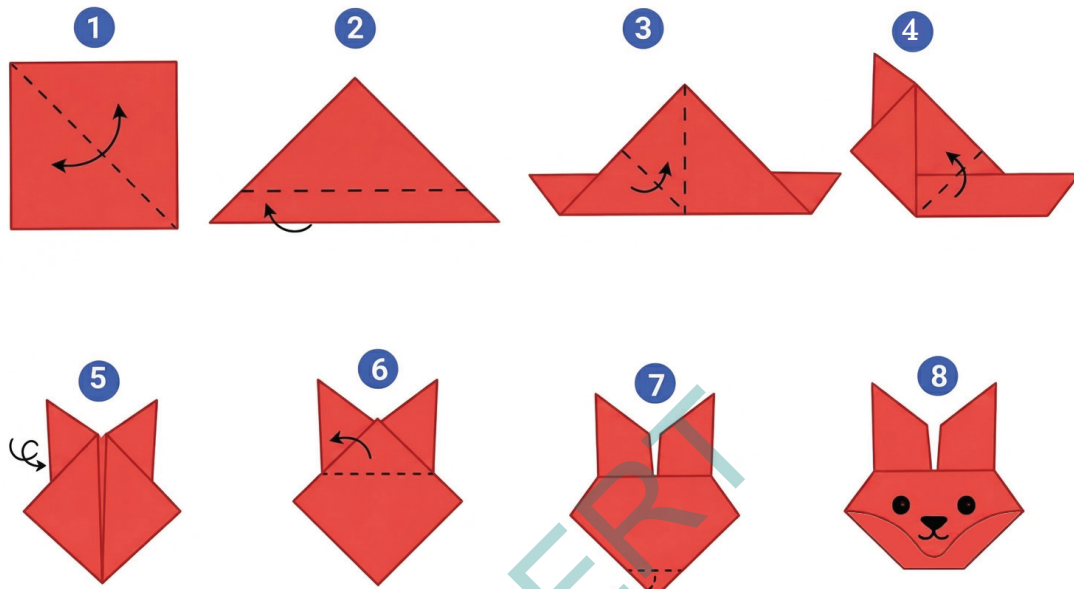
6. Find the degree measures of $\angle BXE$, $\angle CXE$, $\angle AXB$ and $\angle BXC$.



7. Find the degree measures of $\angle PQR$, $\angle PQS$ and $\angle PQT$.



8. Make the paper craft as per the given instructions. Then, unfold and open the paper fully. Draw lines on the creases made and measure the angles formed.



9. Measure all three angles of the triangle shown in Fig. 2.21 (a), and write the measures down near the respective angles. Now add up the three measures. What do you get? Do the same for the triangles in Fig. 2.21 (b) and (c). Try it for other triangles as well, and then make a conjecture for what happens in general! We will come back to why this happens in a later year.

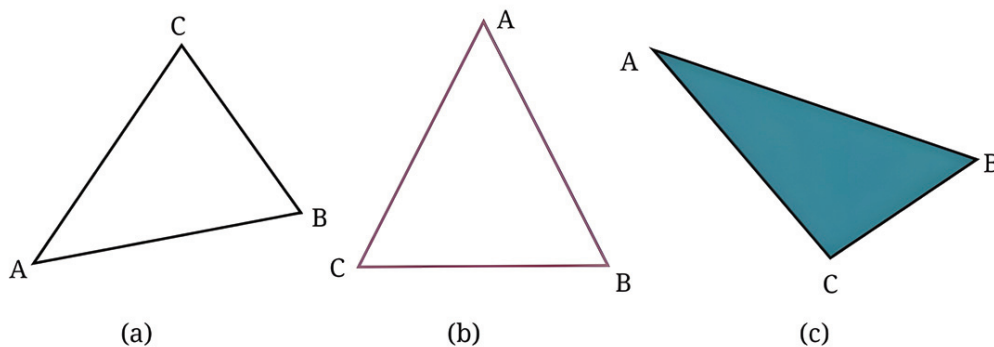
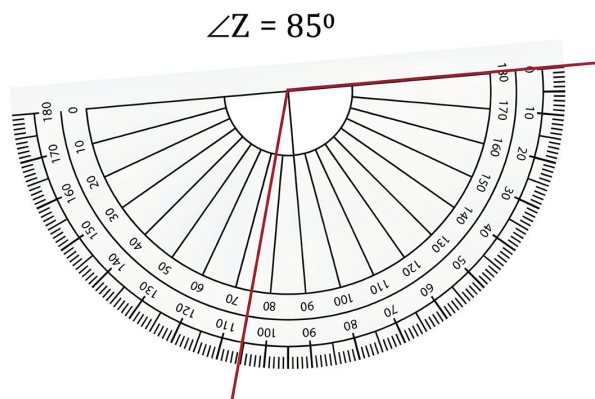
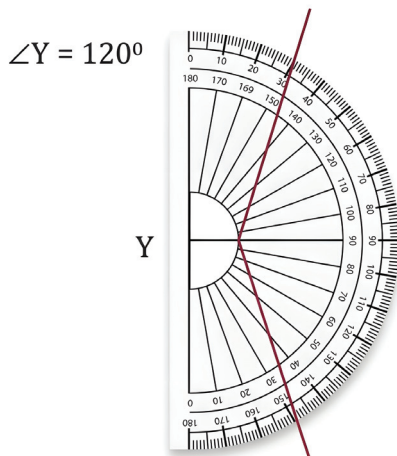
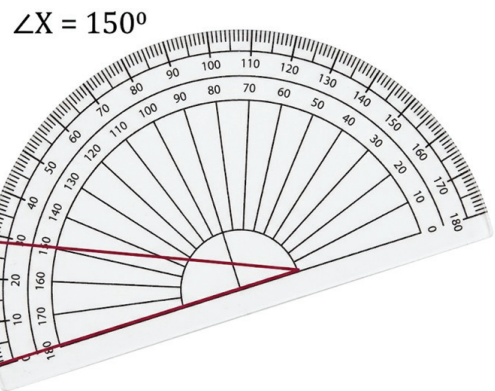
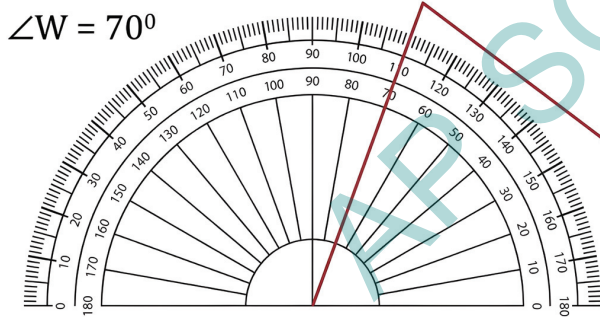
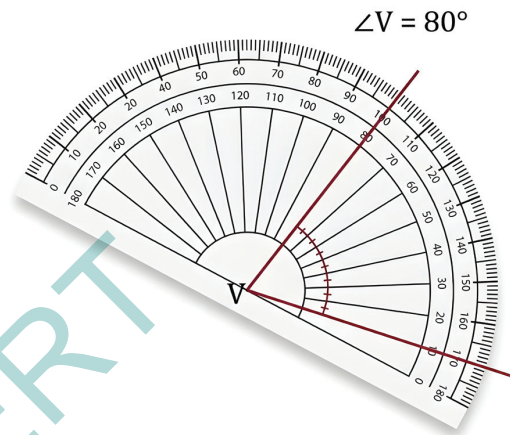
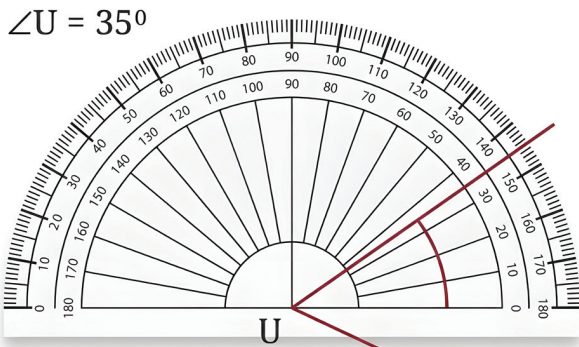


Fig. 2.21

Mind the Mistake, Mend the Mistake!

A student used a protractor to measure the angles as shown below. In each figure, identify the incorrect usage(s) of the protractor and discuss how the reading could have been made and think how it can be corrected.



Angles in Real world

Architecture and Construction

Angles and lines are used to design safe, functional buildings and infrastructure. Architects use precise angles for roof slopes, walls, windows, and doorways to ensure drainage, stability, and aesthetic appeal.

Carpenters and engineers rely on angle measurements while planning furniture, stairs, bridges, or even basic room corners.

Art, Design, and Sports

Artists and designers use angles to develop depth, perspective, and visual balance in their work. Angles provide the foundation for composition, leading the viewer's eye and helping shape forms in paintings, sculptures, or product design. Athletes consider angles for throws, shots, or movements to optimize performance.

Everyday Activities

Angles and lines can be seen in daily objects and tasks:

- The hands of a clock form angles throughout the day.
- Opening doors, books, or folding objects involves angles.
- Parking vehicles or setting ladders against walls all require angle awareness.

DURGAM CHERUVU

Cable Bridge

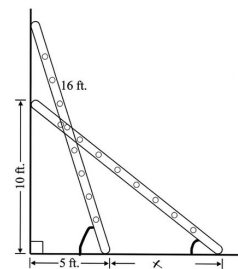
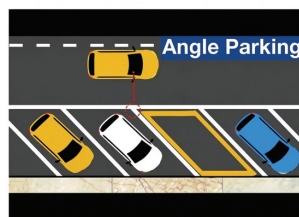
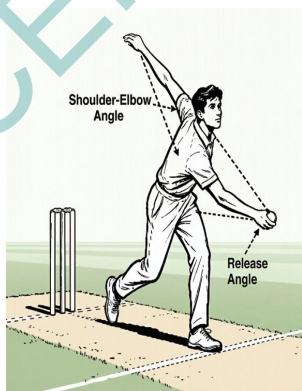
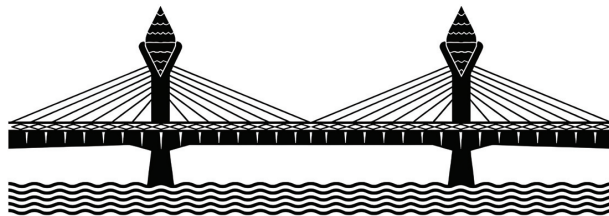




Figure it Out - 2.7

1. Angles in a clock :

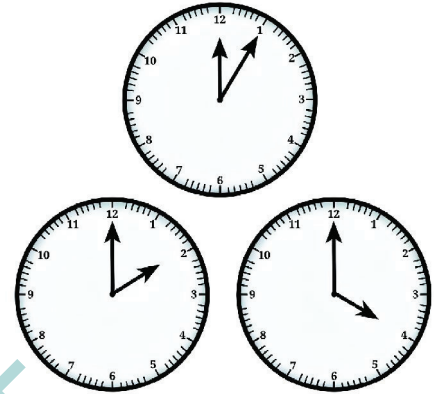
- a. The hands of a clock make different angles at different times. At 1 o'clock, the angle between the hands is 30° .

Why?

- b. What will be the angle at 2 o'clock?

And at 4 o'clock? 6 o'clock?

- c. Explore other angles made by the hands of a clock.



2. The angle of a door:

Is it possible to express the amount by which a door is opened using an angle?

What will be the vertex of the angle and what will be the arms of the angle?



3. Vidya is enjoying her time on the swing. She notices that the greater the angle with which she starts the swinging, the greater is the speed she achieves on her swing. But where is the angle? Are you able to see any angle?



4. Here is a toy with slanting slabs attached to its sides; the greater the angles or slopes of the slabs, the faster the balls roll. Can angles be used to describe the slopes of the slabs? What are the arms of each angle? Which arm is visible and which is not?



5. Observe the images below where there is an insect and its rotated version. Can angles be used to describe the amount of rotation? How? What will be the arms of the angle and the vertex?

Hint: Observe the horizontal line touching the insects.



Teacher's Note

It is important that students see the application of each mathematical concept in their daily lives. Teacher can organise some activities where students can appreciate the practical applications of angles in real-life situations, e.g., clocks, doors, swings, concepts of uphill and downhill, location of the sun, the giving of directions, etc.

2.10 Drawing Angles

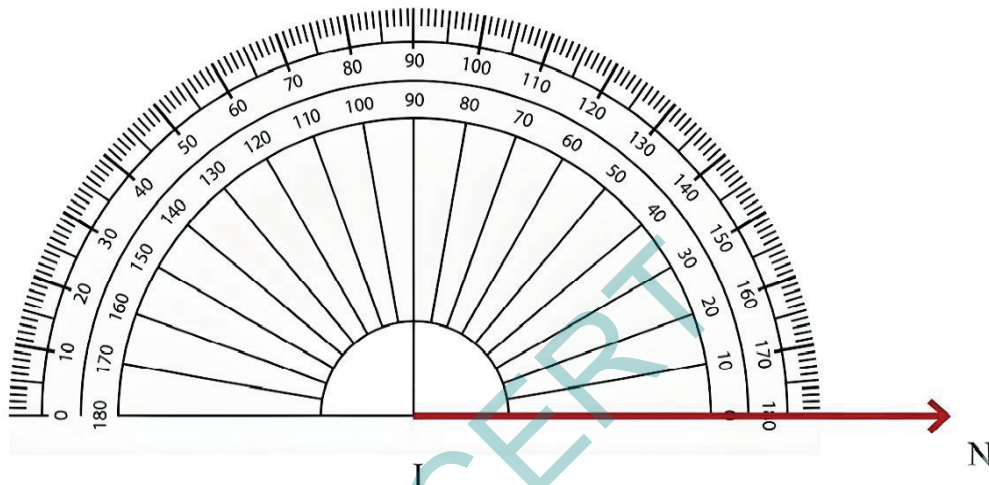
Vidya wants to draw a 30° angle and name it $\angle TIN$ using a protractor.

In $\angle TIN$, T will be the vertex, IT and IN will be the arms of the angle. Keeping one arm, say IN , as the reference (base), the other arm IT should take a turn of 30° .

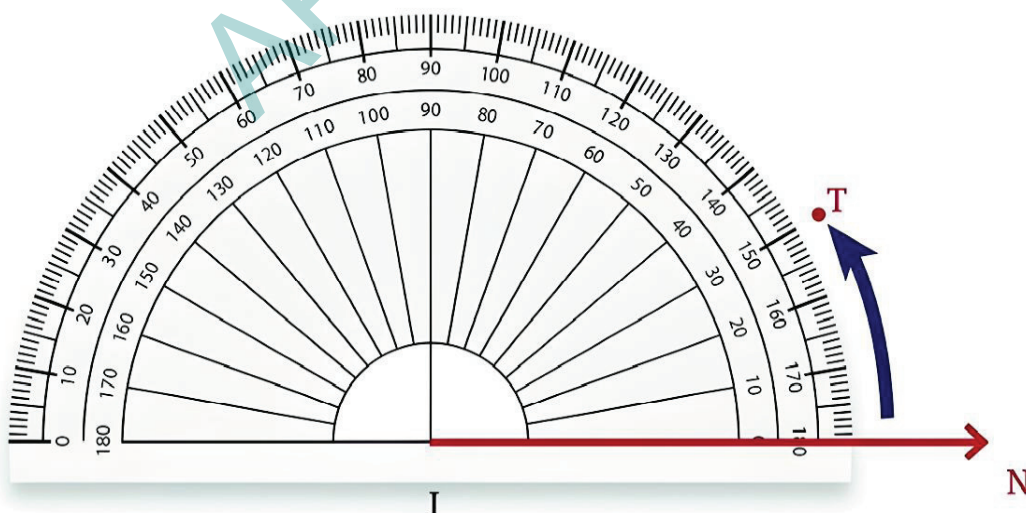
Step 1: We begin with the base and draw \overrightarrow{IN} :



Step 2: We will place the centre point of the protractor on I and align IN to the 0 line.



Step 3: Now, starting from 0, count your degrees (0, 10, 30) up to 30 on the protractor. Mark point T at the label 30°.



Step 4: Using a ruler join the point I and T.

$\angle TIN = 30^\circ$ is the required angle.

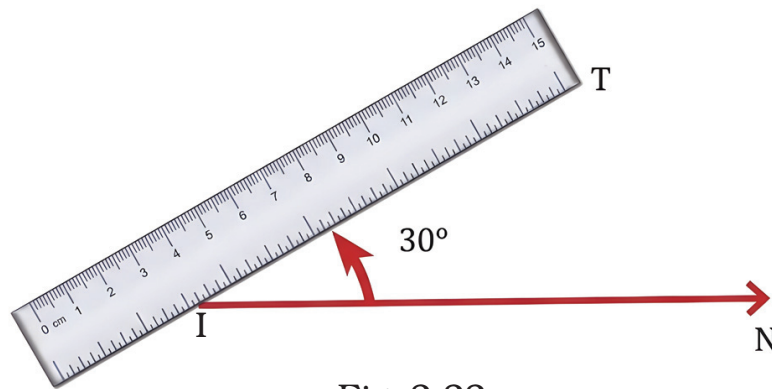


Fig. 2.22

✿ Let's Play a Game #1

This is an angle guessing game! Play this game with your classmates by making two teams, Team 1 and Team 2. Here are the instructions and rules for the game:

- **Team 1** secretly choose an angle measure, for example, 49° and makes an angle with that measure using a protractor without Team 2 being able to see it.
- **Team 2** now gets to look at the angle. They have to quickly discuss and guess the number of degrees in the angle (without using a protractor!).
- **Team 1** now demonstrates the true measure of the angle with a protractor.
- **Team 2** scores the number of points that is the absolute difference in degrees between their guess and the correct measure. For example, if Team 2 guesses 39° , then they score 10 points ($49^\circ - 39^\circ$).
- Each team gets five turns. The winner is the team with the lowest score!

✿ Let's Play a Game #2

We now change the rules of the game a bit. Play this game with your classmates by again making two teams, Team 1 and Team 2. Here are the instructions and rules:

- **Team 1** announces to all, an angle measure, e.g., 34° .
- A player from **Team 2** must draw that angle on the board without using a protractor. Other members of **Team 2** can help the player by speaking words like 'Make it bigger!' or 'Make it smaller!'.
- A player from **Team 1** measures the angle with a protractor for all to see.
- **Team 2** scores the number of points that is the absolute difference in degrees between **Team 2's** angle size and the intended angle size. For example, if player's angle from **Team 2** is measured to be 25° , then **Team 2** scores 9 points ($34^\circ - 25^\circ$).
- Each team gets five turns. The winner is again the team with the lowest score.

Teacher's Note

These games are important to play to build intuition about angles and their measures. Return to this game at least once or twice on different days to build practice in estimating angles. Note that these games can also be played between pairs of students.



Figure it Out - 2.8

1. In Fig. 2.23, list all the angles possible. Did you find them all? Now, guess the measures of all the angles. Then, measure the angles with a protractor. Record all your numbers in a table. See how close your guesses are to the actual measures.

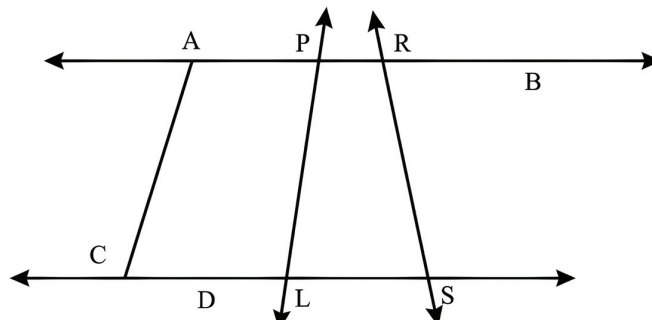
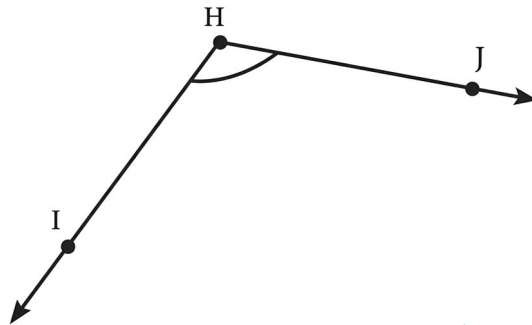


Fig. 2.23

2. Use a protractor to draw angles having the following degree measures:
 a. 110° b. 40° c. 75° d. 112° e. 134°
3. Draw an angle whose degree measure is the same as the angle given below:

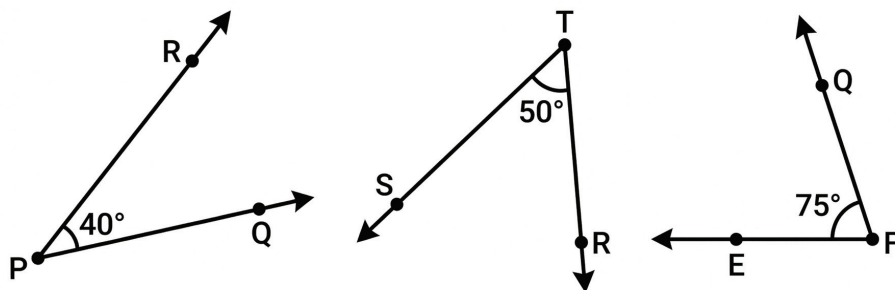


Also, write down the steps you followed to draw the angle.

2.11 Types of Angles and their Measures

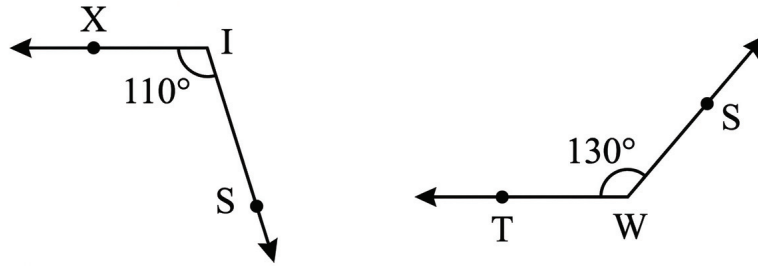
We have read about different types of angles in this chapter. We have seen that a straight angle is 180° and a right angle is 90° . How can other types of angles — acute and obtuse — be described in terms of their degree measures?

Acute Angle: Angles that are smaller than the right angle, i.e., less than 90° and are greater than 0° , are called acute angles.



Examples of acute angles

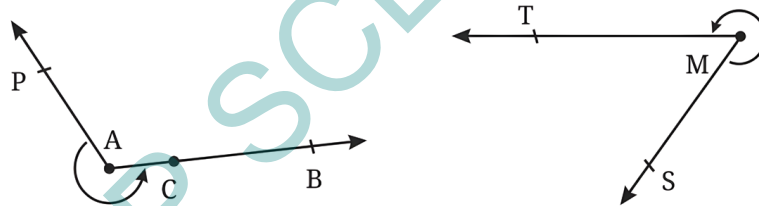
Obtuse Angle: Angles that are greater than the right angle and less than the straight angle, i.e., greater than 90° and less than 180° , are called **obtuse** angles.



Examples of obtuse angles

Have we covered all the possible measures that an angle can take? Here is another type of angle.

Reflex angle: Angles that are greater than the straight angle and less than the whole angle, i.e., greater than 180° and less than 360° , are called **reflex** angles.

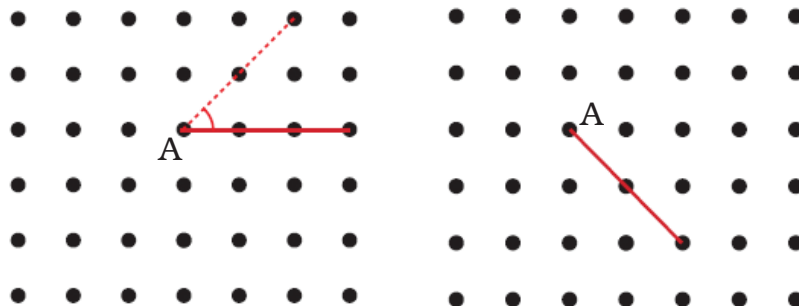


Examples of reflex angles

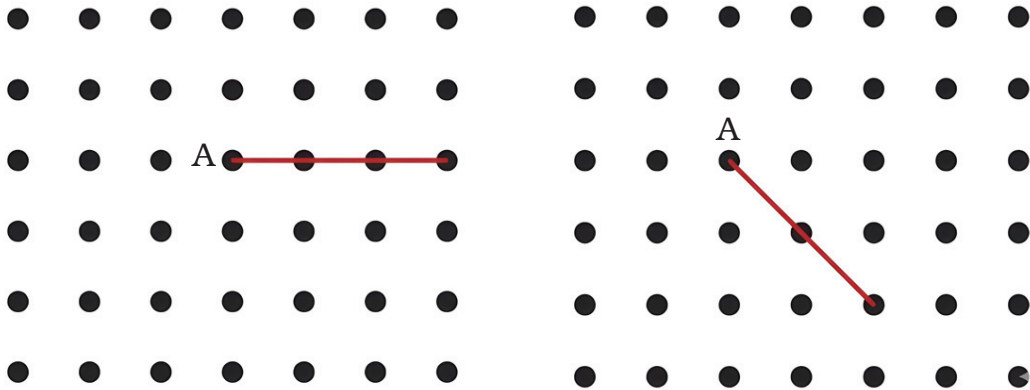


Figure it Out - 2.9

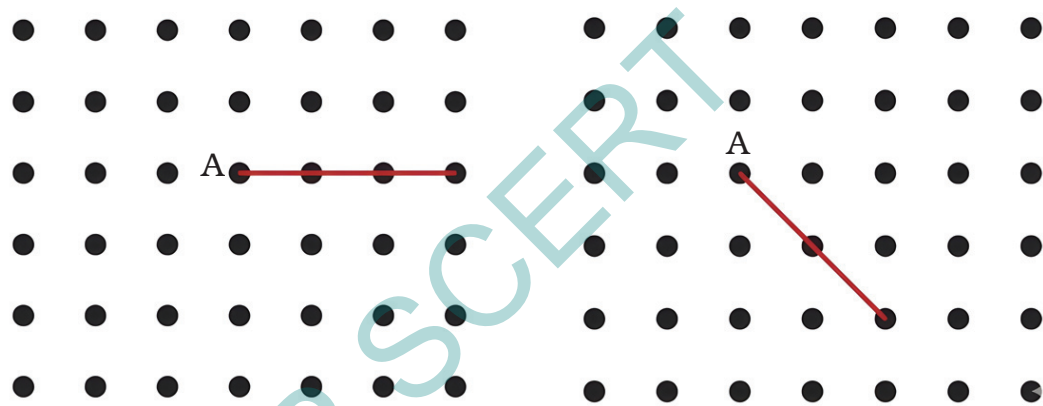
1. In each of the below grids, join A to other grid points in the figure by a straight line to get:
 - a. An acute angle



b. An obtuse angle



c. An reflex angle



Mark the intended angles with curves to specify the angles. One has been done for you.

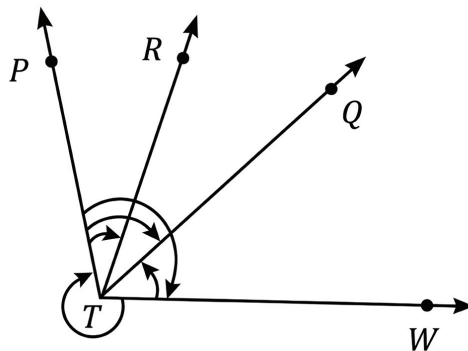
2. Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, right, or reflex.

a. $\angle PTR$

b. $\angle PTQ$

c. $\angle PTW$

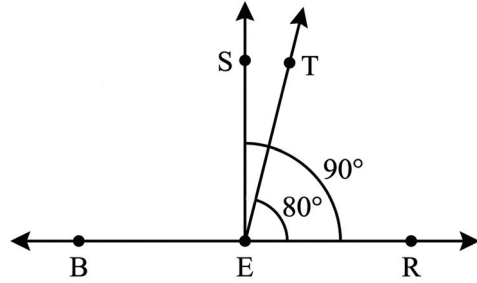
d. $\angle WTQ$





Let's Explore

In this figure, $\angle TER = 80^\circ$. What is the measure of $\angle BET$? What is the measure of $\angle SET$?

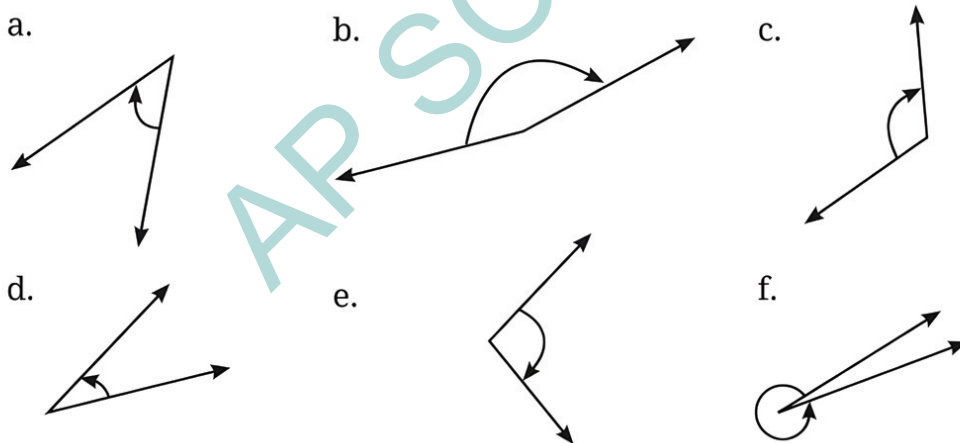


Hint: Observe that $\angle REB$ is a straight angle. Hence, the degree measure of $\angle REB = 180^\circ$ of which 80° is covered by $\angle TER$. A similar argument can be applied to find the measure of $\angle SET$.



Figure it Out - 2.10

- Draw angles with the following degree measures:
 - 140°
 - 82°
 - 195°
 - 70°
 - 35°
- Estimate the size of each angle and then measure it with a protractor:



Classify these angles as acute, right, obtuse or reflex angles.

- Make any figure with three acute angles, one right angle and two obtuse angles.
- Draw the letter 'M' such that the angles on the sides are 40° each and the angle in the middle is 60° .
- Draw the letter 'Y' such that the three angles formed are 150° , 60° and 150° .

6. The Ashoka Chakra has 24 spokes. What is the degree measure of the angle between two spokes next to each other? What is the largest acute angle formed between two spokes?



7. **Puzzle:** I am an acute angle. If you double my measure, you get an acute angle. If you triple my measure, you will get an acute angle again. If you quadruple (four times) my measure, you will get an acute angle yet again! But if you multiply my measure by 5, you will get an obtuse angle measure. What are the possibilities for my measure?



Project Work

Angle - Clock : Use Clocks to visually demonstrate angles formed by the minute and hour hands at different times. Measure and discuss acute, right straight and Obtuse angles based on clock positions.

CHAPTER MASTERY

1) **How many lines can be drawn through given two points?**

[]

(A) Only one (B) 2 (C) 4 (D) Countless

2) **Find "False" statement.**

[]

- (A) Two lines intersect in a point
- (B) The line segment has two end points.
- (C) The ray has one initial point.
- (D) A Ray has two end points

3) Match the following properties with their correct geometrical figures []

Column A

- (i) Indefinite length in both the directions
- (ii) Has no size but shows the position
- (iii) Countless collection of points which is part of line with two end points
- (iv) Indefinite length in one direction

Column B

- (a) Ray
- (b) Line
- (c) Point
- (d) Line Segment

(A) (i) → (a), (ii) → (b), (iii) → (c), (iv) → (d)

(B) (i) → (b), (ii) → (c), (iii) → (d), (iv) → (a)

(C) (i) → (a), (ii) → (c), (iii) → (d), (iv) → (b)

(D) (i) → (b), (ii) → (a), (iii) → (c), (iv) → (d)

4) Assertion (A) : If angles a and b form a straight angle and $a = 40^\circ$ then $b = 150^\circ$

Reason (R) : Sum of straight-line pair of angles is always 180°

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true and R is not the correct explanation of A

(C) A is true, R is false

(D) A is False, R is true

5) Match the following. []

Column A

- (i) Straight Angle
- (ii) Right Angle
- (iii) Obtuse Angle
- (iv) Reflex Angle

Column B

- (a) Less than one-fourth of a revolution
- (b) More than half a revolution
- (c) Half of a revolution
- (d) One-fourth of a revolution
- (e) Between $1/4$ and $1/2$ of a revolution

(A) (i) → (c), (ii) → (d), (iii) → (e), (iv) → (b)

(B) (i) → (c), (ii) → (b), (iii) → (e), (iv) → (d)

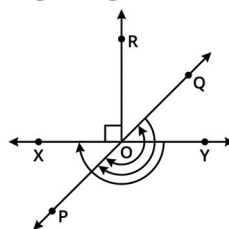
(C) (i) → (a), (ii) → (d), (iii) → (e), (iv) → (b)

(D) (i) → (d), (ii) → (b), (iii) → (e), (iv) → (c)

6) Write two differences between line and line segment.

7) In the given figure, name the following angles as acute, obtuse, right, reflex or straight

- (a) $\angle QOY$ (b) $\angle YOP$
 (c) $\angle ROX$ (d) $\angle QOX$
 (e) $\angle POQ$



8) What fraction of a clockwise revolution does the hour hand of a clock turn through, when it goes from

- (A) 3 to 9 (B) 4 to 7 (C) 7 to 10 (D) 12 to 9

9) The number of right angles turned through by the hour hand of a clock when it goes from 3 to 6 in clockwise direction is _____

10) Draw the rough sketch of the following:

- (A) Acute angle (B) Obtuse angle

SUMMARY

- A point determines a location. It is denoted by a capital letter. A line segment corresponds to the shortest distance between two points. The line segment joining points S and T is denoted by \overline{ST} .
- A line is obtained when a line segment like \overline{ST} is extended on both sides indefinitely; it is denoted by \overleftrightarrow{ST} or sometimes by a single small letter like m.
- A ray is a portion of a line starting at a point D and going in one direction indefinitely. It is denoted by \overrightarrow{DP} where P is another point on the ray. An angle can be visualised as two rays starting from a common starting point. Two rays \overrightarrow{OP} and \overrightarrow{OM} form the angle $\angle POM$ (also called $\sphericalangle MOP$); here, O is called the vertex of the angle, and the rays \overrightarrow{OP} and \overrightarrow{OM} are called the arms of the angle.
- The size of an angle is the amount of rotation or turn needed about the vertex to rotate one ray of the angle onto the other ray of the angle.
- The sizes of angles can be measured in degrees. One full rotation or turn is considered as 360 degrees and denoted as 360° .
- Degree measures of angles can be measured using a protractor.
- Angles can be straight (180°), right (90°), acute (more than 0° and less than 90°), obtuse (more than 90° and less than 180°), and reflex (more than 180° and less than 360°).

3

NUMBER PLAY



Learning Outcomes:

Learners will be able to

- Explore and appreciate patterns, properties, and curiosities of numbers. (CG-1)
- Use logical reasoning and pattern recognition to solve puzzles and mathematical games. (CG-7)
- Apply mental computation and estimation in daily mathematical situations. (CG-1)
- Appreciate the creative and investigative nature of Mathematics beyond routine calculation. (CG-7)

3.0 Introduction

Welcome to one of the most exciting chapters in mathematics!

Today, we step away from routine lessons and dive into the playful world of Number Play, where every number you see is a clue in a fascinating puzzle. Imagine two friends, Ravi and Raju, standing by a busy road.

They weren't just watching the traffic - they were playing a game with the numbers on vehicle plates!

Ravi noted the number on one vehicle : 4534.

He added all the digits: $4 + 5 + 3 + 4 = 16$.

Then he added those digits again: $1 + 6 = 7$.

The next vehicle passed. Raju quickly noted its number: 2133.

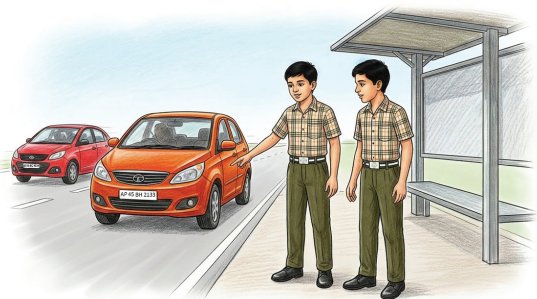
He added the digits: $2 + 1 + 3 + 3 = 9$, and cheered loudly, "I win!"

As they continued their game, they noticed fascinating patterns:

Some numbers were increasing (like 1234) or decreasing (like 4321).

Some had repeated digits (like 7744 or 5050).

Some were palindromic - the same when read forwards or backwards (like 1221).



Ravi and Raju realized that number patterns are everywhere - on number plates, clocks, mobile screens, and calendars.

They aren't just figures; they are clues waiting to be observed and understood.

So get ready to turn everyday observations into thrilling mathematical discoveries.

Sharpen your mind, open your eyes - and let the Number Play begin!

3.1 Numbers can Tell us Things

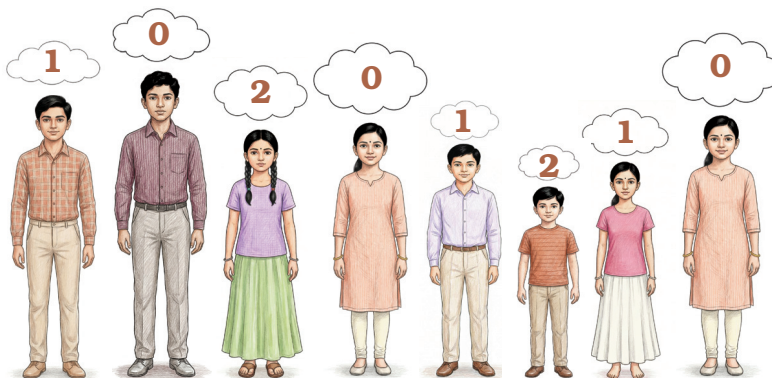
What are these numbers telling us?

Some children in a park are standing in a line. Each one says a number.



✿ What do you think these numbers mean?

The children now rearrange themselves, and again each one says a number based on the arrangement.



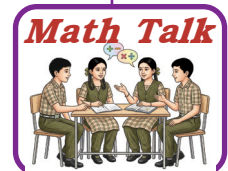
Did you figure out what these numbers represent?

Hint: Could their heights be playing a role?

A child says '1' if there is only one taller child standing next to them. A child says '2' if both the children standing next to them are taller. A child says '0', if neither of the children standing next to them are taller. That is each person says the number of taller neighbours they have.

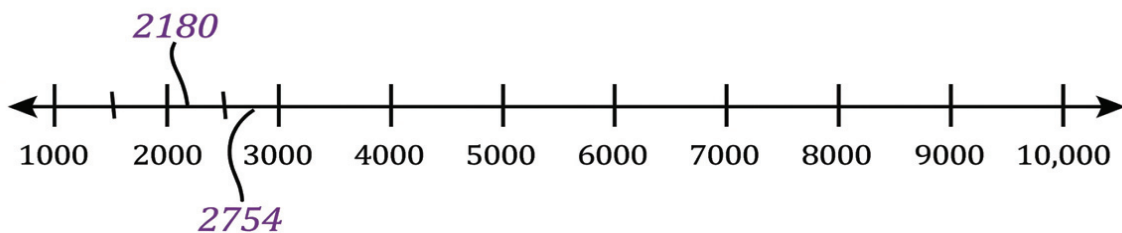
✿ Try answering the questions below and share your reasoning.

1. Can the children rearrange themselves so that the children standing at the ends say '2'?
2. Can we arrange the children in a line so that all would say only 0s?
3. Can two children standing next to each other say the same number?
4. There are 5 children in a group, all of different heights. Can they stand such that four of them say '1' and the last one says '0'? Why or why not?
5. For this group of 5 children, is the sequence 1, 1, 1, 1, 1 possible?
6. Is the sequence 0, 1, 2, 1, 0 possible? Why or why not?
7. How would you rearrange the five children so that the maximum number of children say '2'?



3.2 Patterns of Numbers on the Number Line

✿ We are quite familiar with number lines now. Let's see if we can place some numbers in their appropriate positions on the number line.

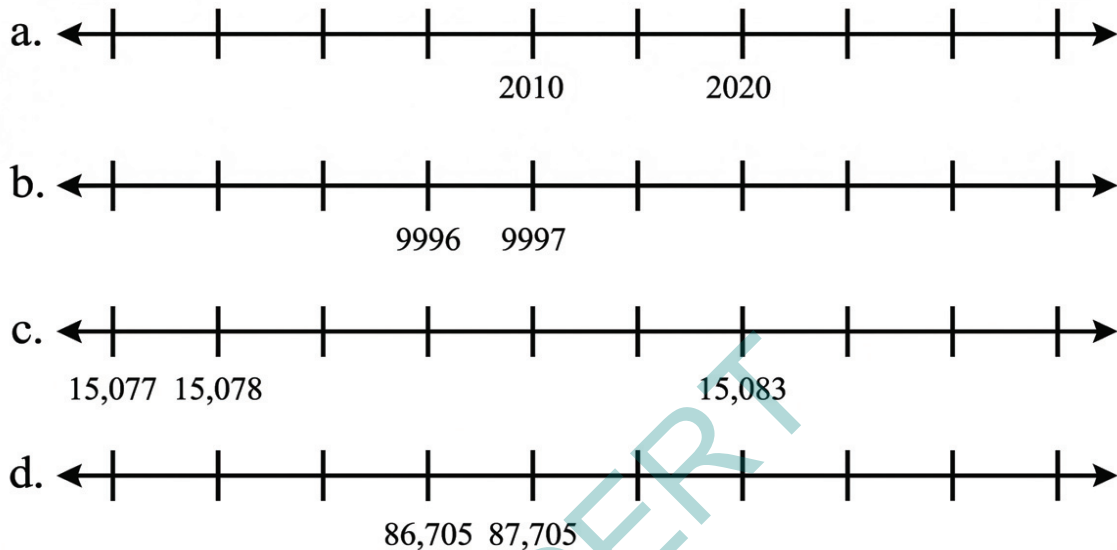


Here are the numbers: 2180, 2754, 1500, 3600, 9950, 9590, 1050, 3050, 5030, 5300 and 8400.



Figure it Out - 3.1

Identify the numbers marked on the number lines below, and label the remaining positions.



Put a circle around the smallest number and a box around the largest number in each of the sequences above.

3.3 Supercells

Observe the numbers written in the table below. Why are some numbers coloured? Discuss.

43	79	75	63	10	29	28	34
200	577	626	345	790	694	109	198

A cell is coloured if the number in it is larger than its adjacent cells. The number 626 is coloured as it is larger than 577 and 345, whereas 200 is not coloured as it is smaller than 577. The number 198 is coloured as it has only one adjacent cell with 109 in it, and 198 is larger than 109.



Figure it Out - 3.2

1. Colour or mark the supercells in the table below.

6828	670	9435	3780	3708	7308	8000	5583	52
------	-----	------	------	------	------	------	------	----

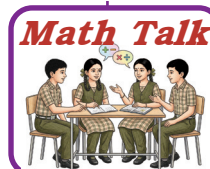
2. Fill the table below with only 4-digit numbers such that the supercells are exactly the coloured cells.

5346							9635	
------	--	--	--	--	--	--	------	--

3. Fill the table below such that we get as many supercells as possible. Use numbers between 100 and 1000 without repetitions.

--	--	--	--	--	--	--	--	--

- (a) Out of the 9 cells, how many supercells are there in the above table?
- (b) Find out how many supercells are possible for different numbers of cells.
- (c) Do you notice any pattern? What is the method to fill a given table to get the maximum number of supercells? Explore and share your strategy.
4. Can you fill a supercell table without repeating numbers such that there are no supercells? Why or why not?
5. Will the cell having the largest number in a table always be a supercell? Can the cell having the smallest number in a table be a supercell? Why or why not?
6. Fill a table such that the cell having the second largest number is not a supercell.
7. Fill a table such that the cell having the second largest number is not a supercell but the second smallest number is a supercell. Is it possible?
8. Make other variations of this puzzle and challenge your classmates.



Try This

Let's do the supercells activity with more rows.

Here the neighbouring cells are those that are immediately to the left, right, top and bottom.

The rule remains the same: a cell becomes a supercell if the number in it is greater than all the numbers in its neighbouring cells. In Table 1, 8632 is greater than all its neighbours 4580, 8280, 4795 and 1944.

- Complete Table 2 with 5-digit numbers whose digits are '1', '0', '6', '3', and '9' in some order. Only a coloured cell should have a number greater than all its neighbours.

The biggest number in the table is _____.

The smallest even number in the table is _____.

The smallest number greater than 50,000 in the table is _____.

Once you have filled the table above, put commas appropriately after the thousands digit.

Table 1

2430	7500	7350	9870
3115	4795	9124	9230
4580	8632	8280	3446
5785	1944	5805	6034

Table 2

	96,301	36,109	
	13,609	60,319	19,306
		60,193	
	10,963		

3.4 Playing with Digits

We start writing numbers from 1, 2, 3 ... and so on. There are nine 1-digit numbers.

- Find out how many numbers have two digits, three digits, four digits, and five digits.

1-digit numbers From 1-9 -----	2-digit numbers -----	3-digit numbers -----	4-digit numbers -----	5-digit numbers -----
9				

Digit sums of numbers

Lasya observes that when she adds up digits of certain numbers the sum is the same.

For example, adding the digits of the number 68 will be same as adding the digits of 176 or 545.

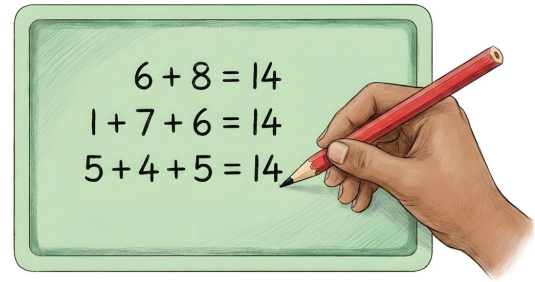
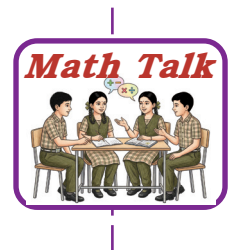


Figure it Out - 3.3

Digit sum 14

1. Write other numbers whose digits add up to 14.
2. What is the smallest number whose digit sum is 14?
3. What is the largest 5-digit whose digit sum is 14?
4. How big a number can you form having the digit sum of 14? Can you make an even bigger number?
5. Find out the digit sums of all the numbers from 40 to 70. Share your observations with the class.
6. Calculate the digit sums of 3-digit numbers whose digits are consecutive (for example, 345). Do you see a pattern? Will this pattern continue?



Digit Detectives

After writing numbers from 1 to 100, Achyuth wondered how many times he would have written the digit '7'.

- ✿ Among the numbers 1–100, how many times will the digit '7' occur? Among the numbers 1–1000, how many times will the digit '7' occur?



3.5 Pretty Palindromic Patterns

What pattern do you see in these numbers: 66, 848, 575, 797, 1111? These numbers read the same from left to right and from right to left. Try and see. Such numbers are called **palindromes** or **palindromic numbers**.

All palindromes using 1, 2, 3

The numbers 121, 313, 222 are some examples of palindromes using the digits '1', '2', '3'.

✿ Write all possible 3-digit palindromes using these digits.

Mahaveeracharya's "Necklaces" (Kantabharana)

The Indian mathematician Mahaveeracharya (9th Century CE), in his work **Ganita Sara Sangraha**, presented numbers with beautiful, rhythmic patterns, which he called Kantabharana (**Necklaces**).

1. Kantabharana: 12345654321

$$27,994,681 \times 441 = 12,345,654,321$$

2. Rajakanthikabharana: 12345678987654321

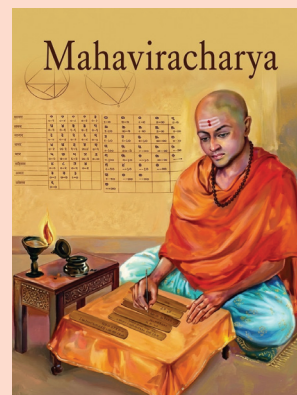
(A complete palindromic number with non-Zero digits)

$$111,111,111 \times 111,111,111 = 12,345,678,987,654,321$$

3. The Repeating Pattern :

$$333,333,666,667 \times 33 = 11,0000,11,0000,11$$

These examples demonstrate that numbers possess intrinsic beauty and hidden rhythm, key aspects of Number Play.



Reverse-and-add palindromes

Now, look at these additions. Try to figure out what is happening.

Steps to follow: Start with a 2-digit number. Add this number to its reverse. Stop if you get a palindrome or else repeat the steps of reversing the digits and adding.

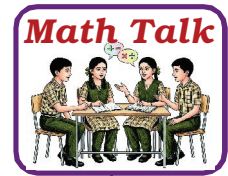
Try the same procedure for some other numbers, and perform the same steps. Stop if you get a palindrome. There are numbers for which you have to repeat this a large number of times.

34	29	48	76
43	92	84	67
77	121	132	143
		231	341
		363	484

Are there numbers for which you do not reach a palindrome at all?



Let's Explore



1. Will reversing and adding numbers repeatedly, starting with a 2-digit number, always give a palindrome? Explore and find out.
2. Will reversing and adding numbers repeatedly, 196, always give a palindrome?

❁ Puzzle time

tth th h t u

--	--	--	--	--

Write the number in words.

I am a 5-digit palindrome.

I am an odd number.

My 'tens' digit is double of my 'unit' digit.

My 'hundreds' digit is double of my 'tens' digit.

Who am I? _____

3.6 Clock and Calendar Numbers

On the usual 12-hour clock, there are timings with different patterns. For example, 4:44, 10:10, 12:21.

- ❁ Try and find out all possible times on a 12-hour clock of each of these types.

Araadhya has her birthday on 11/02/2011 where the digits reads (from left to right and right to left) the same forwards and backwards. It is a palindromic date. Aditya has his birthday on 20/12/2012 where the digits '2', '0', '1', and '2' repeat in that order. It is not a palindromic date.

- ❁ Gopi is now looking at a calendar for the future. He imagines he is a "Time Detective" searching for special dates hidden in the future years.

The Mystery of the Mirror Dates

Gopi notices that palindromic dates are very rare.

He found one in the past: 02/02/2020.

He found one in the future 14/02/2041

Can you help Gopi find the next ones?

- ❁ Find all possible dates of this form from the past.

Gopi was looking at this year's calendar. He started wondering, "Why should we change the calendar every year? Can we not reuse a calendar?".

What do you think?

You might have noticed that last year's calendar was different from this year's. Also, next year's calendar will also be different from the previous years.

- ❁ But, will any year's calendar repeat again after some years? Will all dates and days in a year match exactly with that of another year?



Let's Explore

- ❁ If the date format is DD/MM/YYYY, what is the next palindromic date after 03/02/2030?

Are there any palindromic dates in the year 2031? Why or why not?

Challenge: What is the last palindromic date of the 21st Century (up to the year 2100)?

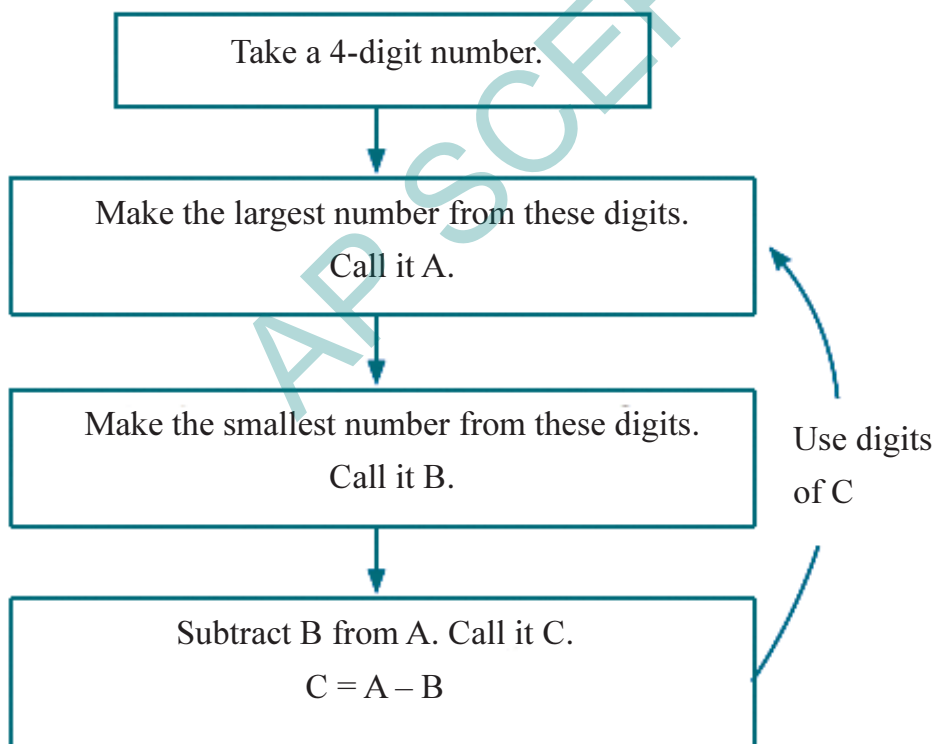
3.7 The Magic Number of Kaprekar

D.R. Kaprekar was a mathematics teacher in a government school in Devlali, Maharashtra. He liked playing with numbers very much and found many beautiful patterns in numbers that were previously unknown.



In 1949, he discovered a fascinating and magical phenomenon when playing with 4-digit numbers.

Follow these steps and experience the magic for yourselves! Pick any 4-digit number having at least two different digits, say 6382.



What happens if we continue doing this?

$$A = 8632$$

$$B = 2368$$

$$C = 8632 - 2368 \\ = 6264$$

$$A = 6642$$

$$B = 2466$$

$$C = 6642 - 2466 \\ = 4176$$

$$A = 7641$$

$$B = 1467$$

$$C = 7641 - 1467 \\ = 6174$$

$$A =$$

$$B =$$

$$C =$$



Let's Explore

Take different 4-digit numbers and try carrying out these steps. Find out what happens. Check with your friends what they got.

You will always reach the magic number '6174'! The number '6174' is now called the 'Kaprekar constant'.

Carry out these same steps with a few 3-digit numbers. What number will start repeating?



Figure it Out - 3.4

- Sarala uses the digits '4', '7', '3' and '2', and makes the smallest and largest 4-digit numbers with them: 2347 and 7432. The difference between these two numbers is $7432 - 2347 = 5085$.
The sum of these two numbers is 9779.
Choose 4 - digits to make:
 - The difference between the largest and smallest numbers greater than 5085.
 - The difference between the largest and smallest numbers less than 5085.
 - The sum of the largest and smallest numbers greater than 9779.
 - The sum of the largest and smallest numbers less than 9779.
- What is the sum of the smallest and largest 5-digit palindrome? What is their difference?
- The time now is 10:01. How many minutes until the clock shows the next palindromic time? What about the one after that?
- How many rounds does the number 5683 take to reach the Kaprekar constant?

3.8 Simple Estimation

At times, we may not know or need an exact count of things and an estimate is sufficient for the purpose at hand. For example, your school headmaster might know the exact number of students enrolled in your school, but you may only know an estimated count. How many students are in your school? About 150? 400? A thousand?

Lasya's class section has 32 children. The other 2 sections of her class have 29 and 35 children. So, she estimated the number of children in her class to be about 100. Along with Class 6, her school also has Classes 7–10 and each class has 3 sections each. She assumed a similar number in each class and estimated the number of students in her school to be around 500.



Figure it Out - 3.5

We shall do some simple estimates. It is a fun exercise, and you may find it amusing to know the various numbers around us. Remember, we are not interested in the exact numbers for the following questions. Share your methods of estimation with the class.

1. Steps you would take to walk:
 - a. From the place you are sitting to the classroom door
 - b. Across the school ground from start to end
 - c. From your classroom door to the school gate
 - d. From your school to your home
2. Number of times you blink your eyes or number of breaths you take:
 - a. In a minute
 - b. In an hour
 - c. In a day

3. Name some objects around you that are:

- a. a few thousand in number
- b. more than ten thousand in number

❁ **Estimate the answer**

Try to guess within 30 seconds. Check your guess with your friends.

4. Number of words in your maths textbook:

- a. More than 5000
- b. Less than 5000

5. Number of students in your school who travel to school by bus:

- a. More than 200
- b. Less than 200

6. Achyuth wants to buy milk and 3 types of fruit to make fruit custard for 5 people. He estimates the cost to be ₹100. Do you agree with him? Why or why not?

7. Estimate the distance between Amaravati (in Andhra Pradesh) to Hyderabad (in Telangana).

Hint: Look at the map of India to locate these cities.

8. Amara is in Grade 6 and says she has spent around 13,000 hours in school till date. Do you agree with her? Why or why not?

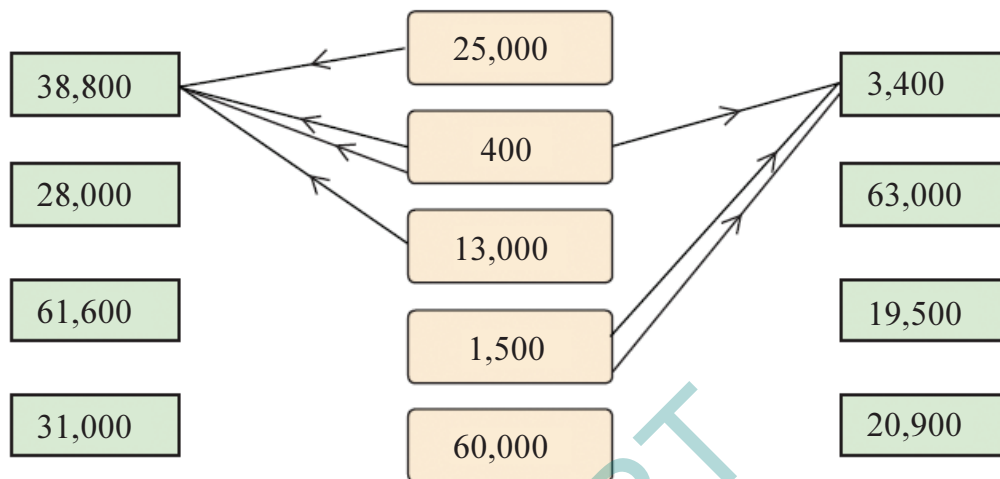
9. Earlier, people used to walk long distances as they had no other means of transport. Suppose you walk at your normal pace. Approximately, how long would it take you to go from:

- a. Your current location to one of your favourite places nearby.
- b. Your current location to any neighbouring state's capital city.
- c. The southernmost point in India to the northernmost point in India.

10. Make some estimation questions and challenge your classmates!

3.9 Mental Math

Observe the figure below. What can you say about the numbers and the lines drawn?



Numbers in the middle column are added in different ways to get the numbers on the sides ($1500 + 1500 + 400 = 3400$). The numbers in the middle can be used as many times as needed to get the desired sum. Draw arrows from the middle to the numbers on the sides to obtain the desired sums.

Two examples are given. It is simpler to do it mentally!

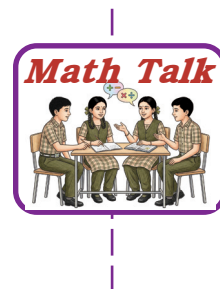
$$38,800 = 25,000 + 400 \times 2 + 13,000$$

$$3400 = 1500 + 1500 + 400$$

❖ Can we make 1,000 using the numbers in the middle?

Why not? What about 14,000, 15,000 and 16,000?

Yes, it is possible. Explore how. What thousands cannot be made?



Adding and Subtracting

Here, using the numbers in the boxes, we are allowed to use both addition and subtraction to get the required number. An example is shown.

40,000	7,000
300	1,500
12,000	800

$$39,800 = 40,000 - 800 + 300 + 300$$

$$45,000 =$$

$$5,900 =$$

$$17,500 =$$

$$21,400 =$$

Digits and Operations

An example of adding two 5-digit numbers to get another 5-digit number is $12,350 + 24,545 = 36,895$.

An example of subtracting two 5-digit numbers to get another 5-digit number is $48,952 - 24,547 = 24,405$.



Figure it Out - 3.6

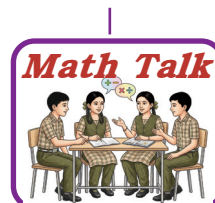
- Write an example for each of the below scenarios whenever possible.

5-digit + 5-digit to give a 5-digit sum more than 90,250	5-digit + 3-digit to give a 6-digit sum	4-digit + 4-digit to give a 6-digit sum	5-digit + 5-digit to give a 6-digit sum	5-digit + 5-digit to give 18,500
5-digit – 5-digit to give a difference less than 56,503	5-digit – 3-digit to give a 4-digit difference	5-digit – 4-digit to give a 4-digit difference	5-digit – 5-digit to give a 3-digit difference	5-digit – 5-digit to give 91,500

Could you find examples for all the cases? If not, think and discuss what could be the reason. Make other such questions and challenge your classmates.

- Always, Sometimes, Never?

Below are some statements. Think, explore and find out if each of the statement is 'Always true', 'Only sometimes true' or 'Never true'. Why do you think so? Write your reasoning and discuss this with the class.

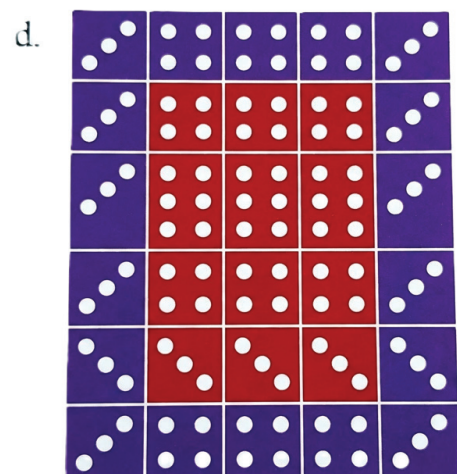
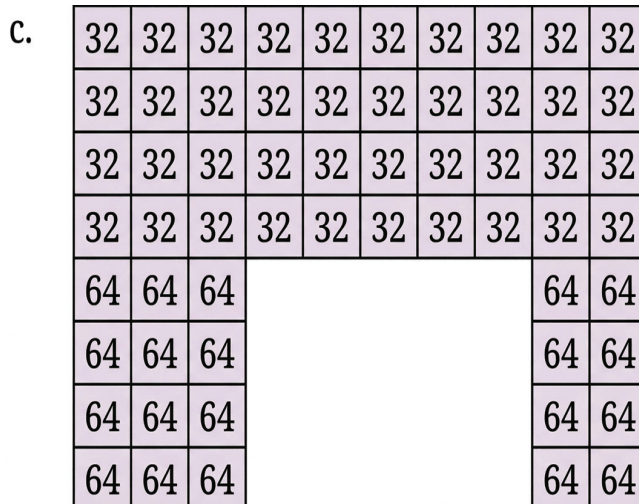
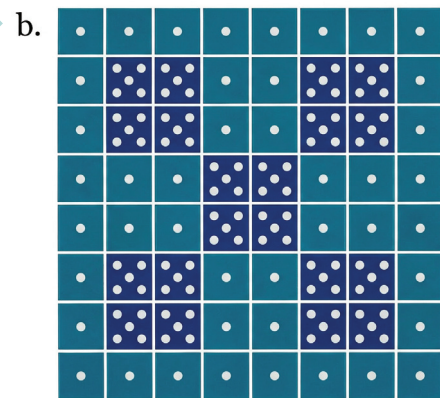
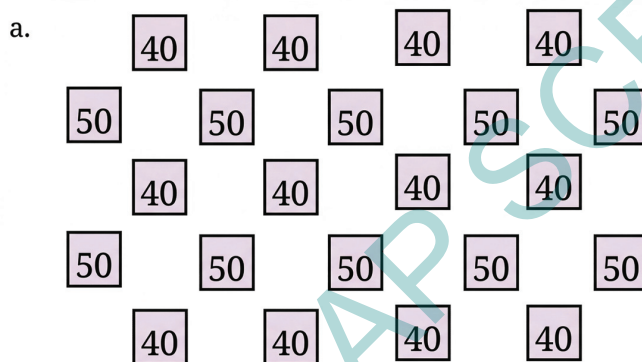


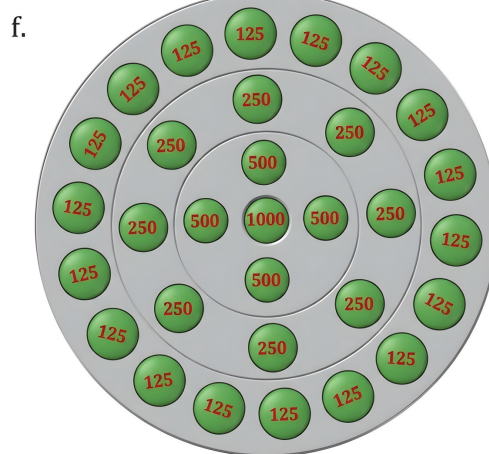
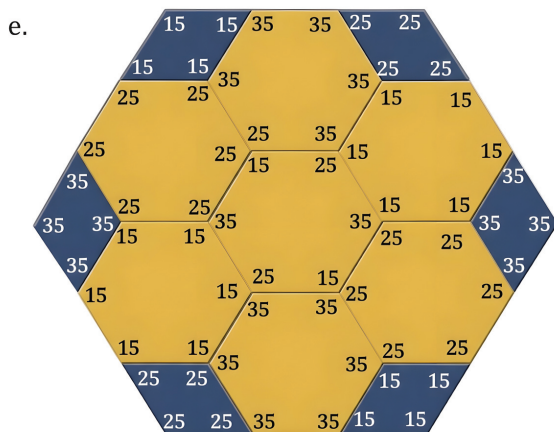
- a. 5-digit number + 5-digit number gives a 5-digit number
- b. 4-digit number + 2-digit number gives a 4-digit number
- c. 4-digit number + 2-digit number gives a 6-digit number
- d. 5-digit number – 5-digit number gives a 5-digit number
- e. 5-digit number – 2-digit number gives a 3-digit number

3.10 Playing with Number Patterns

Here are some numbers arranged in some patterns. Find out the sum of the numbers in each of the below figures. Should we add them one by one or can we use a quicker way?

- ✿ Share and discuss in class the different methods each one of you used to solve these questions.





3.11 An Unsolved Mystery – the Collatz Conjecture!

Look at the sequences below—the same rule is applied in all the sequences:

a. 12, 6, 3, 10, 5, 16, 8, 4, 2, 1

b. 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

c. 21, 64, 32, 16, 8, 4, 2, 1

d. 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

Do you see how these sequences were formed?

The rule is: one starts with any number; if the number is even, take half of it; if the number is odd, multiply it by 3 and add 1; repeat.

Notice that all four sequences above eventually reached the number 1. In 1937, the German mathematician, Lothar Collatz conjectured that the sequence will always reach 1, regardless of the whole number you start with. Even today — despite many mathematicians working on it — it remains an unsolved problem as to whether Collatz's conjecture is true! Collatz's conjecture is one of the most famous unsolved problems in mathematics.

- ✿ Make some more Collatz sequences like those above, starting with your favourite whole numbers.

Do you always reach 1? Do you believe the conjecture of Collatz that all such sequences will eventually reach 1? Why or why not?

3.12 Games and Winning Strategies

Numbers can also be used to play games and develop winning strategies.

Here is a famous game called 50. Play it with a classmate. Then try it at home with your family!

✿ **Game #1:** Half-Century Chase - Who will reach 50 first?

Rules of the Game:

Two players are batting partners - let's call them Player A and Player B.

The score board starts at 0.

Player A begins the innings and can score 1, 2, 3, or 4 runs on the first ball.

Then, the players take turns adding 1, 2, 3, or 4 runs to the previous total on the scoreboard.

The first player to make the scoreboard reach exactly 50 runs is the Half-Century Hero.

Example:

A says 3

B adds 2 and says 5

A adds 4 and says 9

B adds 1 and says 10

... and the match continues until one reach 50.

Which player can always win if they play correctly?

What is the pattern of numbers that the winning player should say?

There are many variations of this game. Here is another common variation:

✿ **Game #2 :** Century Clash - Who Will Make the 100 First?

Rules of the Game:

Two players are now star batsmen - Player A and Player B.

The scoreboard starts at 0.

Player A starts the innings and can score any number of runs from 1 to 10.

Then the two players take turns adding 1 to 10 runs to the previous total.

The first player to make the total exactly 100 becomes the Century Hero



Example:

A says 8

B adds 7 and says 15

A adds 9 and says 24

B adds 8 and says 32

...and the match continues until someone reaches 100

Play this game several times with your classmate. See if you can figure out the corresponding winning strategy in this case! Which player can always win? What is the pattern of numbers that the winning player should say this time?

Make your own variations of this game - decide how much one can add at each turn, and what number is the winning number. Then play your game several times, and figure out the winning strategy and which player can always win.

❁ Game #3 : The Decrementer**Rules of the Game:**

The scoreboard starts at 40 runs.

Two players (Player A and Player B) take turns.

On your turn, you can subtract either 1 run, 3 runs, or 5 runs from the current score.

The player who brings the total down to exactly 0 wins the match!

(That player "takes the final wicket")

Example:

Scoreboard: 40

A subtracts 3 → score = 37

B subtracts 5 → score = 32

A subtracts 1 → score = 31

... continue till one player says "0" - that player wins!



Figure it Out - 3.7

1. There is only one supercell (number greater than all its neighbours) in this grid. If you exchange two digits of one of the numbers, there will be 4 supercells. Figure out which digits to swap.

16,200	39,344	29,765
23,609	62,871	45,306
19,381	50,319	38,408

Try This

2. How many rounds does your year of birth take to reach the Kaprekar constant?
3. We are the group of 5-digit numbers between 35,000 and 75,000 such that all of our digits are odd. Who is the largest number in our group? Who is the smallest number in our group? Who among us is the closest to 50,000?
4. Estimate the number of holidays you get in a year including weekends, festivals and vacation. Then, try to get an exact number and see how close your estimate is.
5. Estimate the number of liters a mug, a bucket and an overhead tank can hold.
6. Write one 5-digit number and two 3-digit numbers such that their sum is 18,670.
7. Choose a number between 210 and 390. Create a number pattern similar to those shown in Section 3.10 that will sum up to this number.
8. Recall the sequence of Powers of 2 from Chapter 1, Table 1. Why is the Collatz conjecture correct for all the starting numbers in this sequence?
9. Check if the Collatz Conjecture holds for the starting number 100.
10. Starting with 0, players alternate adding numbers between 1 and 3. The first person to reach 22 wins. What is the winning strategy now?

CHAPTER MASTERY

A. True / False

- 575 is a palindromic number. (True / False)
- 6174 is called the Kaprekar constant for 4-digit numbers. (True / False)
- In a supercell table, the smallest number can be a supercell. (True / False)
- Every Collatz sequence eventually reaches 1. (True / False)

B. Multiple Choice Questions

- What is a palindromic number? ()
 - A number divisible by 10
 - A number read the same forwards and backwards
 - A number with repeated digits
 - A prime number
- The final digit sum of 1729 is: ()
 - 19
 - 10
 - 1
 - 7
- The 3-digit Kaprekar constant is: ()
 - 495
 - 999
 - 174
 - 100

C. Fill in the Blanks

- In the Collatz rule, if the number is even, we divide it by _____.

D. Assertion-Reason

- Assertion (A): 1221 is a palindromic number.

Reason (R): It reads the same from left to right and right to left.

Choose the correct option:

- Both A and R are true and R is the correct explanation of A
- Both A and R are true but R is not the correct explanation of A
- A is true but R is false
- A is false but R is true

10. In the "Supercells" activity, what specific condition must a number meet to be considered a Supercell? (understand)
11. Explain the two rules used to generate a sequence in the Collatz Conjecture. (understand)
12. Write an example for the following statement.
(4-digit number) – (4-digit number) = 3-digit number.
13. Compare the "Half-Century Chase" (reaching 50) and the "Century Clash" (reaching 100). Identify the inning strategy for the first player in the "Half-Century Chase" if they can add 1, 2, 3, or 4 runs each turn. (Analysis)

SUMMARY

- Numbers can be used for many different purposes including, to convey information, make and discover patterns, estimate magnitudes, pose and solve puzzles, and play and win games.
- Thinking about and formulating set procedures to use numbers for these purposes is a useful skill and capacity (called 'computational thinking').
- Many problems about numbers can be very easy to pose, but very difficult to solve. Indeed, numerous such problems are still unsolved (for example, Collatz's Conjecture).

4

DATA HANDLING AND PRESENTATION



Learning Outcomes:

Learners will be able to

- Collect and organize the data from the surroundings (CG-5)
- Organizes the recorded data in a convenient way for tabulation and represents the tabular data in to vernal form (CG-5)
- Compare different types of interpretation of data (CG-5)
- Select appropriate scale, draw and create new problems using the concept of representation of data (CG-5)
- Recognize interaction of mathematics with multiple subjects across science, social sciences, sports (CG-10)

4.0 Introduction

If you ask your classmates about their favourite colours, you will get a list of colours. This list is an example of data. Similarly, if you measure the weight of each student in your class, you would get a collection of measures of weight—again data.

Any collection of facts, numbers, measures, observations or other descriptions of things that convey *information* about those things is called **data**.

We live in an age of information. We constantly see large amounts of data presented to us in new and interesting ways. In this chapter, we will explore some of the ways that data is presented, and how we can use some of those ways to correctly display, interpret and make inferences from such data!

4.1 Collecting and Organising Data

Navya and Naresh are discussing their favourite games.



Cricket is my favourite game!

I play cricket sometimes but hockey is the game I like the most.



I think cricket is the most popular game in our class.

I am not sure. How can we find the most popular game in our class?



To figure out the most popular game in their class, what should Navya and Naresh do? Can you help them?

- Naresh and Navya decided to go to each student in the class and ask what their favourite game is. Then they prepared a list.

Navya is showing the list in **Table.1**



Karthik – Kabaddi	Pushkar - Football	Ananya - Kabaddi
Aadya – Hockey	Densy - Badminton	Kushal - Badminton
Simran – Kabaddi	Janvi - Football	Rajesh - Football
Nandu – Badminton	Leela - Hockey	Thara - Football
Ankita – Kabaddi	Afshan - Hockey	Yadaiah - Cricket
Yasodha – Hockey	Sankar - Cricket	Sanvi - Hockey
Neeraja – Cricket	Gurpreet - Hockey	Johnpal- Badminton
Rehana – Hockey	Teja - Kabaddi	Yuvraj - Football
Bhaskar– Badminton	Bhavya - Cricket	Harsha - Hockey
Monika – Football	Mohan - Kabaddi	Jyothi - Cricket
Tahira – Cricket		

Table 1

Navya says (happily), “I have collected the data. I can figure out the most popular game now!”.

A few other children are looking at the list and wondering, “We can’t yet see the most popular game. How can we get it from this list?”.

❁ **Answer the following from Table.1**

1. What would you do to find the most popular game among Naresh’s and Navya’s classmates?
2. What is the most popular game in their class?
3. Try to find out the most popular game among your classmates.
4. Haritha wants to respond to the questions given below. Put a tick (✓) for the questions where she needs to carry out data collection and put a cross (×) for the questions where she doesn't need to collect data. Discuss your answers in the classroom.
 - a. What is the most popular TV show among her classmates?
 - b. When did India get independence?
 - c. How much water is getting wasted in her locality?
 - d. What is the capital of India?

- ❁ Nileshe sir is a teacher. He decided to bring sweets to the class to celebrate the new year. The sweets shop nearby has jilebi, gulab jamun, laddu, barfi, and rasagulla. He wanted to know the choices of the children. He wrote the names of the sweets on the board and asked each child to tell him their preference. He put a tally mark 'I' for each student and when the count reached 5, he put a line through the previous four and marked it as $\overline{\text{||||}}$ in **Table 2**.

Table 2

Sweets	Tally Marks	No. of Students
Jilebi		6
Gulab jamun		9
Laddu		
Barfi		
Rasgulla		

❖ **Answer the following from Table.2**

1. Complete the table to help Nilesh sir to purchase the correct numbers of sweets:

a. How many students chose jilebi?

b. Barfi was chosen by students.

c. How many students chose Laddu?

d. Rasgulla was chosen by students.

e. How many students chose gulab jamun?

Nilesh Sir requested one of the staff members to bring the sweets as given in the table. The above table helped him to purchase the correct numbers of sweets.

2. Is the above table sufficient to distribute each type of sweet to the correct student? Explain. If it is not sufficient, what is the alternative?

*Frequency is the number of times a specific value or item appears in a data set. The numbers 6,9 etc., are the frequencies of the sweet preferences for jilebi, gulab jamun... respectively and the table prepared by Nilesh sir is called **Frequency distribution table.***

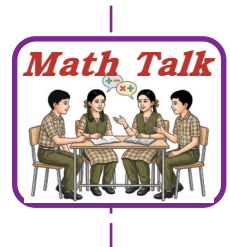
- ❁ Sandhya madam asked her students about the sizes of the shoes they wear. She noted the data on the board.

4	5	3	4	3	4	5	5	4
5	5	4	5	6	4	3	5	6
4	6	4	5	7	5	6	4	5

She then arranged the shoe sizes of the students in ascending order —
3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7

- ❁ **Answer the following from the above ascending order:**

- Help her to figure out the following:
 - The largest shoe size in the class is _____.
 - The smallest shoe size in the class is _____.
 - There are _____ students who wear shoe size 5.
 - There are _____ students who wear shoe sizes larger than 4.
- How did arranging the data in ascending order help to answer these questions?
- Are there other ways to arrange the data?



Project Work-1

Collect and write the names of a few flowers you see around. You record the data and fill in the following table:

Flower	No. of Flowers
Rose	
Jasmin	
...	
....	

- Which type of flower was found in the greatest number?
- Which type of flower was found in the smallest number?
- Were there any two types of flowers found in the same numbers?



Project Work-2

Take a blank piece of paper and paste any small news item from a newspaper. Each student may use a different article. Now, prepare a table on the piece of paper as given below. Count the number of each of the letters 'c', 'e', 'i', 'r' and 'x' in the words of the news article, and fill in the table.

Letter	c	e	i	r	x	Any other letter of your choice
Number of times found in the news item						

- The letter found the most number of times is _____.
- The letter found the least number of times is _____.
- List the five letters 'c', 'e', 'i', 'r', 'x' in ascending order of frequency. Now, compare the order of your list with that of your classmates. Is your order the same or nearly the same as theirs? (Almost everyone is likely to get the order 'x, c, r, i, e'.) Why do you think this is the case?

❁ **Example :** Anjali collected some data on the ways of transport used by students to go to school: walk, bus, auto, walk, auto, bus, walk, bicycle, walk, auto, bus, walk, bicycle, walk, walk, bicycle, bus, walk, bus, bus, walk, bicycle, walk, walk, tricycle, auto, bus, walk, auto, walk. Prepare frequency distribution table for this data.

Solution : The frequency distribution table with tally marks for the above data:



Transport	Tally Marks	Frequency
Walk		13
Bus		7
Bicycle		4
Auto		5
Tricycle		1



Figure it Out - 4.1

1. Chinnu listed the various means of transport that passed across the road in front of his house from 9 a.m. to 10 a.m on a holiday:

bike	car	bike	bus	bike	bike
bike	auto	bicycle	tricycle	bicycle	auto
car	scooter	car	auto	bicycle	bike
car	auto	bike	scooter	bike	car
bicycle	scooter	bicycle	scooter	bike	bus
auto	auto	bike	bicycle	bus	bike
bicycle	scooter	bus	scooter	auto	bike
scooter	bicycle	bike	tricycle	auto	scooter
car	scooter				

- Prepare a frequency distribution table for the data.
 - Which means of transport was used the most?
 - If you were there to collect this data, how could you do it? Write the steps or process.
2. Roll a die 15 times and record the number you obtain each time. Prepare a frequency distribution table using tally marks. Find the number that appeared:
- The minimum number of times.
 - The maximum number of times.
 - Find numbers that appeared an equal number of times.
3. Swapna prepared a frequency distribution table of data on the number of wickets taken by Jaspreet Bumrah in his last 30 matches.



Wickets Taken	Number of Matches
0	2
1	4
2	6
3	8
4	3
5	5
6	1
7	1



- What information is this table giving?
- What may be the title of this table?
- What caught your attention in this table?
- In how many matches has Bumrah taken 4 wickets?
- Siva says, "If we want to know the total number of wickets he has taken in his last 30 matches, we have to add the numbers 0, 1, 2, 3 ..., up to 7."

Can Siva get the total number of wickets taken in this way? Give Reason.







- How would you correctly figure out the total number of wickets taken by Bumrah in his last 30 matches, using this table?

Note to Teacher

Provide more opportunities to collect and organise data. Ask students to guess what is the most popular colour, game, toy, school subject, etc., amongst the students in their classroom and then collect the data for it. It can be a fun activity in which they also learn about their classmates. Discuss how they can organise the data in different ways, each way having its own advantages and limitations. For all these tasks and the tasks under 'Figure it Out', discuss the tasks with the children and let them understand the tasks, and then let them plan and present their research processes and conclusions in the class.

4.2 Pictographs

Pictographs are one visual and suggestive way to represent data without writing any numbers. Look at this picture — you may be familiar with it from previous classes.

Modes of Travelling	Number of Students	1 Student = 
Private Car		
Public Bus		
School Bus		
Cycle		
Walking		

This picture helps you understand at a glance the different modes of travel used by students. Based on this picture, answer the following questions:

- Which mode of travel is used by the most number of students?
- Which mode of travel is used by the least number of students?

A pictograph represents data through pictures of objects. It helps answer questions about data with just a quick glance.

In the above pictograph, one unit or symbol (😊) is used to represent one student. There are also other pictographs where one unit or symbol stands for many people or objects.

- ❁ **Example :** Nanda Kishore collected responses from the children of his school regarding how often they slept at least 9 hours during the night. He prepared a pictograph from the data:

Response	Number of Children (▲ = 10 Children)
Always	▲ ▲ ▲ ▲ ▲
Sometimes	▲ ▲ ▲ ▲ ▲
Never	▲ ▲ ▲ ▲ ▲

Answer the following questions using the pictograph.

1. What is the number of children who always slept at least 9 hours at night?
2. How many children sometimes slept at least 9 hours at night?
3. How many children always slept less than 9 hours each night? Explain how you got your answer.

Solutions:

1. In the table, there are 5 pictures ▲ for 'Always'. Each picture ▲ represents 10 children. Therefore, 5 pictures indicate $5 \times 10 = 50$ children.
2. There are 2 complete pictures ▲ ($2 \times 10 = 20$) and a half picture ▲ (half of $10 = 5$). Therefore, the number of children who sleep at least 9 hours only sometimes is $20 + 5 = 25$.

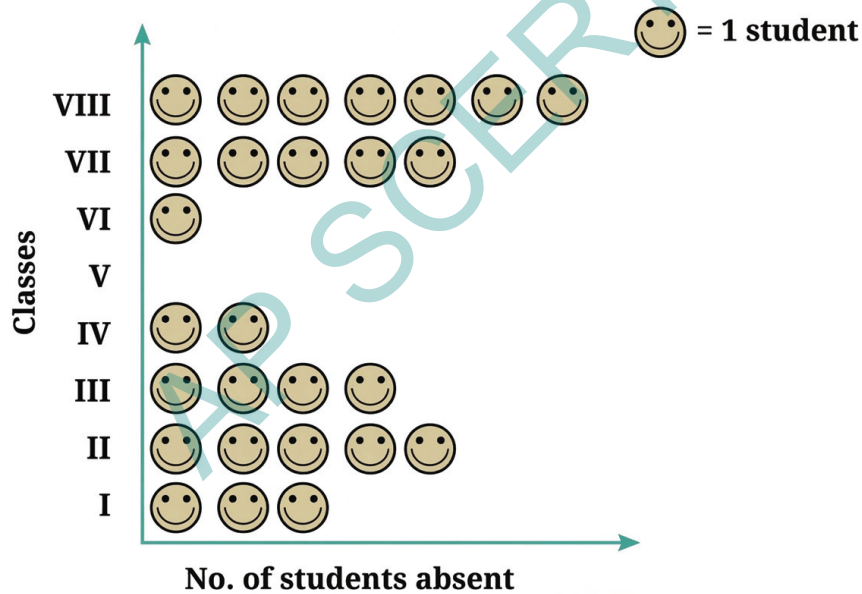
3. There are 4 complete pictures for 'Never'. Hence, $4 \times 10 = 40$ children never sleep at least 9 hours in a night, i.e., they always sleep less than 9 hours.

4.2.1. Drawing a Pictograph

One day, Vijay collected data on how many students were absent in each class:

Class	I	II	III	IV	V	VI	VII	VIII
No. of students	3	5	4	2	0	1	5	7

He created a pictograph to present this data and decided to show 1 student as 😊 in the pictograph.

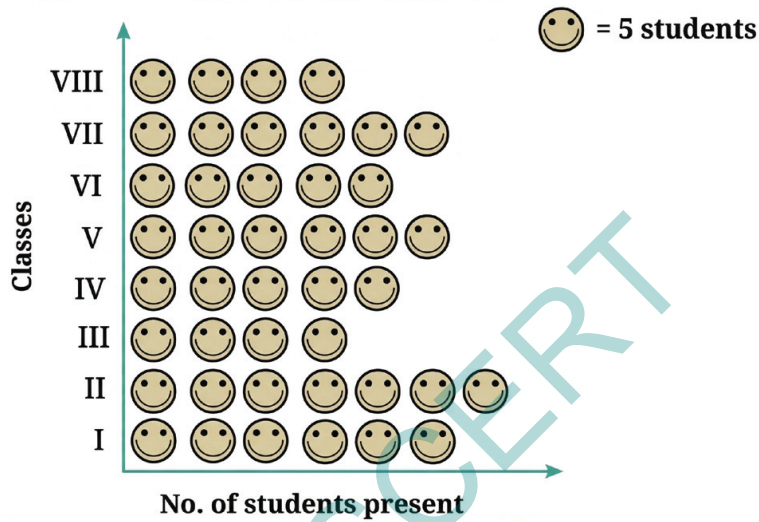


Pictograph-1

Meanwhile, his friends Fhiredouse and Sangita collected data on how many students were present in each class:

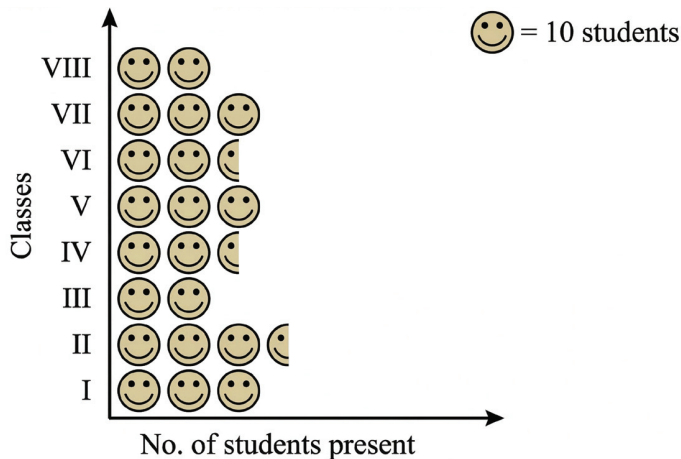
Class	I	II	III	IV	V	VI	VII	VIII
No. of students	30	35	20	25	30	25	30	20

✿ If they want to show their data through a pictograph, where they also use one symbol 😊 for each student, as Vijay did, what are the challenges they might face? Firdhouse made a plan to address this - since there were many students, she decided to use 😊 to represent 5 students. She figured students, she decided to use that would save time and space too. Her pictograph.2 is as follows:



Pictograph-2

Sangita decided to use one 😊 to represent 10 students instead. Since she used one 😊 to show 10 students, she had a problem in showing 25 students and 35 students in the pictograph. Then, she realised she could use 😊 to show 5 students. and her pictograph. 3 is as follows:



Pictograph-3

- ❖ What could be the problems faced in preparing such a pictograph, if the total number of students present in a class is 33 or 27?









Pictographs are a nice visual and suggestive way to represent data. They represent data through pictures of objects.

- Pictographs can help in answering the questions and make inferences about data with just a quick glance (in the examples above— about favourite games, favourite colours, most common modes of conveyance, number of students absent, etc.).
- By reading a pictograph, we can quickly understand the frequencies of the different categories (for example, cricket, hockey, etc.) and the comparisons of these frequencies.
- In a pictograph, the categories can be arranged horizontally or vertically. For each category, simple pictures and symbols are then drawn in the designated columns or rows according to the frequency of that category.
- A scale or key (for example, 😊: 1 student or 😊: 5 students) is added to show what each symbol or picture represents. Each symbol or picture can represent one unit or multiple units.
- It can be more challenging to prepare a pictograph when the amount of data is large or when the frequencies are not exact multiples of the scale or key.



Figure it Out - 4.2

- The following pictograph shows the number of books borrowed by students, in a week, from the library of a high School,

Day	Number of Books Borrowed	( = 1 Book)
Monday		
Tuesday		
Wednesday		
Thursday		
Friday		
Saturday		

- On which day were the minimum number of books borrowed?
- What was the total number of books borrowed during the week?
- On which day were the maximum number of books borrowed?
What may be the possible reason?

2. Anil sells kites in Gandhi street. Six shopkeepers from nearby villages come to purchase kites from him. The number of kites he sold to these six shopkeepers are given below -

Shopkeeper	Number of Kites Sold
Chandu	250
Rani	300
Rukhsana	100
Arjun	450
Mary	250
Poonam	700

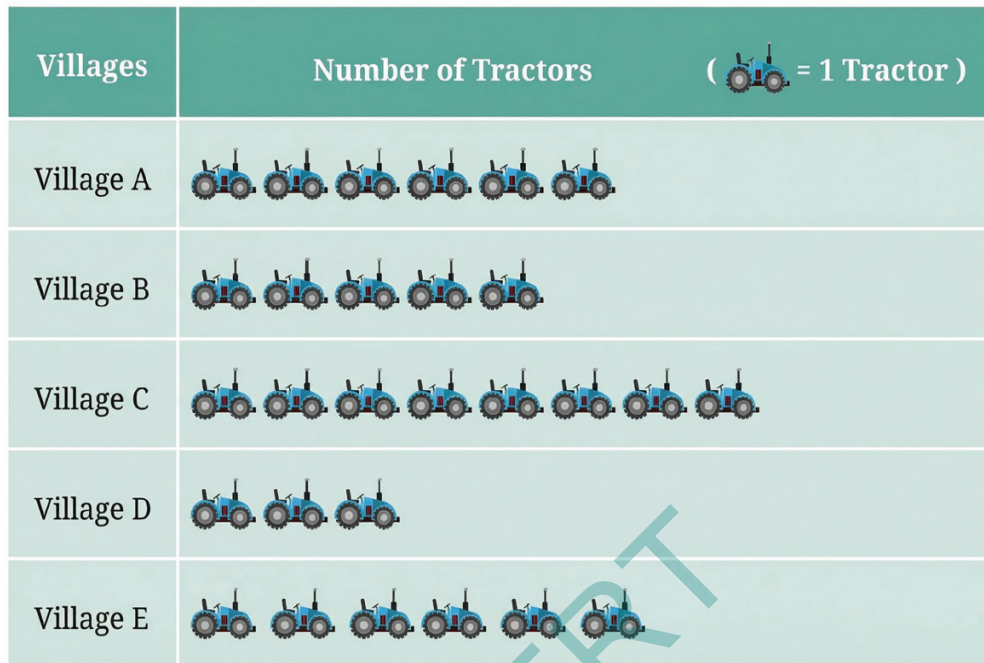


Prepare a pictograph using the symbol  to represent 100 kites.

Answer the following questions:

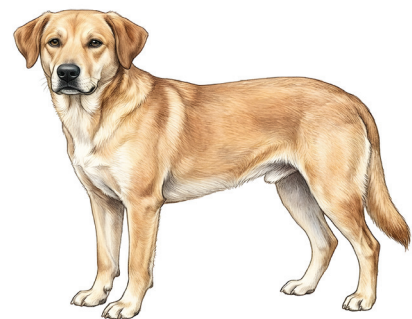
- How many symbols represent the kites that Rani purchased?
- Who purchased the maximum number of kites?
- Who purchased more kites, Arjun or Chandu?
- Rukhsana says Poonam purchased more than double the number of kites that Rani purchased. Is she correct? Why?

3. The following pictograph shows the number of tractors in the five different villages:

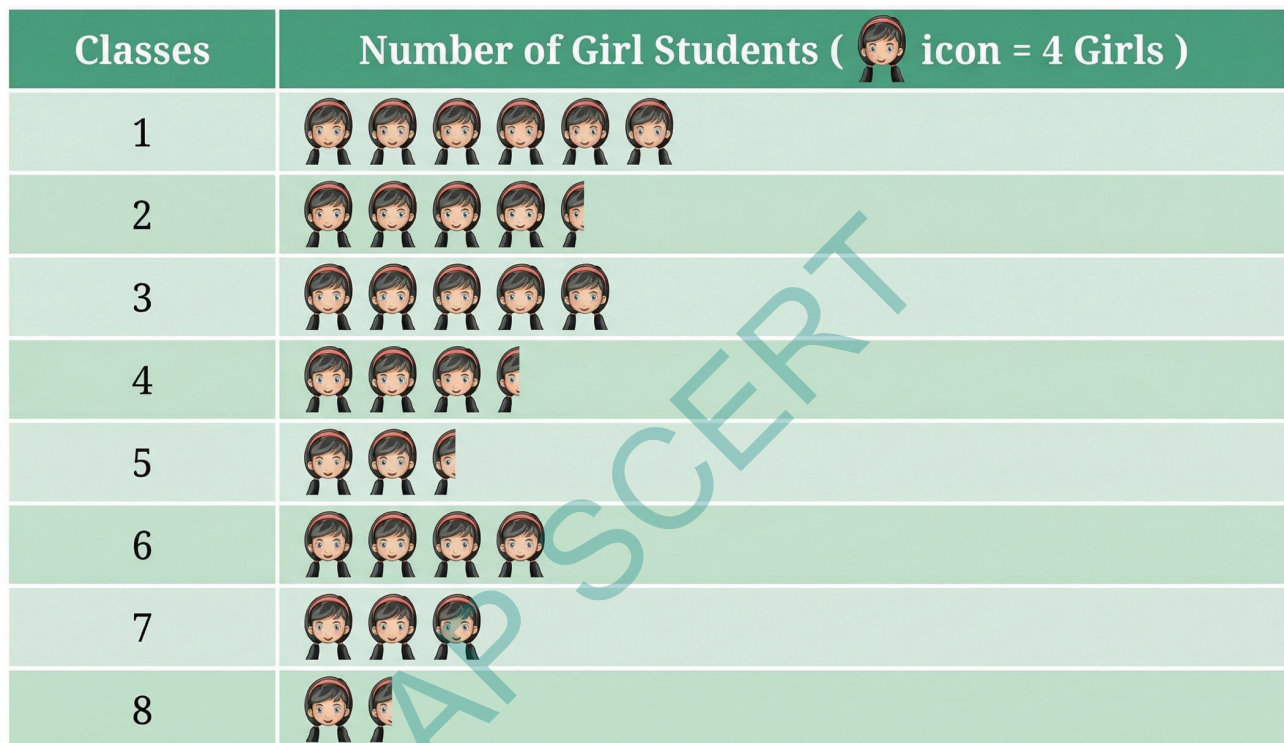


Observe the pictograph and answer the following questions—

- Which village has the least number of tractors?
 - Which village has the most number of tractors?
 - How many more tractors does Village C have than Village B?
 - Kamala says, “Village D has half the number of tractors as Village E.” Is she right?
4. Indian Pariah dogs (a type of breed of Indian dogs) are largely found in Tirupati district. The government took an initiative to protect this breed by providing support to those who adopted these dogs. Due to this initiative, the number of these dogs increased. The number of dogs in six villages of the district are as follows -
 Village A : 18, Village B : 36, Village C : 12,
 Village D : 48, Village E : 18, Village F : 24
 Prepare a pictograph and answer the following questions:
- What will be a useful scale or key to draw this pictograph?
 - How many symbols will you use to represent the dogs in Village B?



- c. Harika said that the number of these dogs in Village B and Village D together will be more than the number of these dogs in the other 4 villages. Is she right? Give reasons for your response.
5. The number of girl students in each class of a school is depicted by the pictograph:



Observe this pictograph and answer the following questions:

- Which class has the least number of girl students?
- What is the difference between the number of girls in Class 5 and 6?
- If two more girls were admitted in Class 2, how would the graph change?
- How many girls are there in Class 7?

4.3 Bar Graphs

Have you seen graphs like this on TV or in a newspaper?

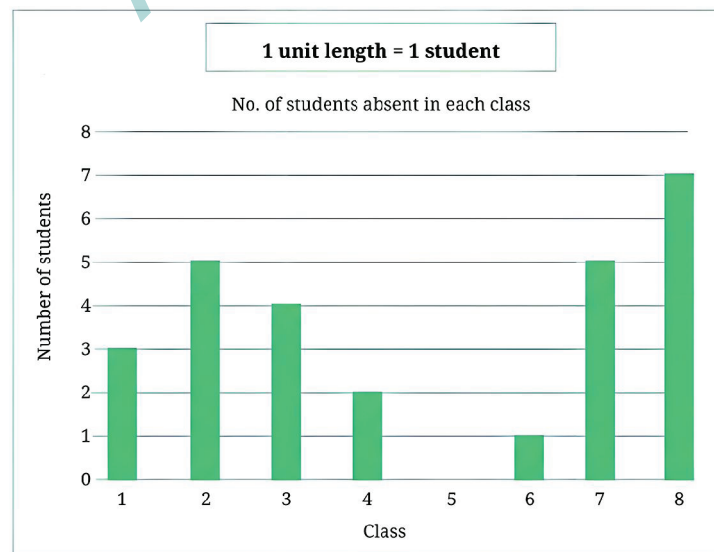
Like pictographs, such bar graphs can help us to quickly understand and interpret information, such as the highest value, the comparison of values of different categories, etc. However, when the amount of data is large, presenting it by a pictograph is not only time consuming but at times difficult

too. Let us see how data can be presented instead using a bar graph.

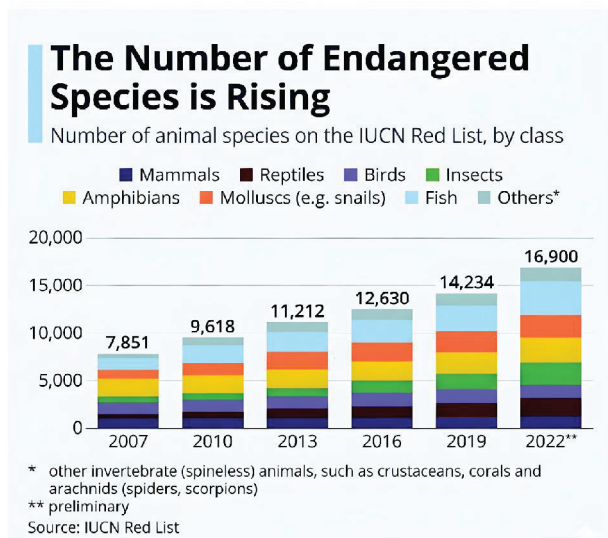
Let's take the data collected by Vijay earlier, regarding the number of students absent on one day in each class:

Class	I	II	III	IV	V	VI	VII	VIII
No. of students	3	5	4	2	0	1	5	7

He presented the same data using a bar graph-1



Bar Graph-1



Note to Teachers

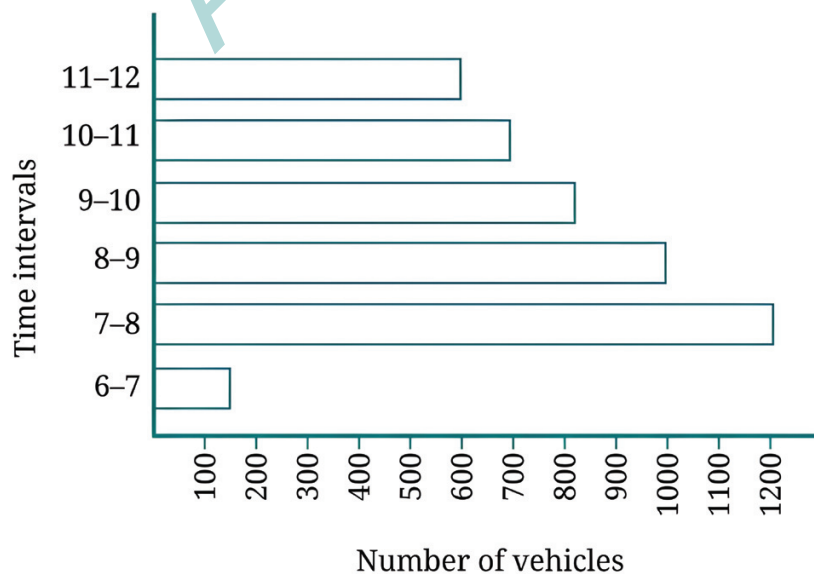
If the students have not noticed, please point out the equally spaced horizontal lines. Explain that this means each pair of consecutive numbers on the left has the same gap.

✿ Answer the following questions using the bar graph :

1. In Class 2, _____ students were absent that day.
2. In which class were the maximum number of students absent?
3. Which class had full attendance that day?

✿ When making bar graphs, bars of uniform width can be drawn horizontally or vertically with equal spacing between them; then the length or height of each bar represents the given number. As we saw in pictographs, we can use a scale or key when the frequencies are larger.

Let us look at an example of vehicular traffic at a busy road crossing in Amaravathi, which was studied by the traffic police on a particular day. The number of vehicles passing through the crossing each hour from 6 a.m. to 12:00 noon is shown in the bar graph.2. One unit of length stands for 100 vehicles.



Bar Graph-2

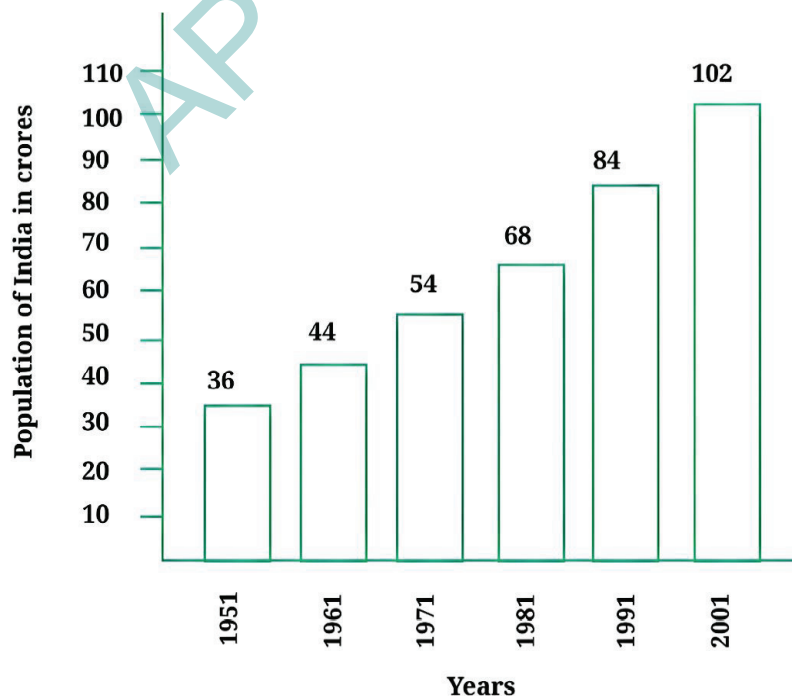
❁ **Observe that**

- The maximum traffic at the crossing is shown by the longest bar, i.e., for the time interval 7-8 a.m. During that time, 1200 vehicles passed through the crossing.
 - The second longest bar is for 8-9 a.m. (1000 vehicles)
 - Similarly, the minimum traffic is shown by the smallest bar, i.e., the bar for the time interval 6-7 a.m.(150 Vehicles)
 - The second smallest bar is that for the 11 a.m.-12 noon, (600 vehicles).
- The total number of cars passing through the crossing during the two-hour interval 8.00-10.00 a.m. as shown by the bar graph is about $1000 + 800 = 1800$ vehicles.

❁ **Answer the following from bar graph.2**

1. How many total cars passed through the crossing between 6 a.m. and 12 noon?
2. Why do you think so little traffic occurred during the hour of 6-7 a.m., as compared to the other hours from 7 a.m. to 12 noon?
3. Why do you think the traffic was the heaviest between 7-8 a.m.?
4. Why do you think the traffic was lesser and lesser each hour after 8 a.m. all the way until 12 noon?

❁ **Example :**



Bar Graph-3

This bar graph.3 shows the population of India in each decade over a period of 50 years. The numbers are expressed in crores. If you were to take 1 unit length to represent one person, drawing the bars will be difficult! Therefore, we choose the scale so that 1 unit represents 10 crores. The bar graph for this choice is shown in the figure. So a bar of length 5 units represents 50 crores and of 8 units represents 80 crores.

- On the basis of this bar graph.3, what may be a few questions you may ask your friends?
- How much did the population of India increase over 50 years? How much did the population increase in each decade?

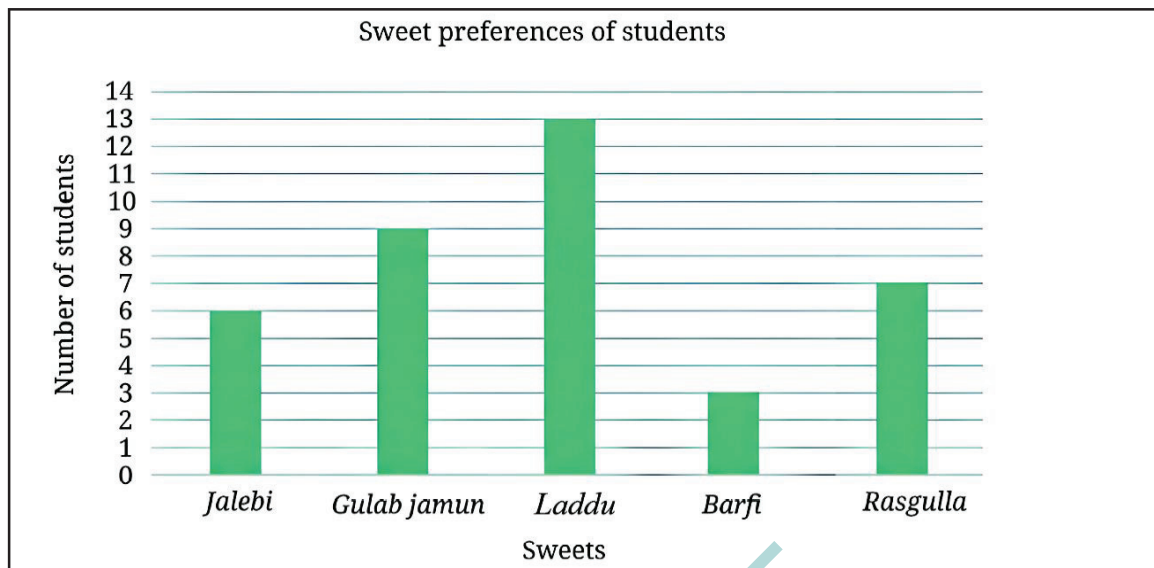
4.3.1 Drawing a Bar Graph

In a previous example, Nilesh Sir prepared a frequency table.2 representing the sweet preferences of the students in his class. Let's try to prepare a bar graph to present his data —

Sweet	No. of Students
Jilebi	6
Gulab jamun	9
Laddu	13
Barfi	3
Rasgulla	7

1. First, we draw a horizontal line and a vertical line. On the horizontal line, we will write the name of each of the sweets, equally spaced, from which the bars will rise in accordance with their frequencies; and on the vertical line we will write the frequencies representing the number of students.
2. We must choose a scale. That means we must decide how many students will be represented by a unit length of a bar so that it fits nicely on our paper. Here, we will take 1 unit length to represent 1 student.
3. For jilebi, we therefore need to draw a bar having a height of 6 units (which is the frequency of the sweet jilebi), and similarly for the other sweets we have to draw bars as high as their frequencies.

4. We, therefore, get a bar graph.4 as shown below —



Bar Graph-4

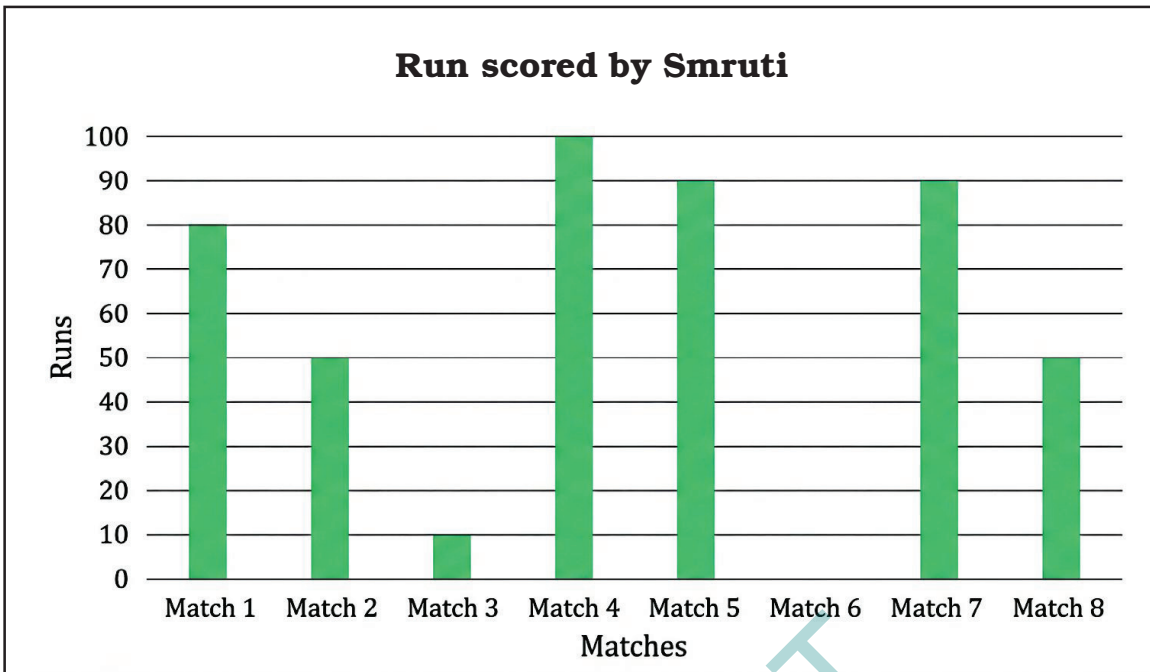
When the frequencies are larger and we cannot use the scale of 1 unit length = 1 number (frequency), we need to choose a different scale like we did in the case of pictographs.

Example : The number of runs scored by Smruti in each of the 8 matches are given in the table below:

Match	Match	Match	Match	Match	Match	Match	Match	Match
1	2	3	4	5	6	7	8	
Runs	80	50	10	100	90	0	90	50

In this example, the minimum score is 0 and the maximum score is 100. Using a scale of 1 unit length = 1 run would mean that we have to go all the way from 0 to 100 runs in steps of 1. This would be unnecessarily tedious. Instead, let us use a scale where 1 unit length = 10 runs. We mark this scale on the vertical line and draw the bars according to the scores in each match. We get the bar graph.5 representing the above data.





Example : The following table shows the monthly expenditure of Imran's family on various items:

Items	Expenditure (in ₹)
House rent	3000
Food	3400
Education	800
Electricity	400
Transport	600
Miscellaneous	1200

To represent this data in the form of a bar graph, here are the steps —

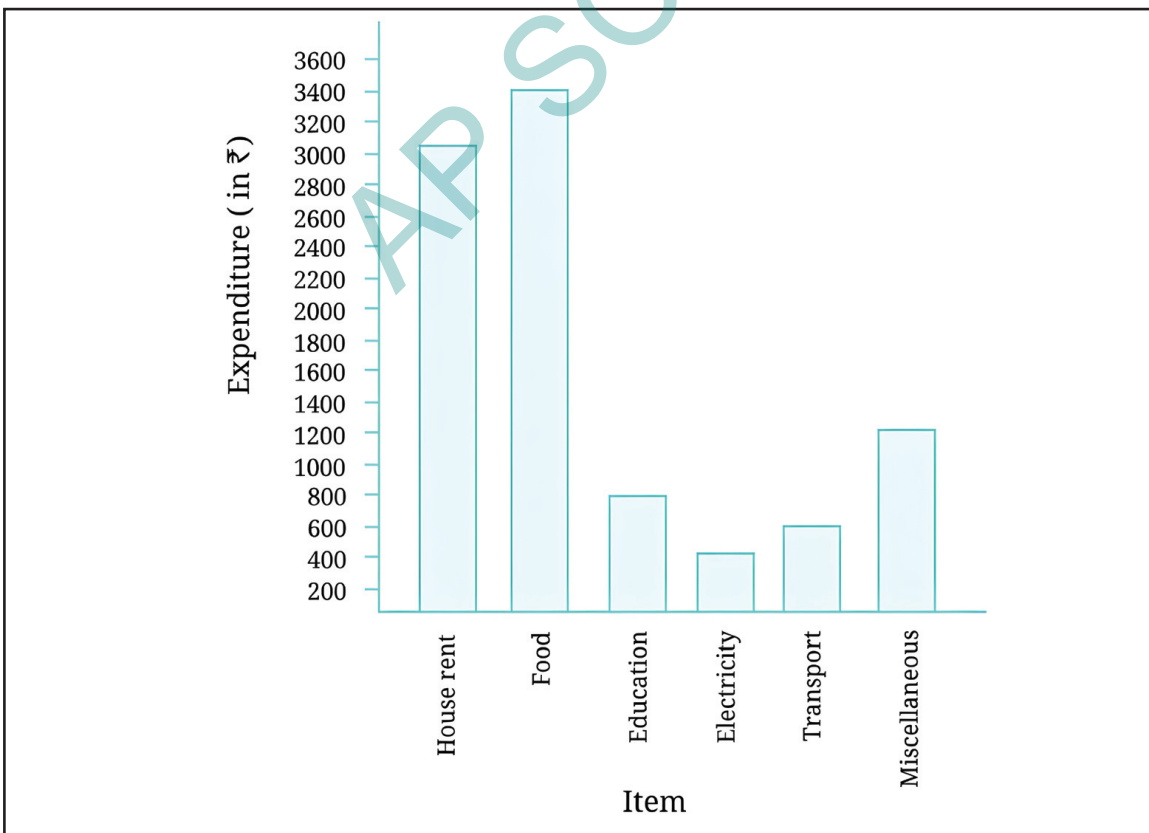
- Draw two perpendicular lines, one horizontal and one vertical.
- Along the horizontal line, mark the 'items' with equal spacing between them and mark the corresponding expenditures along the vertical line.

- Take bars of the same width, keeping a uniform gap between them.
- Choose a suitable scale along the vertical line. Let, 1 unit length = ₹ 200, and then mark and write the corresponding values (₹ 200, ₹ 400, etc.) representing each unit length.

Finally, calculate the heights of the bars for various items as shown below —

House rent	$3000 \div 200$	15 units
Food	$3400 \div 200$	17 units
Education	$800 \div 200$	4 units
Electricity	$400 \div 200$	2 units
Transport	$600 \div 200$	3 units
Miscellaneous	$1200 \div 200$	6 units

Here is the bar graph.6 that we obtain based on the above steps:



Bar Graph-6

✿ Use the bar graph-6 to answer the following questions:

1. On which item does Imran's family spend the most and the second most?
2. Is the cost of electricity about one-half the cost of education?
3. Is the cost of education less than one-fourth the cost of food?

- Like pictographs, bar graphs give a nice visual way to represent data. They represent data through equally-spaced bars, each of equal width, where the lengths or heights give frequencies of the different categories.
- Each category is represented by a bar where the length or height depicts the corresponding frequency or quantity (for example, cost runs).
- The bars have uniform spaces between them to indicate that they are free standing and represent different categories.
- The bars help in interpreting data much faster than a frequency table. By reading a bar graph, we can compare frequencies of different categories at a glance.
- We must decide the scale (for example, 1 unit length = 1 student or 1 unit length = ₹ 200) for a bar graph on the basis of the data including the minimum and maximum frequencies, so that the resulting bar graph fits nicely and looks visually appealing on the paper or poster we are preparing. The markings of the unit lengths as per the scale must start from zero.

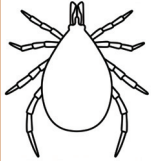

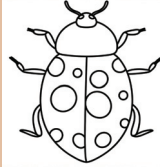

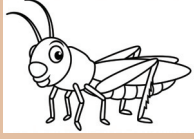
Note to Teacher

The main focus of this chapter is to learn how to handle data to find answers to specific questions or inquiries, to test hypotheses or to take specific decisions. This should be kept in mind when providing practice opportunities to collect, organise and analyse data.



Figure it Out - 4.3

1. Samantha visited a tea garden, and collected data of the insects and critters she saw there. Here is the data she collected:

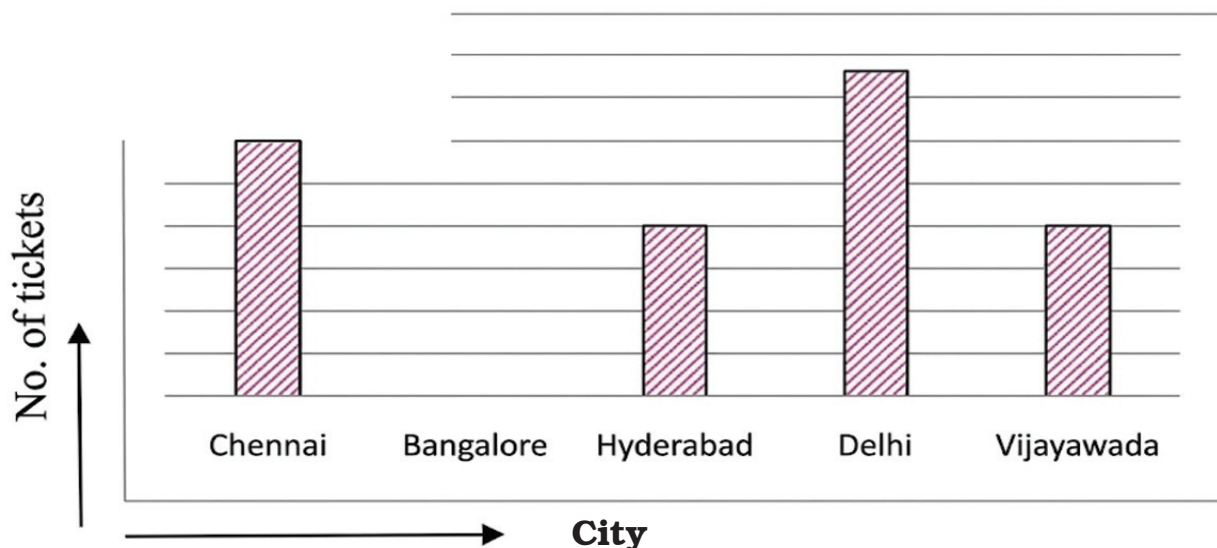
				
Mites	Caterpillars	Beetles	Butterflies	Grasshoppers
6	10	5	3	2

Help her in preparing a bar graph to represent this data.

2. Pooja collected data on the number of tickets sold at the Tirupati railway station for a few different cities of India over a period of two hours.

City	Chennai	Bangalore	Hyderabad	Delhi	Vijayawada
Number of tickets	24	20	16	28	16

She used this data and prepared a bar graph on the board to discuss the data with her students, but someone erased a portion of the graph.

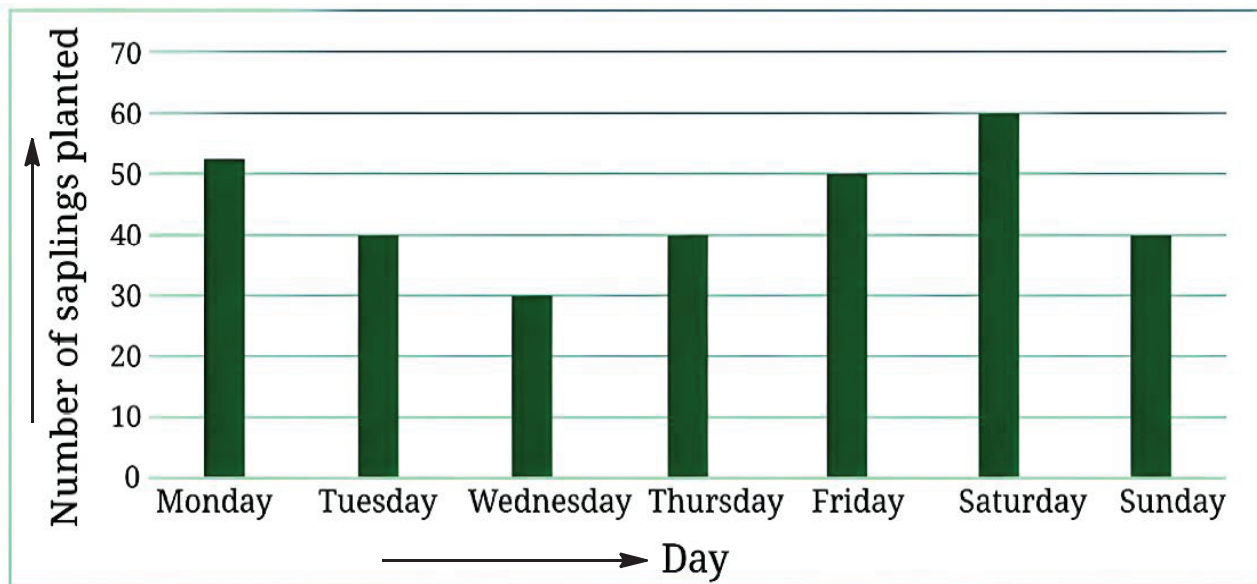


- a. Write the number of tickets sold for Chennai above the bar.
 - b. Write the number of tickets sold for Delhi above the bar.
 - c. The bar for Chennai is 6 unit lengths and the bar for Hyderabad is 4 unit lengths. What is the scale for this graph?
 - d. Draw the correct bar for Bangalore.
 - e. Add the scale of the bar graph by placing the correct numbers on the vertical axis.
 - f. Are the bars for Chennai and Delhi correct in this graph? If not, draw the correct bar(s).
3. A survey of 120 school students was conducted to find out which activity they preferred to do in their free time:

Preferred Activity	Number of Students
Playing	45
Reading story books	30
Watching TV	20
Listening to music	10
Painting	15

Draw a bar graph to illustrate the above data taking the scale of 1 unit length = 5 students. Which activity is preferred by most students other than playing?

4. Students and teachers of a primary school decided to plant tree saplings in the school campus and in the surrounding village during the first week of July. Details of the saplings they planted are as follows —



- The total number of saplings planted on Wednesday and Thursday is _____.
- The total number of saplings planted during the whole week is _____.
- The greatest number of saplings were planted on _____ and the least number of saplings were planted on _____.

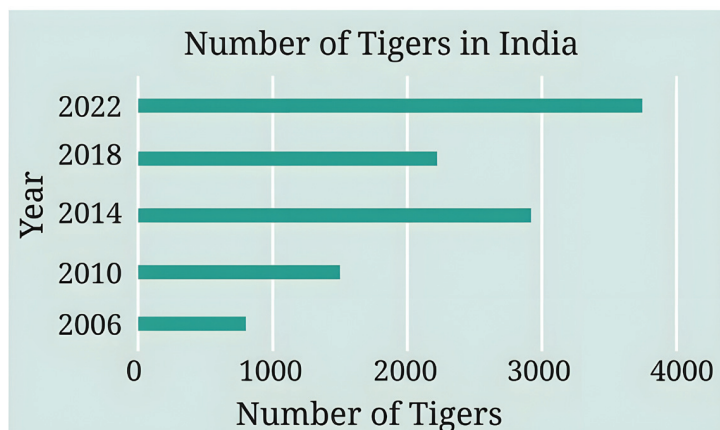
Why do you think that is the case? Why were more saplings planted on certain days of the week and less on others? Can you think of possible explanations or reasons? How could you try and figure out whether your explanations are correct?



- India launched a Project Tiger in 1973 to track and protect the tigers. The data is from 2006 and 2022 in four-year intervals. Murthy and Divya looked up information and prepared a frequency table for this data along with a bar graph. But there are a few mistakes in the graph. Can you find those mistakes and fix them?



Year	Number of Tigers (approx.)
2006	1400
2010	1700
2014	2200
2018	3000
2022	3700



4.4 Artistic and Aesthetic Considerations

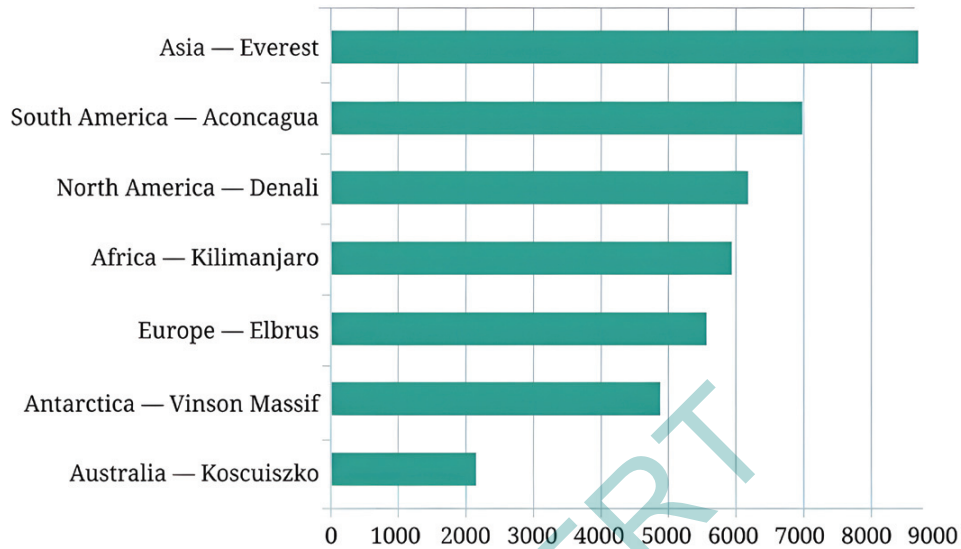
In addition to the steps described in previous sections, there are also some artistic and aesthetic aspects one can consider when preparing visual presentations of data to make them more interesting and effective. While making a visual presentation of data such as a pictograph or bar graph, it is important to make it fit in the intended space by choosing the scale appropriately. It is also desirable to make the data presentation visually appealing and easy-to-understand and appreciate.

Here is an example naming the tallest mountain on each continent, along with the height of each mountain in meters.

Continent	Asia	South America	North America	Africa	Europe	Antarctica	Australia
Tallest Mountain	Everest	Aconcagua	Denali	Kilimanjaro	Elbrus	Vinson Massif	Kosciuszko
Height	8848m	6962m	6194m	5895m	5642m	4892m	2228m

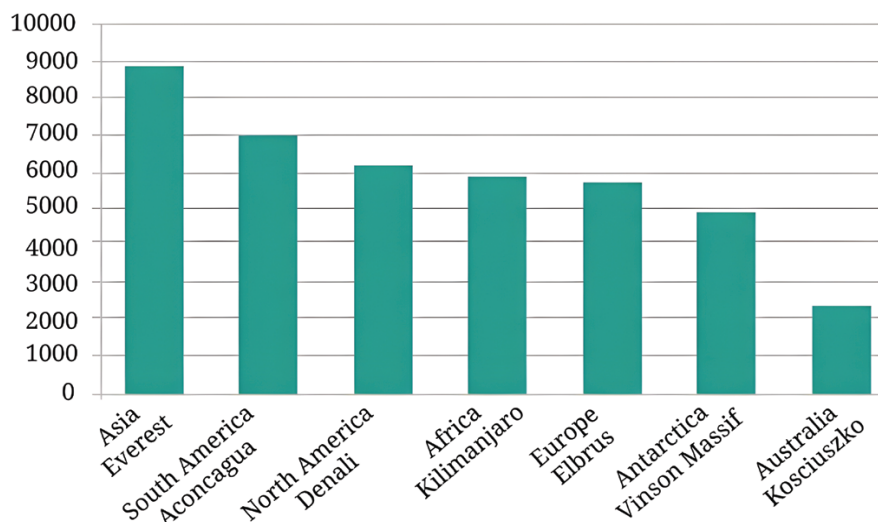
How much taller is Mount Everest than Mount Kosciuszko? Are Mount Denali and Mount Kilimanjaro very different in height? This is not so easy to quickly discern from a large table of numbers.

Now, we can convert the table of numbers into a bar graph, as shown. Here, each value is drawn as a horizontal box. These are longer or shorter depending on the number they represent. This makes it easier to compare the heights of all these mountains at a glance.



However, since the boxes represent heights, it is better and more visually appealing to rotate the picture, so that the boxes grow upward, vertically from the ground like mountains. A bar graph with vertical bars is also called a column graph. Columns are the pillars you find in a building that hold up the roof.

Below is a column graph for our table of tallest mountains. From this column graph, it becomes easier to compare and visualise the heights of the mountains.



In general, it is more intuitive, suggestive and visually appealing to represent heights, that are measured upwards from the ground, using bar graphs that have vertical bars or columns. Similarly, lengths that are parallel to the ground (for example, distances between location on Earth) are usually best represented using bar graphs with horizontal arcs.

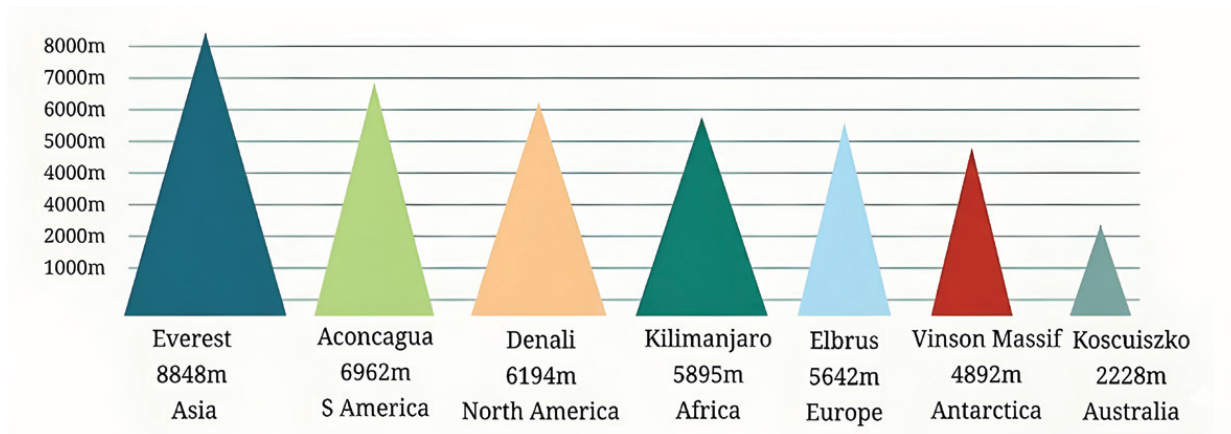


Let's Explore

1. If you wanted to visually represent the data of the heights of the tallest persons in each class in your school, would you use a graph with vertical bars or horizontal bars? Why?
2. If you were making a table of the longest rivers on each continent and their lengths, would you prefer to use a bar graph with vertical bars or with horizontal bars? Why? Try finding out this information, and then make the corresponding table and bar graph! Which continents have the longest rivers?

Infographics

- ❁ Data visualisations such as bar graphs are further beautified with more extensive artistic and visual imagery, they are called information graphics or infographics in short. The aim of infographics is to make use of attention-attracting and engaging visuals to communicate information even more clearly and quickly, in a visually pleasing way.
- ❁ As an example of how infographics can be used to communicate data even more suggestively, let us go back to the table above listing the tallest mountain on each continent. We drew a bar graph with vertical bars (columns) rather than horizontal bars, to be more indicative of mountains. But instead of rectangles, we could use triangles, which look a bit more like mountains. And, we can add a splash of colour as well. Here is the result.



The infographic looks more appealing and suggestive at first glance, yet, it has some issues. The goal of our bar graph earlier was to represent the heights of various mountains - using bars of the appropriate heights but the same widths. The purpose of using the same widths was to make it clear that we are only comparing heights.

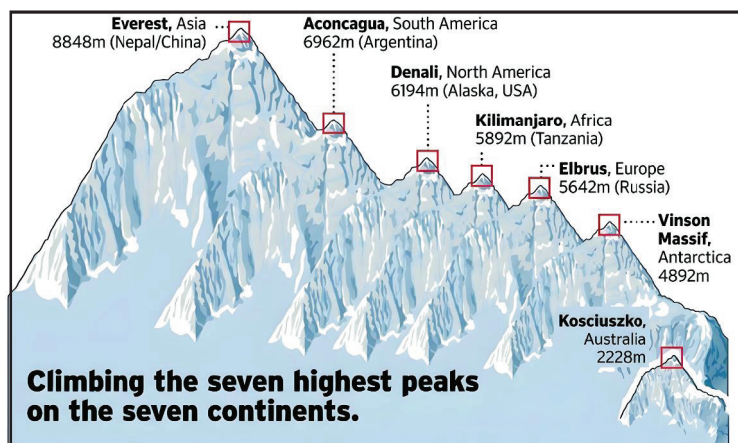
However, in this infographic, the taller triangles are also wider! Are taller mountains always wider? The infographic is implying additional information that may be misleading and may or may not be correct. Sometimes going for more appealing pictures can also accidentally mislead.

Taking this idea further, and to make the picture even more visually stimulating and suggestive, we can further change the shapes of the mountains to make them look even more like mountains, and add other details, while attempting to preserve the heights.

For example, we can create an imaginary mountain range that contains all these mountains.

Is the infographic below better than the column graph with rectangular columns of equal width? The mountains look more realistic, but is the picture accurate?

For example, Everest appears to be twice as tall as Elbrus.



What is 5642×2 ?

While preparing visually-appealing presentations of data, we also need to be careful that the pictures we draw do not mislead us about the facts. In general, it is important to be careful when making or reading infographics.



CHAPTER MASTERY

I. Answer the following questions :

- Using tally marks which one of the following represents the number 7
 -
 -
 -
 -
- The marks (out of 10) obtained by 15 students in a mathematics test are listed as below: 8, 0, 5, 8, 3, 0, 8, 10, 10, 3, 4, 8, 7, 8, 9 The number of students who obtained marks more than or equal to 8 is
 - 5
 - 8
 - 6
 - 7
- Fill the following table which represents shirt size of 40 students in a school






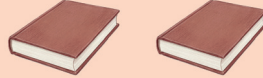
Shirt size	Tally Marks	Number of students
30		3
32		—
34	—	8
36		—
38	—	10
40	—	7

II. True or False :

- 4. In a pictograph, if a symbol  represents 20 flowers in a basket then  stands for 30 flowers.
- 5. In a bar graph, the width of bars may be unequal
- 6. In a bar graph, the gap between two consecutive bars may not be the same.

III. Solve the following :

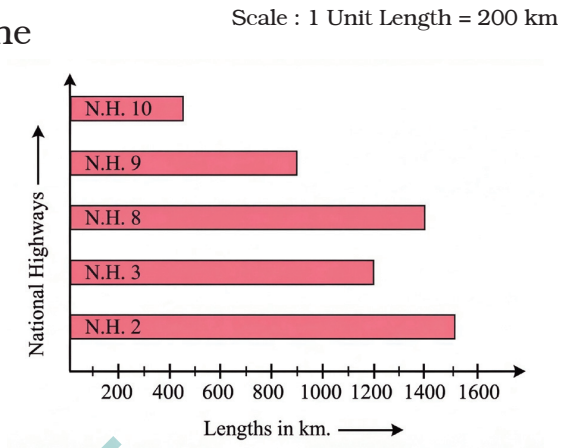
- 7. In an examination, the grades achieved by 30 students of a class are given below. Arrange these grades in a table using tally marks: B, C, C, E, A, C, B, B, D, D, D, D, B, C, C, C, A, C, B, E, A, D, C, B, E, C, B, E, C, D
- 8. A survey was carried out in a certain school to find out the popular school subjects among students of Classes VI to VIII. The data in this regard is displayed as pictograph given below:

Subject	Number of Students  = 50 students
Hindi	
English	
Mathematics	
Science	
Social Studies	

- a) Which subject is most popular among the students?
- b) How many students like Mathematics?
- c) Find the number of students who like subjects other than Mathematics and Science.

9. The bar graph given below represents approximate length (in kilometres) of some National Highways in India. Study the bar graph and answer the following questions

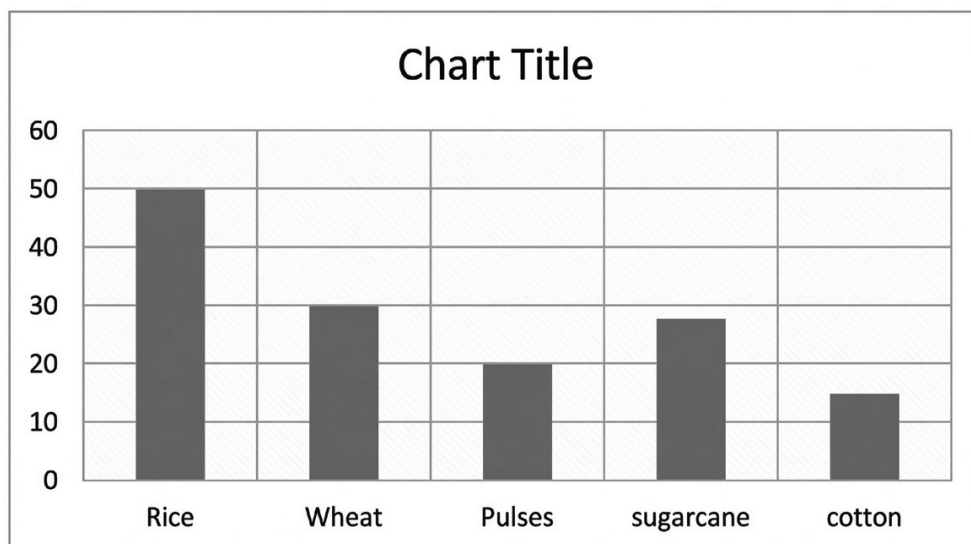
- Which National Highway (N.H.) is the longest among the above?
- Which National Highway is the shortest among the above?
- What is the length of National Highway 9?
- Length of which National Highway is about three times the National Highway 10?



10. The following table shows the area of the land on which different crops were grown

Type of crop	Rice	Wheat	Pulses	Sugarcane	Cotton
Area of land (sq.u)	50	30	20	25	10

Babji draw the bar graph for this information as given below. Tulasi checked it and told that something is wrong. Can you find what is wrong and correct it?



11. The number of ATMs of different banks in a city is shown below:

Bank	Number of ATMs
HDFC Bank	5
Union Bank	15
Indian Bank	20
State Bank of India	25
Canara Bank	10

Draw a bar graph to represent the above information by choosing the scale of your choice.

SUMMARY

- Data can be organised in a tabular form using tally marks for easy analysis and interpretation.
- Frequencies are the counts of the occurrences of values, measures or observations.
- Pictographs represent data in the form of pictures, or objects or parts of objects. Each picture represents a frequency which can be 1 or more than 1 — this is called the scale and it must be specified.
- Bar graphs have bars of uniform width; the length or height that indicates the total frequency of occurrence. The scale that is used to convert length or height to frequency again, must be specified.
- Choosing the appropriate scale for a pictograph or bar graph is important to accurately and effectively convey the desired information or data and to also make it visually appealing.
- Other aspects of a graph also contribute to its effectiveness and visual appeal such as how colours are used, what accompanying pictures are drawn, and whether the bars are horizontal or vertical. These aspects correspond to the artistic and aesthetic side of data handling and presentation.

5

PRIME TIME



Learning Outcomes:

Learners will be able to

- Define and list the factors and multiples of given numbers. (CG-1)
- Find common factors and common multiples of two or more numbers. (CG-1)
- Identify prime and composite numbers up to 100. (CG-1)
- Use prime factorization to represent numbers and solve the related problems. (CG-2)
- Use divisibility rules to check quickly. (CG-8)
- Participate in activities like factor and multiple games to build number sense. (CG-7)

5.0 Introduction

This chapter introduces the idea of factors and multiples through fun games like the idli-vada game, helping students notice patterns in numbers. By observing when numbers repeat in different lists, children learn what common multiples, common factors, prime numbers, composite numbers, and co-prime numbers are. The chapter also shows how every number (except 1) can be broken down into prime factors, and how this helps in checking divisibility and understanding the structure of numbers. Through puzzles, diagrams, and activities, students build a strong foundation for number sense in an enjoyable and intuitive way.

5.1 Common Multiples and Common Factors



Idli-Vada Game

Children sit in a circle and play a game of numbers.

One of the children starts by saying '1'. The second player says '2', and so on. But when it is the turn of 3, 6, 9, ... (multiples of 3), the player should say 'idli' instead of the number. When it is the turn of 5, 10, ... (multiples of 5), the player should say 'vada' instead of the number. When a number is both a multiple of 3 and a multiple of 5, the player should say 'idli-vada'. If a player makes any mistake, they are out.

The game continues in rounds till only one person remains.

For which numbers should the players say 'idli' instead of saying the number? These would be 3, 6, 9, 12, 18, ... and so on.

For which numbers should the players say 'vada'? These would be 5, 10, 15, 20, ... and so on.

Which is the first number for which the players should say, 'idli-vada'? It is 15, which is a multiple of 3, and also a multiple of 5. Find out other such numbers that are multiples of both 3 and 5. These numbers are called _____.





Figure it Out - 5.1

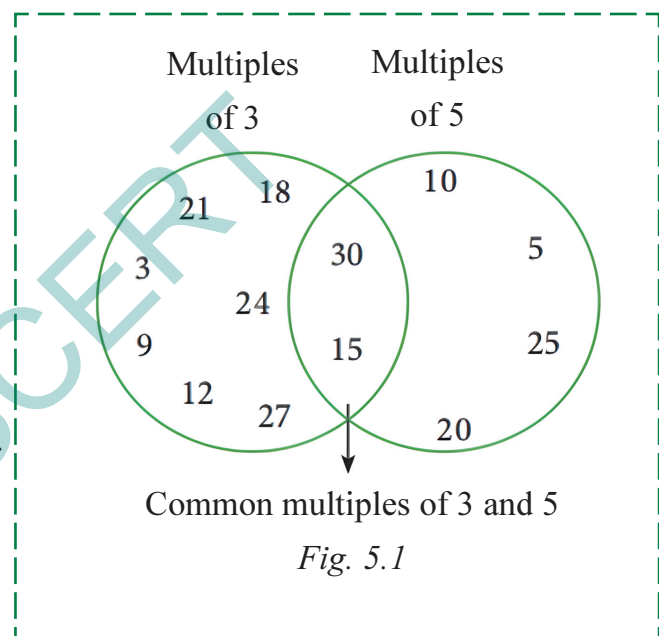
1. At what number is 'idli-vada' said for the 10th time?
2. If the game is played for the numbers 1 to 90, find out:
 - a. How many times would the children say 'idli' (including the times they say 'idli-vada')?
 - b. How many times would the children say 'vada' (including the times they say 'idli-vada')?
 - c. How many times would the children say 'idli-vada'?
3. What if the game was played till 900? How would your answers change?
4. Is this figure somehow related to the 'idli-vada' game?

Hint: Imagine playing the game till 30. Draw the figure if the game is played till 60.

Let us now play the 'idli-vada' game with different pairs of numbers:

- a. 2 and 5,
- b. 3 and 7,
- c. 4 and 6.

We will say 'idli' for multiples of the smaller number, 'vada' for multiples of the larger number and 'idli-vada' for common multiples. Draw a figure similar to Fig. 5.1 if the game is played up to 60.



Yesterday, we played this game with two numbers. We ended up saying just 'idli' or 'idli-vada' and nobody said just 'vada'!



One of the numbers was 4.

Oh, what could those numbers be!?



- ❁ Which of the following could be the other number:
2, 3, 5, 8, 10?

Jump Jackpot

Jumpy and Grumpy play a game.

- Grumpy places a treasure on some number. For example, he may place it on 24.
- Jumpy chooses a jump size. If he chooses 4, then he has to jump only on multiples of 4, starting at 0.
- Jumpy gets the treasure if he lands on the number where Grumpy placed it.

Which jump sizes will get Jumpy to land on 24?

If he chooses 4: Jumpy lands on $4 \rightarrow 8 \rightarrow 12 \rightarrow 16 \rightarrow 20 \rightarrow 24 \rightarrow 28 \rightarrow \dots$

Other successful jump sizes are 2, 3, 6, 8 and 12.



What about the jump of sizes 1 and 24? Yes, they also will land on 24. The numbers 1, 2, 3, 4, 6, 8, 12, 24 all divide 24 exactly. Recall that such numbers are called **factors or divisors** of 24.

Grumpy increases the level of the game. Two treasures are kept on two different numbers. Jumpy has to choose a jump size and stick to it. Jumpy gets the treasures only if he lands on both the numbers with the chosen jump size. As before, Jumpy starts at 0.

Grumpy has kept the treasures on 14 and 36. And, Jumpy chooses a jump size of 7.

Will Jumpy land on both the treasures? Starting from 0, he jumps to $7 \rightarrow 14 \rightarrow 21 \rightarrow 28 \rightarrow 35 \rightarrow 42 \dots$ We see that he landed on 14 but did not land on 36.

so he does not get the treasure. What jump size should he have chosen? The factors of 14 are: 1, 2, 7, 14. So, these jump sizes will land on 14. The factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36. These jump sizes will land on 36.

So, the jump sizes of 1 or 2 will land on both 14 and 36. Notice that 1 and 2 are the common factors of 14 and 36.

The jump sizes using which both the treasures can be reached are the **common factors** of the two numbers where the treasures are placed.

- ✿ What jump size can reach both 15 and 30? There are multiple jump sizes possible. Try to find them all.
- ✿ Look at the table below. What do you notice?

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70

In the table,

1. Is there anything common among the shaded numbers?
2. Is there anything common among the circled numbers?
3. Which numbers are both shaded and circled? What are these numbers called?



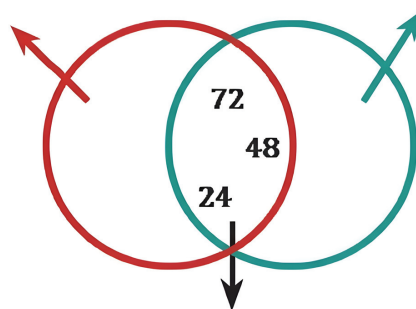
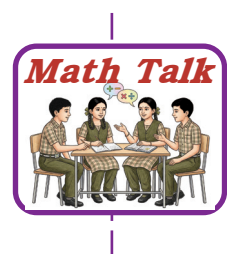
Figure it Out - 5.2

1. Find all multiples of 40 that lie between 310 and 410.

2. Who am I?
 - a. I am a number less than 40. One of my factors is 7. The sum of my digits is 8.
 - b. I am a number less than 100. Two of my factors are 3 and 5. One of my digits is 1 more than the other.
3. A number for which the sum of all its factors is equal to twice the number is called a **perfect number**. The number 28 is a perfect number. Its factors are 1, 2, 4, 7, 14 and 28. Their sum is 56 which is twice 28. Find a perfect number between 1 and 10.
4. Find the common factors of:
 - a. 20 and 28
 - b. 35 and 50
 - c. 4, 8 and 12
 - d. 5, 15 and 25
5. Find any three numbers that are multiples of 25 but not multiples of 50.
6. Anshu and his friends play the 'idli-vada' game with two numbers, which are both smaller than 10. The first time anybody says 'idli-vada' is after the number 50. What could the two numbers be which are assigned 'idli' and 'vada'?
7. In the treasure hunting game, Grumpy has kept treasures on 28 and 70. What jump sizes will land on both the numbers?
8. In the diagram below, Rishi has erased all the numbers except the common multiples. Find out what those numbers could be and fill in the missing numbers in the empty regions.

Multiples of _____

Multiples of _____



9. Find the smallest number that is a multiple of all the numbers from 1 to 10, except for 7.
10. Find the smallest number that is a multiple of all the numbers from 1 to 10.



5.2 Prime Numbers

Rishi and Anusha want to pack Mangoes (anjeer) that grow in their farm. Rishi wants to put 12 Mangoes in each box and Anusha wants to put 7 Mangoes in each box.

How many arrangements are possible?

Think and find out the different ways how —

1. Rishi can arrange 12 Mangoes in a rectangular manner.
2. Anusha can arrange 7 Mangoes in a rectangular manner.

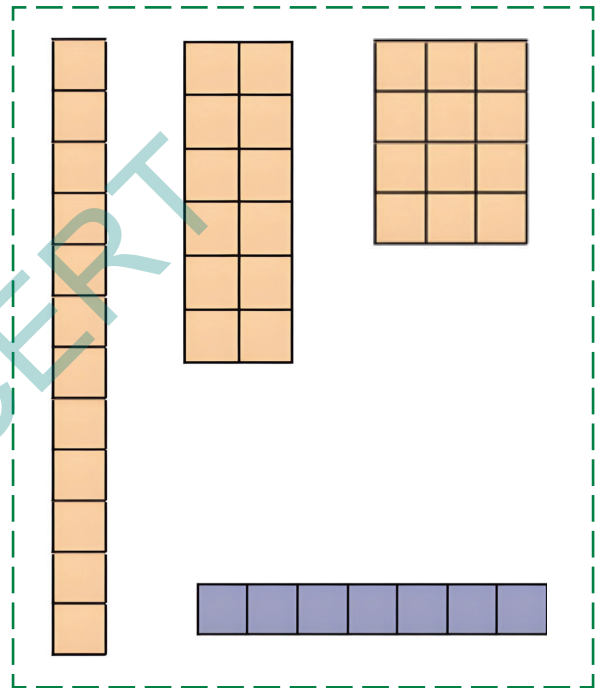
Rishi has listed out these possibilities.

Observe the number of rows and columns in each of the arrangements. How are they related to 12?

In the second arrangement, for example, 12 Mangoes are arranged in two columns of 6 each or $12 = 2 \times 6$.

Anusha could make only one arrangement: 7×1 or 1×7 . There are no other rectangular arrangements possible.

In each of Rishi's arrangements, multiplying the number of rows by the number of columns gives the number 12. So, the number of rows or columns are factors of 12.



We saw that the number 12 can be arranged in a rectangle in more than one way as 12 has more than two factors. The number 7 can be arranged in only one way, as it has only two factors — 1 and 7. Numbers that have only two factors are called prime numbers or primes.

Numbers that have only two factors are called prime numbers or primes. Here are the first few primes — 2, 3, 5, 7, 11, 13, 17, 19. Notice that the factors of a prime number are 1 and the number itself.

What about numbers that have more than two factors? They are called composite numbers. The first few composite numbers are — 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20.

What about 1, which has only one factor? The number 1 is neither a prime nor a composite number.

- ❖ How many prime numbers are there from 21 to 30? How many composite numbers are there from 21 to 30?

Can we list all the prime numbers from 1 to 100?

Here is an interesting way to find prime numbers. Just follow the steps given below and see what happens.

Step 1: Cross out 1 because it is neither prime nor composite.

Step 2: Circle 2, and then cross out all multiples of 2 after that, i.e., 4, 6, 8, and so on.

Step 3: You will find that the next uncrossed number is 3. Circle 3 and then cross out all the multiples of 3 after that, i.e., 6, 9, 12, and so on.

Step 4: The next uncrossed number is 5. Circle 5 and then cross out all the multiples of 5 after that, i.e., 10, 15, 20, and so on.

Step 5: Continue this process till all the numbers in the list are either circled or crossed out.

All the circled numbers are prime numbers. All the crossed out numbers, other than 1, are composite numbers. This method is called the Sieve of Eratosthenes.

This procedure can be carried on for numbers greater than 100 also.

Eratosthenes was a Greek mathematician who lived around 2200 years ago and developed this method of listing primes.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Table 1

It is definitely not some magic; there should be a reason why it works.



Rishi and Anusha started wondering how this simple method is able to find prime numbers! Think how this method works. Read the steps given above again and see what happens after each step is carried out.



Figure it Out - 5.3

1. We see that 2 is a prime and also an even number. Is there any other even prime?
2. Look at the list of primes till 100. What is the smallest difference between two successive primes? What is the largest difference?
3. Are there an equal number of primes occurring in every row in the **table 1** on the previous page? Which decades have the least number of primes? Which have the most number of primes?

Primes through the Ages

Prime numbers are the building blocks of all whole numbers. Starting from the time of the Greek civilisation (more than 2000 years ago) to this day, mathematicians are still struggling to uncover their secrets!

Food for thought: is there a largest prime number? Or does the list of prime numbers go on without an end? A mathematician named Euclid found the answer and so will you in a later class!

Fun fact: The largest prime number that anyone has 'written down' is so large that it would take around 6500 pages to write it! So they could only write it on a computer!

4. Which of the following numbers are prime: 23, 51, 37, 26?
5. Write three pairs of prime numbers less than 20 whose sum is a multiple of 5.
6. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.
7. Find seven consecutive composite numbers between 1 and 100.
8. **Twin primes** are pairs of primes having a difference of 2. For example, 3 and 5 are twin primes. So are 17 and 19. Find the other twin primes between 1 and 100.

9. Identify whether each statement is true or false. Explain.
- There is no prime number whose units digit is 4.
 - A product of primes can also be prime.
 - Prime numbers do not have any factors.
 - All even numbers are composite numbers.
 - 2 is a prime and so is the next number, 3. For every other prime, the next number is composite.
10. Which of the following numbers is the product of exactly three distinct prime numbers: 45, 60, 91, 105, 330?
11. How many three-digit prime numbers can you make using each of 2, 4 and 5 once?
12. Observe that 3 is a prime number, and $2 \times 3 + 1 = 7$ is also a prime. Are there other primes for which doubling and adding 1 gives another prime? Find at least five such examples.

5.3 Co-prime numbers for safe keeping treasures

Which pairs are safe?

Let us go back to the treasure finding game. This time, treasures are kept on two numbers. Jumpy gets the treasures only if he is able to reach both the numbers with the same jump size. There is also a new rule a jump size of 1 is not allowed.

- ✿ Where should Grumpy place the treasures so that Jumpy cannot reach both the treasures?

Will placing the treasure on 12 and 26 work? No! If the jump size is chosen to be 2, then Jumpy will reach both 12 and 26.

What about 4 and 9? Jumpy cannot reach both using any jump size other than 1. So, Grumpy knows that the pair 4 and 9 is safe.

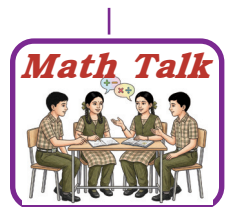
Check if these pairs are safe.

- | | |
|--------------|--------------|
| a. 15 and 39 | b. 4 and 15 |
| c. 18 and 29 | d. 20 and 55 |

What is special about safe pairs? They don't have any common factor other than 1. Two numbers are said to be co-prime to each other if they have no common factor other than 1.

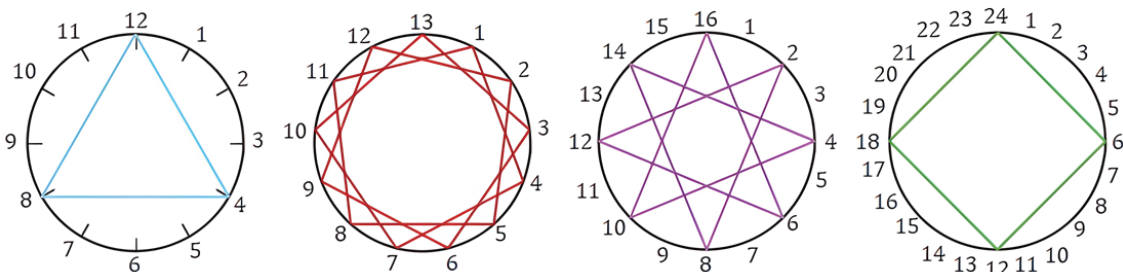
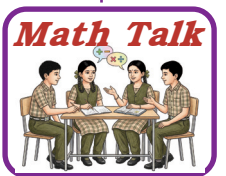
Example: As 15 and 39 have 3 as a common factor, they are not co-prime. But 4 and 9 are co-prime.

- ❖ Which of the following pairs of numbers are co-prime?
 - a. 18 and 35 b. 15 and 37 c. 30 and 415
 - d. 17 and 69 e. 81 and 18
- ❖ While playing the 'idli-vada' game with different number pairs, Anusha observed something interesting!
 1. Sometimes the first common multiple was the same as the product of the two numbers.
 2. At other times the first common multiple was less than the product of the two numbers.
 Find examples for each of the above. How is it related to the number pair being co-prime?



Co-prime art

- ❖ Observe the following thread art. The first diagram has 12 pegs and the thread is tied to every fourth peg (we say that the thread-gap is 4). The second diagram has 13 pegs and the thread-gap is 3. What about the other diagrams? Observe these pictures, share and discuss your findings in class.



In some diagrams, the thread is tied to every peg. In some, it is not. Is it related to the two numbers (the number of pegs and the thread-gap) being co-prime?

When making thread art, you choose a thread gap - how many pegs you jump each time you wrap the thread.

If the number of pegs and the thread gap only have 1 as a common factor, the thread will touch every peg and make one big, beautiful pattern.

If they have another common factor, the thread will not touch all pegs, will make small repeating loops instead of one big design.



Make Thread-art pictures for the following:

- a. 15 pegs, thread-gap of 10
c. 14 pegs, thread-gap of 6

- b. 10 pegs, thread-gap of 7
d. 8 pegs, thread-gap of 3

5.4 Prime Factorisation

Checking if two numbers are co-prime

Teacher : Are 56 and 63 co-prime?

Anusha and Rishi : If they have a common factor other than 1, then they are not co-prime. Let us check.

Anusha : I can write $56 = 14 \times 4$ and $63 = 21 \times 3$. So, 14 and 4 are factors, of 56. Further, 21 and 3 are factors of 63. So, there are no common factors. The numbers are co-prime.

Rishi : Hold on. I can also write $56 = 7 \times 8$ and $63 = 9 \times 7$. We see that 7 is a factor of both numbers, so, they are not co-prime.

Clearly Rishi is right, as 7 is a common factor.

✿ But where did Anusha go wrong?

Writing $56 = 14 \times 4$ tells us that 14 and 4 are both factors of 56, but it does not tell all the factors of 56. The same holds for the factors of 63.

Try another example: 80 and 63. There are many ways to factorise both numbers.

$$80 = 40 \times 2 = 20 \times 4 = 10 \times 8 = 16 \times 5 = ???$$

$$63 = 9 \times 7 = 3 \times 21 = ???$$

We have written '???' to say that there may be more ways to factorise these numbers. But if we take any of the given factorisations, for example, $80 = 16 \times 5$ and $63 = 9 \times 7$, then there are no common factors. Can we conclude that 80 and 63 are coprime? As Anusha's mistake above shows, we cannot conclude that as there may be other ways to factorise the numbers.

What this means is that we need a more systematic approach to check if two numbers are co-prime.

Prime factorisation

Take a number such as 56. It is composite, as we saw that it can be written as $56 = 4 \times 14$. So, both 4 and 14 are factors of 56. Now take one of these, say 14. It is also composite and can be written as $14 = 2 \times 7$.

Therefore, $56 = 4 \times 2 \times 7$. Now, 4 is composite and can be written as $4 = 2 \times 2$. Therefore, $56 = 2 \times 2 \times 2 \times 7$. All the factors appearing here, 2 and 7, are prime numbers. So, we cannot divide them further.

In conclusion, we have written 56 as a product of prime numbers. This is called **prime factorisation** of 56. The individual factors are called prime factors. For example, the prime factors of 56 are 2 and 7.

Every number greater than 1 has a prime factorisation. The idea is the same: keep breaking the composite numbers into factors till only primes are left.

The number 1 does not have any prime factorisation. It is not divisible by any prime number.

What is the prime factorisation of a prime number like 7? It is just 7 (we cannot break it down any further).

Let us see a few more examples.

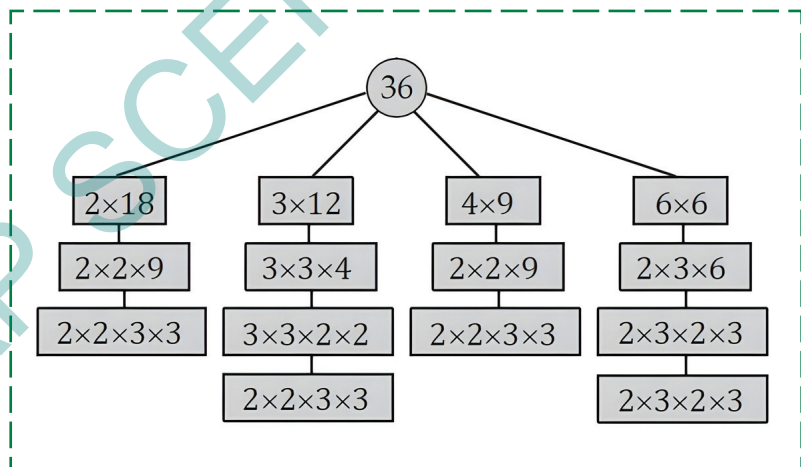
By going through different ways of breaking down the number, we wrote 63 as $3 \times 3 \times 7$ and as

$3 \times 7 \times 3$. Are they different? Not really! The same prime numbers 3 and 7 occur in both cases. Further, 3 appears two times in both and 7 appears once.

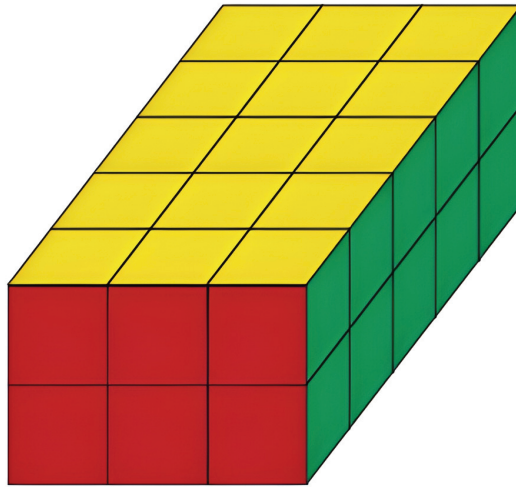
Here, you see four different ways to get prime factorisation of 36. Observe that in all four cases, we get two 2s and two 3s.

Multiply back to see that you get 36 in all four cases.

For any number, it is a remarkable fact that there is only one prime factorisation, except that the prime factors may come in different orders. As we explain below, the order is not important. However, as we saw in these examples, there are many ways to arrive at the prime factorisation!



Does the order matter?



Using this diagram,

can you explain why $30 = 2 \times 3 \times 5$, no matter which way you multiply 2, 3, and 5?

When multiplying numbers, we can do so in any order. The end result is the same. That is why, when two 2s and two 3s are multiplied in any order, we get 36. In a later class, we shall study this under the names of **commutativity and associativity of multiplication**.

Thus, the order does not matter. Usually we write the prime numbers in increasing order. For example, $225 = 3 \times 3 \times 5 \times 5$ or $30 = 2 \times 3 \times 5$.

Prime factorisation of a product of two numbers

When we find the prime factorisation of a number, we first write it as a product of two factors. For example, $72 = 12 \times 6$. Then, we find the prime factorisation of each of the factors. In the above example, $12 = 2 \times 2 \times 3$ and $6 = 2 \times 3$. Now, can you say what the prime factorisation of 72 is?

The prime factorisation of the original number is obtained by putting these together.

$$72 = 2 \times 2 \times 3 \times 2 \times 3$$

We can also write this as $2 \times 2 \times 2 \times 3 \times 3$. Multiply and check that you get 72 back!

Observe how many times each prime factor occurs in the factorisation of 72.

Compare it with how many times it occurs in the factorisations of 12 and 6 put together.



Figure it Out - 5.4

1. Find the prime factorisations of the following numbers:
64, 104, 105, 243, 320, 141, 1728, 729, 1024, 1331, 1000.
2. The prime factorisation of a number has one 2, two 3s, and one 11. What is the number?
3. Find three prime numbers, all less than 30, whose product is 1955.
4. Find the prime factorisation of these numbers without multiplying first
 - a. 56×25
 - b. 108×75
 - c. 1000×81
5. What is the smallest number whose prime factorisation has.
 - a. three different prime numbers?
 - b. four different prime numbers?

Prime factorisation is of fundamental importance in the study of numbers. Let us discuss two ways in which it can be useful.

Using prime factorisation to check if two numbers are co-prime

Let us again take the numbers 56 and 63. How can we check if they are co-prime? We can use the prime factorisation of both numbers

$$56 = 2 \times 2 \times 2 \times 7 \text{ and } 63 = 3 \times 3 \times 7$$

Now, we see that 7 is a prime factor of 56 as well as 63. Therefore, 56 and 63 are not co-prime.

What about 80 and 63. Their prime factorisations are as follows

$$80 = 2 \times 2 \times 2 \times 2 \times 5 \text{ and } 63 = 3 \times 3 \times 7$$

There are no common prime factors. Can we conclude that they are co-prime? Suppose they have a common factor that is composite. Would the prime factors of this composite common factor appear in the prime factorisation of 80 and 63.

Therefore, we can say that if there are no common prime factors, then the two numbers are co-prime.

Let us see some examples.

Example : Consider 40 and 231. Their prime factorisations are as follows:

$$40 = 2 \times 2 \times 2 \times 5 \text{ and } 231 = 3 \times 7 \times 11$$

We see that there are no common primes that divide both 40 and 231. Indeed, the prime factors of 40 are 2 and 5 while, the prime factors of 231 are 3, 7, and 11. Therefore, 40 and 231 are co-prime!

Example : Consider 242 and 195. Their prime factorisations are as follows:

$$242 = 2 \times 11 \times 11 \text{ and } 195 = 3 \times 5 \times 13$$

The prime factors of 242 are 2 and 11. The prime factors of 195 are 3, 5, and 13. There are no common prime factors. Therefore, 242 and 195 are co-prime.

Using prime factorisation to check if one number is divisible by another

We can say that if one number is divisible by another, the prime factorisation of the second number is included in the prime factorisation of the first number.

We say that 48 is divisible by 12 because when we divide 48 by 12, the remainder is zero. How can we check if one number is divisible by another without carrying out long division?

Example : Is 168 divisible by 12? Find the prime factorisations of both:

$$168 = 2 \times 2 \times 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3$$

Since we can multiply in any order, now it is clear that,

$$168 = 2 \times 2 \times 3 \times 2 \times 7 = 12 \times 14$$

Therefore, 168 is divisible by 12.

Example : Is 75 divisible by 21? Find the prime factorisations of both:

$$75 = 3 \times 5 \times 5 \text{ and } 21 = 3 \times 7$$

As we saw in the discussion above, if 75 was a multiple of 21, then all prime factors of 21 would also be prime factors of 75. However, 7 is a prime factor of 21 but not a prime factor of 75. Therefore, 75 is not divisible by 21.

Example: Is 42 divisible by 12? Find the prime factorisations of both:

$$42 = 2 \times 3 \times 7 \text{ and } 12 = 2 \times 2 \times 3$$

All prime factors of 12 are also prime factors of 42. But the prime factorisation of 12 is not included in the prime factorisation of 42. This is because 2 occurs twice in the prime factorisation of 12 but only once in the prime factorisation of 42. This means that 42 is not divisible by 12.

We can say that if one number is divisible by another, then the prime factorisation of the second number is included in the prime factorisation of the first number.



Figure it Out - 5.5

1. Are the following pairs of numbers co-prime? Guess first and then use prime factorisation to verify your answer.
 - a. 30 and 45
 - b. 57 and 85
 - c. 121 and 1331
 - d. 343 and 216
2. Is the first number divisible by the second? Use prime factorisation.
 - a. 225 and 27
 - b. 96 and 24
 - c. 343 and 17
 - d. 999 and 99
3. The first number has prime factorisation $2 \times 3 \times 7$ and the second number has prime factorisation $3 \times 7 \times 11$. Are they co-prime? Does one of them divide the other?
4. Rishi says, "Any two prime numbers are co-prime?". Is he right?

5.5 Divisibility Tests

So far, we have been finding factors of numbers in different contexts, including to determine if a number is prime or not, or if a given pair of numbers is co-prime or not.

It is easy to find factors of small numbers. How do we find factors of a large number?

Let us take 8560. Does it have any factors from 2 to 10 (2, 3, 4, 5, ..., 9, 10)?

It is easy to check if some of these numbers are factors or not without doing long division. Can you find them?

Divisibility by 10

Let us take 10. Is 8560 divisible by 10? This is another way of asking if 10 is a factor of 8560.

For this, we can look at the pattern in the multiples of 10.

The first few multiples of 10 are: 10, 20, 30, 40, ... Continue this sequence and observe the pattern.

Is 125 a multiple of 10? Will this number appear in the previous sequence? Why or why not?

Can you now answer if 8560 is divisible by 10?

✿ **Consider this statement:**

Numbers that are divisible by 10 are those that end with '0'. Do you agree?



Divisibility by 5

The number 5 is another number whose divisibility can easily be checked. How do we do it?

Explore by listing down the multiples: 5, 10, 15, 20, 25, ... What do you observe about these numbers? Do you see a pattern in the last digit?

What is the largest number less than 399 that is divisible by 5? Is 8560 divisible by 5?

✿ **Consider this statement:**

Numbers that are divisible by 5 are those that end with either a '0' or a '5'. Do you agree?



Divisibility by 2

The first few multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

What do you observe? Do you see a pattern in the last digit?

Is 682 divisible by 2? Can we answer this without doing the long division?

Is 8560 divisible by 2? Why or why not?

✿ **Consider this statement:**

Numbers that are divisible by 2 are those that end with '0', '2', '4', '6' or '8'.

Do you agree?

What are all the multiples of 2 between 399 and 411?



Divisibility by 4

Checking if a number is divisible by 4 can also be done easily!

Look at its multiples: 4, 8, 12, 16, 20, 24, 28, 32, ...

Are you able to observe any patterns that can be used? The multiples of 10, 5 and 2 have a pattern in their last digits which we are able to use to check for divisibility. Similarly, can we check if a number is divisible by 4 by looking at the last digit?

It does not work! Look at 12 and 22. They have the same last digit, but 12 is a multiple of 4 while 22 is not. Similarly 14 and 24 have the same last digit, but 14 is not a multiple of 4 while 24 is. Similarly, 16 and 26 or 18 and 28. What this means is that by looking at the last digit, we cannot tell whether a number is a multiple of 4.

Can we answer the question by looking at more digits? Make a list of multiples of 4 between 1 and 200 and search for a pattern.

✿ Find numbers between 330 and 340 that are divisible by 4. Also, find numbers between 1730 and 1740, and 2030 and 2040, that are divisible by 4. What do you observe?

✿ Is 8536 divisible by 4?

✿ **Consider these statements:**

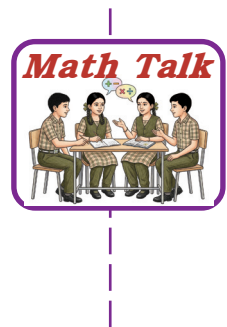
1. Only the last two digits matter when deciding if a given number is divisible by 4.
2. If the number formed by the last two digits is divisible by 4, then the original number is divisible by 4.
3. If the original number is divisible by 4, then the number formed by the last two digits is divisible by 4.

Do you agree? Why or why not?

Divisibility by 8

Interestingly, even checking for divisibility by 8 can be simplified. Can the last two digits be used for this?

- ✿ Find numbers between 120 and 140 that are divisible by 8. Also find numbers between 1120 and 1140, and 3120 and 3140, that are divisible by 8. What do you observe?
- ✿ Change the last two digits of 8560 so that the resulting number is a multiple of 8.
- ✿ Consider these statements:
 1. Only the last three digits matter when deciding if a given number is divisible by 8.
 2. If the number formed by the last three digits is divisible by 8, then the original number is divisible by 8.
 3. If the original number is divisible by 8, then the number formed by the last three digits is divisible by 8.



Do you agree? Why or why not?

We have seen that long division is not always needed to check if a number is a factor or not. We have made use of certain observations to come up with simple methods for 10, 5, 2, 4, 8. Do we have such simple methods for other numbers as well? We will discuss simple methods to test divisibility by 3, 6, 7, and 9 in later classes.



Figure it Out - 5.6

1. 2024 is a leap year (as February has 29 days). Leap years occur in the years that are multiples of 4, except for those years that are evenly divisible by 100 but not 400.
 - a. From the year you were born till now, which years were leap years?
 - b. From the year 2024 till 2099, how many leap years are there?
2. Find the largest and smallest 4-digit numbers that are divisible by 4 and are also palindromes.
3. Explore and find out if each statement is always true, sometimes true or never true. You can give examples to support your reasoning.

- a. Sum of two even numbers gives a multiple of 4.
 - b. Sum of two odd numbers gives a multiple of 4.
4. Find the remainders obtained when each of the following numbers are divided by (a) 10, (b) 5, (c) 2.
- 78, 99, 173, 572, 980, 1111, 2345
5. The teacher asked if 14560 is divisible by all of 2, 4, 5, 8 and 10. Guna checked for divisibility of 14560 by only two of these numbers and then declared that it was also divisible by all of them. What could those two numbers be?
6. Which of the following numbers are divisible by all of 2, 4, 5, 8 and 10: 572, 2352, 5600, 6000, 77622160.
7. Write two numbers whose product is 10000. The two numbers should not have 0 as the units digit.

5.6 Fun with numbers

Special numbers

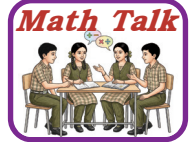
There are four numbers in this box. Which number looks special to you? Why do you say so?

9	16
25	43

Look at the what Rishi's classmates have to share:

- Janaki says: "9 is special because it is a single-digit number whereas all the other numbers are 2-digit numbers".
- Charish says: "9 is special because it is the only number that is a multiple of 3".
- Murugan says: "16 is special because it is the only even number and also the only multiple of 4".
- Gopika says: "25 is special as it is the only multiple of 5".
- Yamuna says: "43 is special because it is the only prime number".
- Radha says: "43 is special because it is the only number that is not a square".

- Below are some boxes with four numbers in each box. Within each box try to say how each number is special compared to the rest. Share with your classmates and find out who else gave the same reasons as you did. Did anyone give different reasons that may not have occurred to you?!



5	7
12	35

3	8
11	24

27	3
123	31

17	27
44	65

A prime puzzle

The figure on the left shows the puzzle. The figure on the right shows the solution of the puzzle. Think what the rules can be to solve the puzzle.



			75
			42
			102
170	30	63	

5	5	3	75
2	3	7	42
17	2	3	102
170	30	63	

Rules

- Fill the grid with prime numbers only.
- The product of each row is the number to the right of the row.
- The product of each column is number at the bottom of the column.

			105
			20
			30
28	125	18	

			8
			105
			70
30	70	28	

			63
			27
			190
45	42	171	

			343
			66
			44
28	154	231	

Insights from the Puzzle:

- Prime numbers are used to form given products in rows and columns.
- Logical reasoning and factorization can be applied using only prime numbers.
- Different combinations of prime numbers can produce the same multiplication results.
- The product of two or more prime numbers always results in a composite number.

CHAPTER MASTERY

MCQ's

- Which pair is NOT co-prime? []
 A) 11, 14 B) 15, 28 C) 8, 9 D) 6, 10
- The common factors of 4, 8, and 12 are: []
 A) 1, 2 and 6 B) 1, 2 and 4 C) 2, 4 and 8 D) 4, 8 and 12

True / False

- Sum of two prime numbers is always even (True / False)

Yes / No

- Using divisibility tests determine which of the following numbers are divisible by 2, 4, 5, 8, 10 (say, Yes or No)

Number	2	4	5	8	10
128	Yes	Yes	No	Yes	No
990
8536
275
8416
639210
429714
2856
3060
406839

Fill in the Blanks

5. First three common multiples of 6 and 8 are _____
6. A number is divisible by 8 if the last _____ digits form a number divisible by 8

Match the Following

7. Match the items in column 1 with the items in column 2.

Column 1

- (i) 35
- (ii) 15
- (iii) 16
- (iv) 20
- (v) 25

Column 2

- (a) Multiple of 8
- (b) Multiple of 7
- (c) Factor of 30
- (d) Factor of 50
- (e) Factor of 20

Assertion & Reason Type

8. Assertion (A): 16420 is divisible by 8 and 10.
Reason (R): A number is divisible by 8 if its last three digits form a number divisible by 8, and it is divisible by 10 if it ends in 0. []
- (A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

Statement Type

9. Statements
- (i) : 21 and 22 are co-prime numbers.
(ii) : Two numbers are co-prime if both numbers are prime. []
- A: Only (i) is correct B: Only (ii) is correct
C: Both are correct D: Both are incorrect

Short Answer Type

10. Express the following as the sum of two odd primes.
(a) 44 (b) 36 (c) 24 (d) 18
11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5.
(Hint: $3 + 7 = 10$)
12. Write the smallest 5-digit number and express it in the form of product of prime factors.
13. Define prime number?
14. Define the divisibility rule for 5.

SUMMARY

- If a number is divisible by another, the second number is called a factor of the first. For example, 4 is a factor of 12 because 12 is divisible by 4 ($12 \div 4 = 3$).
- Prime numbers are numbers like 2, 3, 5, 7, 11, ... that have only two factors, namely 1 and themselves.
- Composite numbers are numbers like 4, 6, 8, 9, ... that have more than 2 factors, i.e., at least one factor other than 1 and themselves. For example, 8 has the factor 4 and 9 has the factor 3, so 8 and 9 are both composite.
- Every number greater than 1 can be written as a product of prime numbers. This is called the number's prime factorisation. For example, $84 = 2 \times 2 \times 3 \times 7$.
- There is only one way to factorise a number into primes, except for the ordering of the factors.
- Two numbers that do not have a common factor other than 1 are said to be co-prime.
- To check if two numbers are co-prime, we can first find their prime factorisations and check if there is a common prime factor. If there is no common prime factor, they are co-prime, and otherwise they are not.
- A number is a factor of another number if the prime factorisation of the first number is included in the prime factorisation of the second number.



INTEGRATING ARTIFICIAL INTELLIGENCE (AI)



The learners will be able to:

1. **Explain** the idea of Data \rightarrow Pattern \rightarrow Output using mathematical examples.
2. **Identify** patterns in numbers and relate them to how AI recognises patterns in data.
3. **Understand** how datasets are organised and how mean and median summarise data.
4. **Compare step-by-step** mathematical rules with algorithms used in AI systems.

Introduction :

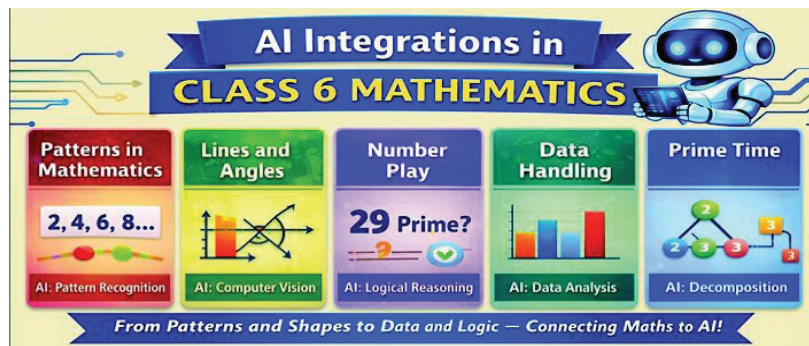
Children We often notice how our calculator gives quick answers, how a digital clock shows time accurately, or how apps help us measure distance and area. These are not magic. They are examples of mathematics working with technology.

You may feel that mathematics is only about numbers, but you already use mathematical thinking in your daily life. When you compare prices, measure length, count objects, or recognise patterns, you are thinking mathematically.

In a similar way, Artificial Intelligence (AI) also works using mathematical ideas. It collects data, looks for patterns, measures values, compares results, and gives correct outputs. Mathematics is the foundation that helps AI systems think logically.



Let us begin our journey of exploring Mathematics with intelligent thinking!



Foundations of AI in Mathematics Learning

The chapters of this semester develop important mathematical skills such as pattern identification, logical reasoning, comparison, **measurement**, **classification**, and data analysis. These skills are also applied in **Artificial Intelligence (AI)**.

Disclaimer : "The use of AI tools by students must be strictly under the direct guidance and supervision of teachers and parents / guardians".

Artificial Intelligence enables machines to analyse data, recognise patterns, and generate outputs based on learned examples. In Mathematics, when students observe a number sequence, they identify a rule and predict the next term. This process reflects the structured thinking that intelligent systems follow.

Mathematical reasoning often follows the structure:

Data → Pattern → Output

When numbers are given (data), a rule is identified (pattern), and a result is predicted (output). This structured reasoning forms the foundation of both mathematical thinking and intelligent systems.

Example 1: Patterns in Numbers



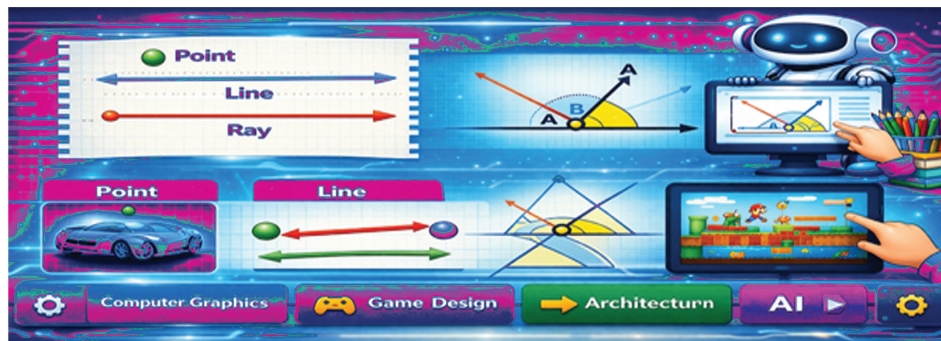
When the sequence 5, 10, 15, 20 is observed, each term increases by 5. The given numbers represent data. The repeated addition forms the pattern. The next term 25 becomes the output. Square numbers, triangular numbers, and visual patterns using shapes further strengthen **pattern recognition**. Identifying relationships among sequences improves **logical reasoning**.

AI systems similarly analyse numerical **data sets** to recognise repeating structures and **predict** future values.

<p>Thinking with AI :</p>	<p>If an AI system is given three different sequences – 5, 10, 15, 20 1, 4, 9, 16 1, 3, 6, 10 how will it decide whether each sequence follows the same pattern or different patterns? Scan the QR code to explore how will it decide whether each sequence follows the same pattern or different patterns?</p>
<p>Key Words: reach a</p>	<p>Logical Reasoning – Thinking step-by-step in a sensible way to correct conclusion. Datasets – A collection of related data organised for analysis. Predict – To tell what will happen next based on a pattern or rule. Pattern Recognition – Identifying repeated structures.</p>

Guidance : Encourage students to first identify the mathematical rule independently before discussing AI pattern recognition. Ensure conceptual clarity in number sequences before linking to digital examples.

Example 2: Lines and Angles



A **point** shows an exact position. A **line** goes on forever in both directions, and a **ray** has one fixed endpoint and extends in one direction. When two rays meet, they form an **angle**. The size of an angle depends on the **rotation**, not on the length of its arms. We can compare angles by **placing one angle over another**, which helps develop spatial thinking. In **digital systems**, **geometric calculations** are used in **computer graphics** and **design applications**.

Thinking with AI :	<p>If an AI system is given an image containing different lines and angles, how will it identify whether two angles are equal?</p> <p>Scan the QR code to explore how will it will it identify whether two angles are equal?</p>
Key Words:	<p>Superimposition – Placing one figure exactly over another to compare them.</p> <p>Spatial Reasoning – The ability to understand and think about shapes and space.</p> <p>Geometric Calculations – Mathematical measurements related to shapes, lines, and angles</p>

Guidance : Demonstrate angle comparison using classroom materials before connecting to digital spatial systems. Maintain focus on textbook definitions.

Example 3: Logical Rules in Number Play



In a classroom activity, children stand in a line and say numbers based on their positions. Each child says the number of taller neighbours standing next to them. For example, a child says '1' if only one taller child stands next to them, '2' if both neighbours are taller, and '0' if none are taller.

This activity shows how numbers describe relationships logically. Students also explore puzzles like arranging children differently and checking whether sequences such as **0, 1, 2, 1, 0** are possible.

Another activity includes identifying **supercells**, where a number is coloured if it is greater than all its neighbouring numbers.

These tasks develop reasoning through observation, comparison, and structured thinking.

If an AI system is given the heights or numbers of children in order, it will:

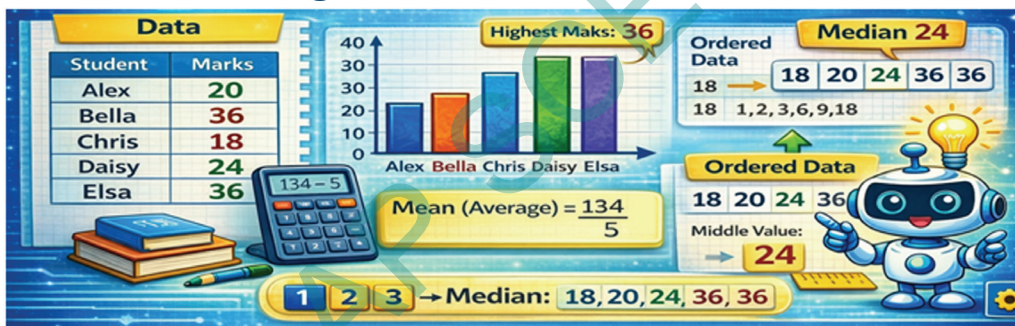
- treat them as a dataset
- compare each value with neighbouring values
- apply rules step-by-step
- produce outputs like 0, 1, or 2 .

This shows **Data → Rule → Output** and computational thinking through logical comparison.

Thinking with AI :	If an AI system is given numbers between 40 and 70, how will it identify all numbers whose digit sum is the same? Write the logical steps the AI will follow ? Scan the QR code for more information.
Key Words:	Supercells - a number that is greater than both of its neighboring numbers in a sequence..

Guidance : Students should practise such activities manually before relating them to algorithmic reasoning..

Example 4 : Data Handling and Presentation



When marks of students are collected, they form data. When arranged in a table, they form a **dataset**. Bar graphs visually represent data trends.

Mean is calculated by dividing the total sum by the number of **observations**. Median is the middle value in ordered data. AI systems also organise datasets, compute averages, and identify **trends**.

Thinking with AI :	If an AI system is given the marks of 7 students – 45, 60, 75, 80, 90, 50, 70 – how will it decide whether the class performance is improving or not? Scan the QR code to explore how will it decide whether the class performance is improving or not?
Key Words:	Observations – Individual values or pieces of data collected in a study. Trends – General patterns or directions shown by data over time.

Guidance : Ensure students first calculate mean and median manually. Use AI only for verification and conceptual clarity.

Example 5 : Understanding Factors through Jump Jackpot



In the **Jump Jackpot activity**, a treasure is placed on a number and a player chooses a jump size. The player lands only on multiples of that jump size starting from 0. For example, if the treasure is placed on **24**, the successful jump sizes are **1, 2, 3, 4, 6, 8, 12, and 24** because all these numbers divide 24 exactly.

In another case, when treasures are placed on **14 and 36**, only jump sizes that reach both numbers (their common factors) will work. For example, **1 and 2** are common factors of both numbers.

This activity helps students understand factors and common factors through logical reasoning.

If an AI system is given two numbers where treasures are placed:

- it lists all factors of each number
- compares both factor list
- identifies common factors
- selects jump sizes that reach both numbers

This shows how AI uses logical steps and data comparison like students..

Thinking with AI :

If an AI system is given two numbers, how will it logically decide whether they are **co-prime** by checking their factors? Write the step-by-step reasoning the AI will follow?
Scan the QR code to explore how will it correctly classify each shape?

Key Words :

Data comparison : The process of examining two or more datasets to identify similarities, differences or patterns.

Guidance : Encourage students to list factors manually before discussing AI comparison.

Exploring with AI

Understanding AI in Mathematics Learning

Artificial Intelligence (AI) is a technology that enables computers and digital systems to analyse information, recognise patterns, and generate outputs based on learned examples. In Mathematics learning, AI can assist students in understanding concepts, practicing problems, and verifying solutions in a structured manner.

Mathematics develops skills such as logical reasoning, pattern recognition, comparison, classification, and data interpretation. These same skills form the foundation of intelligent systems. When students identify a pattern in a number sequence,

calculate the mean of a dataset, or classify shapes based on properties, they are applying structured reasoning. Similarly, AI systems study data, detect patterns, and produce results according to programmed rules and learned examples.

For example, when a student is given the sequence 3, 6, 9, 12, the numbers represent data. By analysing the data, the student recognises the pattern of adding 3 and predicts the next number. In the same way, AI systems analyse datasets to identify repeated relationships and generate outputs.

This process reflects the structure:

Data → Pattern → Output

AI tools can support Mathematics learning in the following ways:

- Providing step-by-step explanations for solving problems.
- Generating additional practice questions.
- Verifying solutions and calculations.
- Explaining alternative methods of solving the same problem.

However, it is important to understand that AI does not think independently like humans. AI systems depend entirely on the data and instructions provided to them. If the dataset used for training is incomplete or incorrect, the AI system may produce inaccurate results. Therefore, AI-generated explanations and answers must always be verified using textbook methods and teacher guidance.

AI should be used as a supportive learning assistant, not as a replacement for manual calculation, logical reasoning, or classroom instruction. Students must first attempt problems independently, apply textbook methods, and then use AI tools for clarification or verification.

Responsible use of AI strengthens discipline, critical thinking, and conceptual clarity. By understanding how AI processes mathematical information, learners develop deeper awareness of both mathematical reasoning and modern technological systems.

Accessing AI for Mathematics Learning

AI tools may be used in school computer laboratories under teacher supervision or at home with parental guidance. AI must be used strictly for educational purposes and only after completing textbook exercises.

Safe and Responsible Use of AI

- Do not share personal information (name, school details, passwords).
 - Do not copy AI responses without understanding.
 - Always verify AI-generated answers with textbook methods.
 - Do not use AI during examinations.
 - Use AI only to improve clarity and reasoning.
- Responsible use of AI strengthens honesty, discipline, and critical thinking.

Myths and Facts About AI

Myth	Truth
AI thinks like humans AI will replace teachers	AI follows instructions. It has no feelings. AI helps learning, but teachers guide students.

Do You Know?

Some Artificial Intelligence systems can now solve very difficult mathematics problems that even challenge expert mathematicians. For more information ask AI about this topic

Writing Effective Prompts in Mathematics

A prompt is a clear instruction given to an AI tool. A good Mathematics prompt should include:

Class level (Class 6)-Chapter name-Clear concept- Request for step-by-step explanation

Example Prompts:

1. "I am a Class 6 student from Chapter 1: Patterns in Mathematics. Explain how to identify the rule in a number sequence like 1, 3, 6, 10."

Here is the sample output for the given prompt.

Let's find the rule in the number sequence 1, 3, 6, 10!
Explain how to identify the rule in a number sequence like 1, 3, 6, 10.

A Look at the sequence: $\rightarrow 1, 3, 6, 10$

$1 \rightarrow 1+2=3 \rightarrow 1+2+3=6 \rightarrow 1+2+3+4=10$

B Recognise the triangular patterns:
Each sequence is in the form:
 $1 \rightarrow 1+2=3 \rightarrow 1+2+3=6 \rightarrow 1+2+3+4=10$

C The rule is: $\star \triangle$ numbers are triangular numbers.
 $1 \rightarrow 1+2=3 \rightarrow 1+2+3=6 \rightarrow 1+2+3+4=10$

We've found the rule! 😊 If you need help with anything else, just let me know, 😊

Types of Prompts in Mathematics Learning:

To obtain clear and meaningful responses from an AI tool, prompts should be written according to the purpose of learning. Prompts may be framed in three main ways:

1. **Concept Understanding Prompt**: This type of prompt helps in understanding a concept clearly in simple language.

Prompt: "I am a Class 6 student from the chapter Lines and Angles. Please explain the difference between a line, a ray, and a line segment with simple diagrams."

2. **Comparison or Analytical Prompt**: This type of prompt helps in comparing ideas and developing reasoning skills.

Prompt: "I am a Class 6 student from the chapter Data Handling and Presentation. Please explain the difference between a pictograph and a bar graph with a simple example and show how each represents data clearly."

3. **Application or Activity-Based Prompt**: This type of prompt helps in applying mathematical knowledge in real-life situations.

Prompt: "I am a Class 6 student from the chapter Number Play. Please create a real-life situation using a number table where I have to identify supercells by comparing neighboring numbers, and explain the solution step-by-step."

Can AI Make Mistakes?

AI systems depend completely on the dataset used to train them. If data is incomplete or incorrect, AI may produce inaccurate outputs.

Therefore, AI-generated solutions must always be verified using textbook methods. AI supports learning but cannot replace teacher guidance or practice.

Chapter-wise AI-Supported Exploration

Tasks for Students to Explore with AI (Mathematics – Semester 1)

Chapter	Foundational Level	Analytical Level	Creative Level
Chapter 1 : Patterns in Mathematics	<ul style="list-style-type: none">• Identify and write the rule for simple number sequences.• Classify numbers as even, odd, square, or triangular numbers.	<ul style="list-style-type: none">• Analyse a given sequence and determine the missing terms.• Compare two different sequences and explain their pattern differences.	<ul style="list-style-type: none">• Create a new number sequence and explain the rule clearly.• Design a visual pattern using shapes and describe its mathematical rule.
Chapter 2 : Lines and Angles	<ul style="list-style-type: none">• Define point, line, ray, and angle with examples.• Identify different types of angles in given diagrams.	<ul style="list-style-type: none">• Compare two angles using reasoning or measurement.• Analyse relationships between intersecting and parallel lines.	<ul style="list-style-type: none">• Construct a geometric figure using different types of angles.• Design a simple pattern using angles and lines.
Chapter 3 : Number Play	<ul style="list-style-type: none">• Identify number relationships using neighbouring comparisons.• Solve puzzles based on taller-neighbour number patterns.	<ul style="list-style-type: none">• Analyse number arrangements logically step-by-step.• Solve reasoning problems using structured comparison.	<ul style="list-style-type: none">• Create a logical puzzle based on number relationships.• Design an activity showing step-by-step logical reasoning.
Chapter 4 : Data Handling and Presentation	<ul style="list-style-type: none">• Collect data from classroom activities and organise it into a table.• Draw a simple bar graph from given data.	<ul style="list-style-type: none">• Calculate mean and median of a small dataset.• Compare two datasets and interpret differences.	<ul style="list-style-type: none">• Design a survey and represent findings graphically.• Create a real-life problem involving average calculation and explain its interpretation.
Chapter 5 : Prime Time	<ul style="list-style-type: none">• Identify factors using number-based activities.• Find common factors of two given numbers.	<ul style="list-style-type: none">• Analyse factor relationships step-by-step.• Solve reasoning problems using common factors.	<ul style="list-style-type: none">• Create a number activity using factors.• Design a logical puzzle based on factors.

Conclusion :

The first five chapters of Semester-1 develop essential mathematical skills such as pattern recognition, logical reasoning, data interpretation, and geometrical understanding. These skills form the foundation of intelligent systems.

With careful study, disciplined practice, and responsible use of technology, mathematical knowledge becomes meaningful and applicable in real-life situations.