CHAPTER 1

AP VIII CLASS-CBSE (2023-24)
RATIONAL NUMBERS (Notes)
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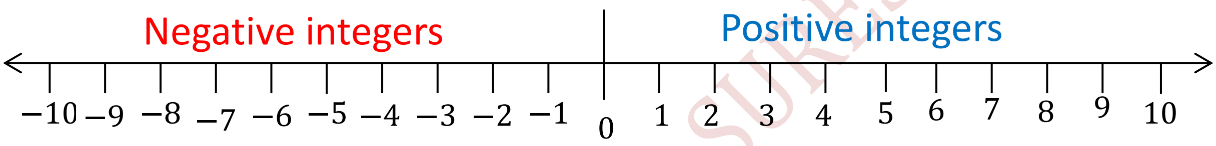
1. **Natural numbers**: The numbers which are used for counting are called Natural numbers and represented with letter N
   N={1,2,3,4,5,……}
2. **Whole numbers**: If ‘0’ is added to Natural numbers then they are called Whole numbers. And is denoted by ‘W’
   W={0,1,2,3,4,5,……}
3. **Integers**: Combination of positive and negative numbers Including 0 are called Integers and represented by ‘Z’ or ‘I’.
   Z=$\{\dots\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\dots\dots\}$
4. Integers number line



5. **Addition of integers**:

(i) When two positive integers are added, we get a positive integer.
e.g. $(+5)+(+6)=+11$

(ii) When two negative integers are added, we get a negative integer.
e.g. $(-5)+(-6)=-11$

(iii) When one positive and one negative integer are added we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained.
e.g. $(+8)+(-5)=3$, $(-8)+(+5)=-3$, $-7+5=-2$

6. **Multiplication of integers**:

(i) If the signs of two integers are same then the product is positive integer.
e.g. $(+3)\times(+5)=15$, $(-4)\times(-3)=12$

(ii) If the signs of two integers are different then the product is negative integer.
e.g. $(+3)\times(-5)=-15$, $(-3)\times(+5)=-15$, $(-4)\times(+3)=-12$, $(+4)\times(-3)=-12$

7. **Division of integers**:

(i) If the signs are same then the quotient is positive.
e.g. $12\div 3=4$, $(-12)\div(-3)=4$

(ii) If the signs are different then the quotient is negative.
e.g. $(-12)\div 3=-4$, $12\div(-3)=-4$

8. Division by zero is not defined

$\frac{1}{0}, \quad \frac{3}{0}, \quad \frac{-51}{0}, \quad \frac{-8}{0}, \quad$ *are not defined*

9. $0 \in W$ (0 belongs to whole numbers)

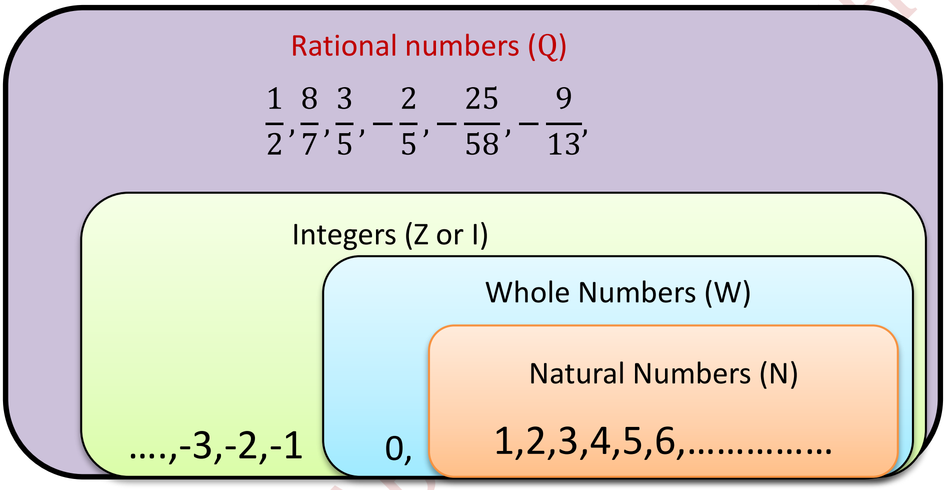
10. $0 \notin N$ (0 does not belong to natural numbers)

11. $-3 \in Z$ (-3 belongs to integers)

12. **Rational numbers**:

A number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Example: $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, −2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers.



1. **Rational numbers are closed under addition** i.e. $a, b \in R \Rightarrow a + b \in R$

$$e.g.\ a = \frac{3}{8}, b = \frac{-5}{7} \text{ are two rational numbers}$$

$$a + b = \frac{3}{8} + \left(\frac{-5}{7}\right) = \frac{3 \times 7 + (-5 \times 8)}{56} = \frac{21 + (-40)}{56} = \frac{-19}{56} \quad \text{is a rational number}$$

2. **Rational numbers are closed under subtraction** i.e. $a, b \in R \Rightarrow a - b \in R$

$$e.g.\ a = \frac{3}{7}, b = \frac{-8}{5} \text{ are two rational numbers.}$$

$$a - b = \frac{3}{7} - \left(\frac{-8}{5}\right) = \frac{3}{7} + \frac{8}{5} = \frac{3 \times 5 + 7 \times 8}{35} = \frac{15 + 56}{35} = \frac{71}{35} \quad \text{is a rational number}$$

3. **Rational numbers are closed under multiplication** i.e. $a, b \in R \Rightarrow a \times b \in R$

$$e.g.\ a = -\frac{4}{5}, b = \frac{-6}{11} \text{ are two rational numbers.}$$

$$a \times b = \left(-\frac{4}{5}\right) \times \left(\frac{-6}{11}\right) = \frac{(-4) \times (-6)}{5 \times 11} = \frac{24}{55} \quad \text{is a rational number}$$

4. For any rational number $a, a \div 0 = \frac{a}{0}$ is not defined

5. Exclude zero then the collection of, all other rational numbers is closed under division

i.e. $a, b(\neq 0) \in R \Rightarrow a \div b = \frac{a}{b} \in R$

$e.g$: $a = \frac{-3}{8}, b = \frac{-9}{2}$ are two rational numbers

$a \div b = \frac{-3}{8} \div \frac{-9}{2} = \left(\frac{-3}{8}\right) \times \left(\frac{-2}{9}\right) = \frac{(-3) \times (-2)}{8 \times 9} = \frac{1}{12}$ is a rational number.

6. **Addition is commutative for rational numbers.** i.e. $a, b \in R \Rightarrow a + b = b + a$

$e.g: a = \frac{-6}{5}, b = \frac{-8}{3}$

$$a + b = \left(\frac{-6}{5}\right) + \left(\frac{-8}{3}\right) = \frac{(-6 \times 3) + (-8 \times 5)}{15} = \frac{(-18) + (-40)}{15} = \frac{-58}{15}$$

$$b + a = \left(\frac{-8}{3}\right) + \left(\frac{-6}{5}\right) = \frac{(-8 \times 5) + (-6 \times 3)}{15} = \frac{(-40) + (-18)}{15} = \frac{-58}{15}$$

$$\therefore a + b = b + a$$

7. **Subtraction will not be commutative for rational numbers** .i.e. $a, b \in R \Rightarrow a - b \neq b - a$

$e.g: a = \frac{2}{3}, b = \frac{5}{4}$

$$a - b = \frac{2}{3} - \frac{5}{4} = \frac{2 \times 4 - 5 \times 3}{12} = \frac{8 - 15}{12} = \frac{-7}{12}$$

$$b - a = \frac{5}{4} - \frac{2}{3} = \frac{5 \times 3 - 2 \times 4}{12} = \frac{15 - 8}{12} = \frac{7}{12}$$

$$a - b \neq b - a$$

8. **Multiplication is commutative for rational numbers.** i.e. $a, b \in R \Rightarrow a \times b = b \times a$

$e.g: a = \frac{-8}{9}, b = \frac{-4}{7}$

$$a \times b = \left(\frac{-8}{9}\right) \times \left(\frac{-4}{7}\right) = \frac{(-8) \times (-4)}{(9) \times (7)} = \frac{32}{63}$$

$$b \times a = \left(\frac{-4}{7}\right) \times \left(\frac{-8}{9}\right) = \frac{(-4) \times (-8)}{(7) \times (9)} = \frac{32}{63}$$

$$\therefore\ a \times b = b \times a$$

9. **Division is not commutative for rational numbers.** i.e. $a, b \in R \Rightarrow \frac{a}{b} \neq \frac{b}{a}$

$e.g: a = \frac{-5}{4}, b = \frac{3}{7}$

$$a \div b = \left(\frac{-5}{4}\right) \div \frac{3}{7}$$
$$= \left(\frac{-5}{4}\right) \times \frac{7}{3}$$
$$= \frac{(-5) \times 7}{4 \times 3} = \frac{-35}{12}$$

$$b \div a = \frac{3}{7} \div \left(\frac{-5}{4}\right)$$
$$= \frac{3}{7} \times \left(\frac{-4}{5}\right)$$
$$= \frac{3 \times (-4)}{7 \times 5} = \frac{-12}{35}$$

$$\therefore a \div b \neq b \div a$$

10. **Additive is associative for rational numbers** i.e. $a, b, c \in R \Rightarrow a + (b + c) = (a + b) + c$

$e.g: a = \frac{-2}{3}, b = \frac{3}{5}, c = \frac{-5}{6}$

$$a + (b + c) = \frac{-2}{3} + \left[\frac{3}{5} + \left(\frac{-5}{6}\right)\right]$$
$$= \frac{-2}{3} + \left[\frac{3 \times 6 + (-5 \times 5)}{30}\right]$$
$$= \frac{-2}{3} + \left[\frac{18 - 25}{30}\right]$$
$$= \frac{-2}{3} + \left(\frac{-7}{30}\right)$$
$$= \frac{(-2 \times 10) + (-7)}{30}$$
$$= \frac{(-20) + (-7)}{30} = \frac{-27}{30}$$

$$(a + b) + c = \left[\frac{-2}{3} + \frac{3}{5}\right] + \left(\frac{-5}{6}\right)$$
$$= \left[\frac{-2 \times 5 + 3 \times 3}{15}\right] + \left(\frac{-5}{6}\right)$$
$$= \left[\frac{-10 + 9}{15}\right] + \left(\frac{-5}{6}\right)$$
$$= \left(\frac{-1}{15}\right) + \left(\frac{-5}{6}\right)$$
$$= \frac{(-1 \times 2) + (-5 \times 5)}{30}$$
$$= \frac{(-2) + (-25)}{30} = \frac{-27}{30}$$

$$\therefore a + (b + c) = (a + b) + c$$

11. **Subtraction is not associative for rational numbers**

$i.e. a, b, c \in R \Rightarrow a - (b - c) \neq (a - b) - c$

$e.g: a = \frac{-2}{3}, b = \frac{-4}{5}, c = \frac{1}{2}$

$$a - (b - c) = \frac{-2}{3} - \left[\frac{-4}{5} - \frac{1}{2}\right]$$
$$= \frac{-2}{3} - \left[\frac{-4 \times 2 - 1 \times 5}{10}\right]$$
$$= \frac{-2}{3} - \left[\frac{-8 - 5}{10}\right]$$
$$= \frac{-2}{3} - \left(\frac{-13}{10}\right) = \frac{-2}{3} + \frac{13}{10}$$

$$= \frac{-2 \times 10 + 13 \times 3}{30}$$

$$= \frac{-20 + 39}{30} = \frac{19}{30}$$

$$(a - b) - c = \left[\frac{-2}{3} - \left(\frac{-4}{5}\right)\right] - \frac{1}{2}$$

$$= \left[\frac{-2}{3} + \frac{4}{5}\right] - \frac{1}{2}$$

$$= \left[\frac{-2 \times 5 + 4 \times 3}{15}\right] - \frac{1}{2}$$

$$= \left[\frac{-10 + 12}{15}\right] - \frac{1}{2}$$

$$= \frac{2}{15} - \frac{1}{2} = \frac{2 \times 2 - 1 \times 15}{30}$$

$$= \frac{4 - 15}{30} = \frac{-11}{30}$$

$$a - (b - c) \neq (a - b) - c$$

12. Multiplication is associative for rational numbers

$i.e. a, b, c \in R \Rightarrow a \times (b \times c) = (a \times b) \times c$

$e.g: \ a = \frac{-7}{3}, b = \frac{5}{4}, c = \frac{2}{9}$

$$a \times (b \times c) = \frac{-7}{3} \times \left[\frac{5}{4} \times \frac{2}{9}\right]$$

$$= \frac{-7}{3} \times \frac{10}{36} = \frac{-7 \times 10}{3 \times 36}$$

$$= \frac{-7 \times 5}{3 \times 18} = \frac{-35}{54}$$

$$(a \times b) \times c = \left[\frac{-7}{3} \times \frac{5}{4}\right] \times \frac{2}{9}$$

$$= \left(\frac{-35}{12}\right) \times \frac{2}{9} = \frac{-35 \times 2}{12 \times 9}$$

$$= \frac{-35 \times 1}{6 \times 9} = \frac{-35}{54}$$

$$\therefore a \times (b \times c) = (a \times b) \times c$$

13. Division is not associative for rational numbers

i.e. $a, b, c \in R \Rightarrow a \div (b \div c) \neq (a \div b) \div c$

$e.g: \ a = \frac{1}{2}, b = \frac{-1}{3}, c = \frac{2}{5}$

$$a \div (b \div c) = \frac{1}{2} \div \left[\frac{-1}{3} \div \frac{2}{5}\right]$$

$$= \frac{1}{2} \div \left[\frac{-1}{3} \times \frac{5}{2}\right]$$

$$= \frac{1}{2} \div \left(\frac{-5}{6}\right)$$

$$= \frac{1}{2} \times \left(\frac{-6}{5}\right) = \frac{-3}{5}$$

$$(a \div b) \div c = \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$$

$$= \left[\frac{1}{2} \times \frac{-3}{1}\right] \div \frac{2}{5}$$

$$= \frac{-3}{2} \div \frac{2}{5}$$

$$= \frac{-3}{2} \times \frac{5}{2} = \frac{-15}{4}$$

$$a \div (b \div c) \neq (a \div b) \div c$$

14. Zero is called the identity for the addition of rational numbers.

$For \ a \in R, a + 0 = 0 + a = a$

15. 1 is the multiplicative identity for rational numbers.

$For\ a \in R, a \times 1 = 1 \times a = a$

16. For a rational number $\frac{a}{b}$, we have,

$$\frac{a}{b} + \left(-\frac{a}{b}\right) = \left(-\frac{a}{b}\right) + \frac{a}{b} = 0$$

We say that $\left(-\frac{a}{b}\right)$ is the additive inverse of $\frac{a}{b}$ and $\frac{a}{b}$ is the additive inverse of $\left(-\frac{a}{b}\right)$

17. $If\ \frac{a}{b} \times \frac{c}{d} = 1\ then\ \frac{c}{d}\ is\ called$ the reciprocal or multiplicative inverse of $\frac{a}{b}$

18. **Distributivity of Multiplication over Addition and Subtraction.**

For all rational numbers a, b, c

$a(b+c) = ab + ac$

$a(b-c) = ab - ac$

$e.g:\ a = \frac{-3}{4}, b = \frac{2}{3}, c = \frac{-5}{6}$

$$a \times (b+c) = \frac{-3}{4} \times \left[\frac{2}{3} + \left(\frac{-5}{6}\right)\right]$$

$$= \frac{-3}{4} \times \left[\frac{2 \times 2 + (-5 \times 1)}{6}\right]$$

$$= \frac{-3}{4} \times \left(\frac{4 + (-5)}{6}\right)$$

$$= \frac{-3}{4} \times \left(\frac{-1}{6}\right) = \frac{3}{24} = \frac{1}{8}$$

$$a \times b + a \times c = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$$

$$= \frac{-1}{2} + \frac{5}{8}$$

$$= \frac{(-1 \times 4) + 5}{8}$$

$$= \frac{-4+5}{8} = \frac{1}{8}$$

$$\therefore a \times (b+c) = a \times b + a \times c$$

Properties of Rational numbers

Property Name	Addition	Subtraction	Multiplication	Division
	$a, b, c \in Q$			a, b, c are non-zero rationale
Closure Property	$a + b \in Q$	$a - b \in Q$	$a \times b \in Q$	$a \div b \in Q$
Commutative law	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative Law	$(a+b)+c = a+(b+c)$	$(a-b)-c \neq a-(b-c)$	$(a \times b) \times c = a \times (b \times c)$	$(a \div b) \div c \neq a \div (b \div c)$
Identity Property	$a + 0 = a$ $0 + a = a$	Not applicable	$a \times 1 = a$ $1 \times a = a$	Not applicable
Inverse Property	$a + (-a) = 0\ and$ $(-a) + a = 0$ $(-a)\ is\ additive\ inverse\ of\ a$ $a\ is\ additive\ inverse\ of\ (-a)$		$a \times \frac{1}{a} = 1\quad and\quad \frac{1}{a} \times a = 1$ $\frac{1}{a}\ is\ multiplicative\ inverse\ of\ a$ $a\ is\ multiplicative\ inverse\ of\ \frac{1}{a}$	

Distributive	$a(b+c)=ab+ac$	$a(b-c)=ab-ac$

Example 1*: *Find $\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\left(\frac{5}{22}\right)$

Sol: $\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\left(\frac{5}{22}\right)$

$$=\frac{3\times 66+(-6)\times 42+(-8)\times 22+5\times 21}{462}$$

$$=\frac{198-252-176+105}{462}$$

$$=\frac{303-428}{462}=\frac{-125}{462}$$

$$\begin{array}{r|l} 7 & 7,11,21,22 \\ \hline 11 & 1,\ 11,\ 3,\ 22 \\ \hline & 1,1,3,\ 2 \end{array}$$

L.C.M $of\, 7,11,21,22=7\times 11\times 3\times 2$
=462

Example 2*: *Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Sol: $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right]$$

$$=\left(\frac{-3}{4}\right)\times\left(\frac{-2}{3}\right)=\frac{1}{2}$$

The role of zero (0)

Where a is a rational number then

$(i)\ a+0=0+a=a$

$(ii)\ a\times 0=0\times a=0$

'Zero' is called the identity for the addition of rational numbers.

The role of '1'

Where 'a' is a rational number then

$(i)\ a\times 1=1\times a=a$

1 is the multiplicative identity for rational numbers

TRY THESE

Find using distributivity

For all rational numbers a, b and c

$(i)\ a\times(b+c)=a\times b+a\times c$

$(ii)\ a\times(b-c)=a\times b-a\times c$

$(i)\ \left\{\frac{7}{5}\times\left(\frac{-3}{12}\right)\right\}+\left\{\frac{7}{5}\times\frac{5}{12}\right\}$

Sol: $\left\{\frac{7}{5}\times\left(\frac{-3}{12}\right)\right\}+\left\{\frac{7}{5}\times\frac{5}{12}\right\}$

$$=\frac{7}{5}\times\left(\frac{-3}{12}+\frac{5}{12}\right)\ \left(by\ distributivity\ a\times b+a\times c=a\times(b+c)\right)$$

$$=\frac{7}{5}\times\frac{2}{12}=\frac{7\times 1}{5\times 6}=\frac{7}{30}$$

(ii) $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}$

Sol: $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}$

$$=\frac{9}{16}\times\left(\frac{4}{12}+\frac{-3}{9}\right)\ \left(by\ distributivity\ a\times b+a\times c=a\times(b+c)\right)$$

$$=\frac{9}{16}\times\left(\frac{1}{3}-\frac{1}{3}\right)=\frac{9}{16}\times 0=0$$

Example* 5: *Find $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

Sol: $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

$$=\frac{-3}{7}\times\frac{2}{5}+\left(-\frac{3}{7}\right)\times\frac{3}{5}-\frac{1}{14}\ \ (by\ commutativity)$$

$$=\frac{-3}{7}\left(\frac{2}{5}+\frac{3}{5}\right)-\frac{1}{14}$$

$$=\frac{-3}{7}\times\left(\frac{5}{5}\right)-\frac{1}{14}$$

$$=\frac{-3}{7}-\frac{1}{14}$$

$$=\frac{-3\times 2-1}{14}=\frac{-6-1}{14}=\frac{-7}{14}=\frac{-1}{2}$$

EXERCISE 1.1

5. Name the property under multiplication used in each of the following.

(i) $\frac{-4}{5}\times 1=1\times\frac{-4}{5}=\frac{-4}{5}$ → Multiplicative identity

(ii) $-\frac{13}{17}\times\frac{-2}{7}=\frac{-2}{7}\times-\frac{13}{17}$ → commutative under multiplication

(iii) $\frac{-19}{29}\times\frac{29}{-19}=1$ → Multiplicative inverse

7. Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ ***as*** $\left(\frac{1}{3}\times 6\right)\times\frac{4}{3}$

Sol: *Associative property under multiplication*.

11. The product of two rational numbers is always a **rational number**

Bits

1. A number can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.
2. The rational number 0 is the additive identity for rational numbers.
3. The rational number 1 is the multiplicative identity for rational numbers.
4. The additive inverse of the rational number $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa.
5. The reciprocal or multiplicative inverse of the rational number $\frac{a}{b}$ is $\frac{b}{a}$.
6. Distributivity of rational numbers : For all rational numbers a, b and c
$$a(b+c) = ab + ac \quad and \quad a(b-c) = ab - ac$$
7. The reciprocal of a positive rational number is positive.
8. The reciprocal of a negative rational number is negative.
9. Zero has no reciprocal.
10. The numbers 1 and -1 are their own reciprocal.
11. The negative of a negative rational number is always a positive rational number.
12. The set of numbers which do not have any additive identity – Natural numbers(N)
13. The rational number that does not have any reciprocal is 0.
14. Commutative under addition: $a + b = b + a$
15. Commutative under multiplication: $a \times b = b \times a$
16. Associative property under addition: $a + (b + c) = (a + b) + c$
17. Associative property under multiplication: $a \times (b \times c) = (a \times b) \times c$
18. Division by zero is not defined
$$\frac{1}{0}, \quad \frac{3}{0}, \quad \frac{-51}{0}, \quad \frac{-8}{0}, \quad are\ not\ defined$$
19. $0 \in W$ (0 belongs to whole numbers)
20. $0 \notin N$ (0 does not belong to natural numbers)
21. $-3 \in Z$ (–3 belongs to integers)
22. A rational number and its additive inverse are opposite in their sign.
23. The multiplicative inverse of a rational number is its reciprocal.
24. Neither a positive nor a negative rational number is 0.
25. The equivalent of $\frac{5}{7}$, whose numerator is 45 is $\frac{45}{63}$
26. The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is $\frac{35}{45}$

CHAPTER 2

AP VIII CLASS-CBSE (20223-24)

LINEAR EQUATIONS IN ONE VARIABLE (Notes)

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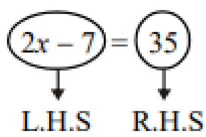
1. Expression: An expression is a constant or a variable or combination of these two, using the mathematical operations (+, –, ×, ÷) i.e., terms are added to form expressions
2. Algebraic expression: If an expression has at least one algebraic term, then that expression is Algebraic expression.

   Ex: d, a + 3, 5c – 4, $2x^2 + 3x - 6$, $\frac{x}{y}$ are Algebraic expressions.
3. Coefficient: A coefficient may be either a numerical or an algebraic factor or a product of both in a term.
4. The terms having the same algebraic factors are like terms and the terms having different algebraic factors are unlike terms.

   Examples:

   (i). The terms $2x, -3x$ and $4x$ are like terms, as they have same algebraic factor 'x'

   (ii). The terms 5t and 8s are unlike terms, as they have different algebraic factors t and s
5. **Monomial**: An expression with only one term is called Monomial.
6. **Binomial**: An expression which contains two unlike terms is called a Binomial.
7. **Trinomial**: An expression which contains three unlike terms is called a Trinomial.
8. **Polynomial** : An algebraic expression in which the exponent of variable is a non-negative integer is called a Polynomial
9. In an expression, if the terms are arranged in such a way that the exponents of the terms are in descending order then the expression is said to be in standard form.
10. **Linear equations in one variable**: An equation of the form $ax + b = 0$ or $ax = b$ where a, b are constants and $a \neq 0$ is called linear equation in one variable or simple equation.
11. If the degree of an equation is one then it is called a linear equation.
12. The expression on the left of the equality sign is called the **L.H.S** (Left Hand Side) of the equation and right of the equality sign is called **R.H.S** (Right Hand Side) of the equation.



13. The value which when substituted for the variable in the given equation makes L.H.S. = R.H.S. is called a solution or root of the given equation.
14. When we transpose terms

   '+' quantity becomes '−'quantity

   '−' quantity becomes '+'quantity

   '×' quantity becomes '÷'quantity

   '÷' quantity becomes '×'quantity

Example 1: Find the solution of 2x – 3 = 7

Sol: Given equation: 2x – 3 = 7

$(Transposing - 3\ to\ R.H.S\ it\ become + 3)$

$2x = 7 + 3.$

$2x = 10$

Divide both sides by 2

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Example 2: Solve $5x + \frac{7}{2} = \frac{3}{2}x - 14$

Sol: Given equation: $\mathbf{5x + \frac{7}{2} = \frac{3}{2}x - 14}$

Multiply both sides by 2

$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$

$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$

$10x + 7 = 3x - 28$

$10x - 3x == -28 - 7$ (transposing $3x$ *to* LHS and $+ 7$ to RHS)

$7x = -35$

Divide both sides by 7

$\frac{7x}{7} = \frac{-35}{7}$

$x = -5$

$\therefore$ Solution $x = -5$

EXERCISE 2.3

Solve the following equations and check your results.

1. $3x = 2x + 18$

Sol: 3x = 2x + 18

$3x - 2x = 18$

$x = 18$

Check:

Putting x=18

LHS = 3 × 18 = 54

RHS = 2 × 18 + 18 = 36 + 18 = 54

LHS = RHS

Hence verified

2. 5t – 3 = 3t – 5

Sol: 5t – 3 = 3t – 5

$5t - 3t = -5 + 3$

$2t = -2$

$t = \frac{-2}{2} = -1$

Check: Putting $t = -1$

LHS $= 5(-1) - 3 = -5 - 3 = -8$

RHS $= 3(-1) - 5 = -3 - 5 = -8$

LHS = RHS $\therefore$ Hence verified

3. $5x + 9 = 5 + 3x$

Sol: $5x + 9 = 5 + 3x$

$5x - 3x = 5 - 9$

$2x = -4$

$x = \frac{-4}{2} = -2$

Check: Putting $x = -2$

LHS $= 5(-2) + 9 = -10 + 9 = -1$

RHS $= 5 + 3(-2) = 5 - 6 = -1$

LHS = RHS, Hence verified

4. $4z + 3 = 6 + 2z$

Sol: 4z + 3 = 6 + 2z

$4z - 2z = 6 - 3$

$2z = 3$

$z = \frac{3}{2}$

Check: Putting $z = \frac{3}{2}$

LHS $= 4\left(\frac{3}{2}\right) + 3 = 6 + 3 = 9$

RHS $= 6 + 2\left(\frac{3}{2}\right) = 6 + 3 = 9$

LHS = RHS, Hence verified.

5. $2x - 1 = 14 - x$

Sol: $2x - 1 = 14 - x$

$2x + x = 14 + 1$

$3x = 15$

$x = \frac{15}{3} = 5$

Check: Putting $x = 5$

LHS $= 2(5) - 1 = 10 - 1 = 9$

RHS $= 14 - 5 = 9$

LHS = RHS . Hence verified

6. $8x + 4 = 3(x - 1) + 7$

Sol: $8x + 4 = 3(x - 1) + 7$

$8x + 4 = 3x - 3 + 7$

$8x + 4 = 3x + 4$

$8x - 3x = 4 - 4$

$5x = 0$

$x = 0$

Check:Putting $x = 0$

LHS $= 8(0) + 4 = 0 + 4 = 4$

RHS $= 3(0 - 1) + 7 = -3 + 7 = 4$

LHS = RHS

Hence verified.

7. $x = \frac{4}{5}(x + 10)$

Sol: $x = \frac{4}{5}(x+10)$

$5x = 4(x+10)$

$5x = 4x + 40$

$5x - 4x = 40$

$x = 40$

Check: Putting $x = 40$

LHS $= 40$

RHS $= \frac{4}{5}(40+10) = \frac{4}{5} \times 50 = 4 \times 10 = 40$

LHS = RHS, Hence verified.

8. $\boldsymbol{\frac{2x}{3} + 1 = \frac{7x}{15} + 3}$

Sol: $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

Multiply with'15'

$15 \times \left(\frac{2x}{3} + 1\right) = 15 \times \left(\frac{7x}{15} + 3\right)$

$15 \times \frac{2x}{3} + 15 \times$

$= 15 \times \frac{7x}{15} + 15 \times 3$

$10x + 15 = 7x + 45$

$10x - 7x = 45 - 15$

$3x = 30$

$x = \frac{30}{3} = 10$

Check: Putting $x = 10$

LHS $= \frac{2 \times 10}{3} + 1 = \frac{20}{3} + 1 = \frac{23}{3}$

RHS $= \frac{7 \times 10}{15} + 3 = \frac{14}{3} + 3 = \frac{23}{3}$

LHS = RHS

Hence verified

9. $\boldsymbol{2y + \frac{5}{3} = \frac{26}{3} - y}$

Sol: $2y + \frac{5}{3} = \frac{26}{3} - y$

Multiply with '3'

$3 \times \left(2y + \frac{5}{3}\right) = 3 \times \left(\frac{26}{3} - y\right)$

$3 \times 2y + 3 \times \frac{5}{3} = 3 \times \frac{26}{3} - 3 \times y$

$6y + 5 = 26 - 3y$

$6y + 3y = 26 - 5$

$9y = 21$

$y = \frac{21}{9} = \frac{7}{3}$

Check: Putting $y = 7$

LHS $= 2\left(\frac{7}{3}\right) + \frac{5}{3} = \frac{14}{3} + \frac{5}{3} = \frac{19}{3}$

RHS $= \frac{26}{3} - \frac{7}{3} = \frac{26-7}{3} = \frac{19}{3}$

LHS = RHS

Hence verified.

10. $\boldsymbol{3m = 5m - \frac{8}{5}}$

Sol: $5m - \frac{8}{5} = 3m$

$5m - 3m = \frac{8}{5}$

$2m = \frac{8}{5}$

$m = \frac{8}{5 \times 2} = \frac{4}{5}$

Check:

Putting $m = \frac{4}{5}$

$$LHS = 3 \times \frac{4}{5} = \frac{12}{5}$$

$$RHS = 5 \times \frac{4}{5} - \frac{8}{5} = \frac{20}{5} - \frac{8}{5} = \frac{12}{5}$$

$$LHS = RHS$$

Hence verified.

Reducing Equations to Simpler Form

$$\boldsymbol{Ex\ 16: Solve\ \frac{6x+1}{3} + 1 = \frac{x-3}{6}}$$

Sol: LCM of 3,6=6

Multiplying both sides of the equation by 6

$$6 \times \left(\frac{6x+1}{3}\right) + 6 \times 1 = 6 \times \left(\frac{x-3}{6}\right)$$

$$2(6x+1) + 6 = x - 3$$

$$12x + 2 + 6 = x - 3$$

$$12x + 8 = x - 3$$

$$12x - x = -3 - 8$$

$$11x = -11$$

$$x = -1$$

$Check:\ put\ x = -1$

$$LHS = \frac{6(-1)+1}{3} + 1$$

$$= \frac{-6+1}{3} + 1 = \frac{-5}{3} + \frac{3}{3} = \frac{-2}{3}$$

$$RHS = \frac{-1-3}{6} = \frac{-4}{6} = \frac{-2}{3}$$

$$LHS = RHS$$

$$\boldsymbol{Ex\ 17: Solve\ 5x - 2(2x-7) = 2(3x-1) + \frac{7}{2}}$$

$$Sol: 5x - 2(2x-7) = 2(3x-1) + \frac{7}{2}$$

$$5x - 4x + 14 = 6x - 2 + \frac{7}{2}$$

$$x + 14 = 6x - 2 + \frac{7}{2}$$

$$14 + 2 - \frac{7}{2} = 6x - x$$

$$16 - \frac{7}{2} = 5x$$

$$5x = \frac{32-7}{2} = \frac{25}{2}$$

$$x = \frac{25}{2 \times 5} = \frac{5}{2}$$

EXERCISE 2.5

Solve the following linear equations.

1. $\boldsymbol{\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}}$

$$Sol: \frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$$

$$\frac{5x-2}{10}=\frac{4x+3}{12}$$

$$12(5x-2)=10(4x+3)$$

$$12\times 5x-12\times 2=10\times 4x+10\times 3$$

$$60x-24=40x+30$$

$$60x-40x=30+24$$

$$20x=54$$

$$x=\frac{54}{20}=\frac{27}{10}$$

2. $\boldsymbol{\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21}$

$sol:$ $\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$

$$\frac{6n-9n+10n}{12}=21$$

$$\frac{7n}{12}=21$$

$$7n=21\times 12$$

$$n=\frac{21\times 12}{7}=36$$

3. $\boldsymbol{x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}}$

$Sol:$ $x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}$

LCM of 3,6,2 =6

Multiply with '6'

$$6\times x+6\times 7-6\times\frac{8x}{3}=6\times\frac{17}{6}-6\times\frac{5x}{2}$$

$$6x+42-16x=17-15x$$

$$42-10x=17-15x$$

$$15x-10x=17-42$$

$$5x=-25$$

$$x=\frac{-25}{5}=-5$$

4. $\boldsymbol{\frac{x-5}{3}=\frac{x-3}{5}}$

$Sol:$ $\frac{x-5}{3}=\frac{x-3}{5}$

$$5(x-5)=3(x-3)$$

$5x - 25 = 3x - 9$

$5x - 3x = -9 + 25$

$2x = 16$

$x = \frac{16}{2} = 8$

5. $\boldsymbol{\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t}$

Sol: $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

LCM of 4,3=12

Multiply with'12'

$12 \times \frac{3t-2}{4} - 12 \times \frac{2t+3}{3} = 12 \times \frac{2}{3} - 12 \times t$

$3(3t-2) - 4(2t+3) = 4 \times 2 - 12t$

$9t - 6 - 8t - 12 = 8 - 12t$

$t - 18 = 8 - 12t$

$t + 12t = 8 + 18$

$13t = 26$

$t = \frac{26}{13} = 2$

6. $\boldsymbol{m - \frac{m-1}{2} = 1 - \frac{m-2}{3}}$

Sol: $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

LCM of 2,3=6

Multiply with '6'

$6 \times m - 6 \times \left(\frac{m-1}{2}\right) = 6 \times 1 - 6 \times \left(\frac{m-2}{3}\right)$

$6m - 3(m-1) = 6 - 2(m-2)$

$6m - 3m + 3 = 6 - 2m + 4$

$3m + 3 = 10 - 2m$

$3m + 2m = 10 - 3$

$5m = 7$

$m = \frac{7}{5}$

Simplify and solve the following linear equations.

7. $\boldsymbol{3(t - 3) = 5(2t + 1)}$

Sol: $3(t - 3) = 5(2t + 1)$

$3t - 9 = 10t + 5$

$3t - 10t = 5 + 9$

$-7t = 14$

$t = \frac{14}{-7} = -2$

8. $\mathbf{15(y - 4) - 2(y - 9) + 5(y + 6) = 0}$

Sol: $15(y - 4) - 2(y - 9) + 5(y + 6) = 0$

$15y - 60 - 2y + 18 + 5y + 30 = 0$

$18y - 12 = 0$

$18y = 12$

$y = \frac{12}{18} = \frac{2}{3}$

9. $\mathbf{3(5z - 7) - 2(9z - 11) = 4(8z - 13) - 17}$

Sol: $3(5z - 7) - 2(9z - 11) = 4(8z - 13) - 17$

$15z - 21 - 18z + 22 = 32z - 52 - 17$

$-3z + 1 = 32z - 69$

$32z + 3z = 1 + 69$

$35z = 70$

$z = \frac{70}{35} = 2$

10. $\mathbf{0.25(4f - 3) = 0.05(10f - 9)}$

Sol: $0.25(4f - 3) = 0.05(10f - 9)$

$\frac{25}{100}(4f - 3) = \frac{5}{100}(10f - 9)$

$25(4f - 3) = 100 \times \frac{5}{100}(10f - 9)$

$100f - 75 = 50f - 45$

$100f - 50f = -45 + 75$

$50f = 30$

$f = \frac{30}{50} = 0.6$

CHAPTER 3

AP VIII CLASS-CBSE (2023-24)

Understanding Quadrilaterals (Notes)

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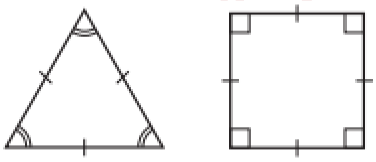
1. **Simple curve**: A simple curve is a curve that does not cross itself
2. **Simple closed curve**: A curve which starts and ends at the same point without crossing itself is called a simple closed curve.
3. **Polygons**: A simple closed curve made up of only line segments is called a polygon
4. **Convex polygon**: A Convex polygon is defined as a polygon with no portions of their diagonals in their exteriors.
5. **Concave polygon**: a polygon that has at least one interior angle greater than 180 degrees

(or) A concave polygon is a polygon which is not convex.

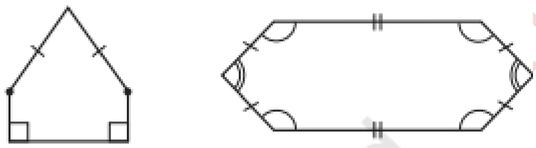
6. **Regular polygon**: Regular polygons have all sides equal in length and all angles are equal.

Ex: Equilateral triangle , Square,...



7. **Irregular polygon**: An irregular polygon does not have all sides equal also all angles are not equal.

Ex: Scalene triangle, Right triangle,....

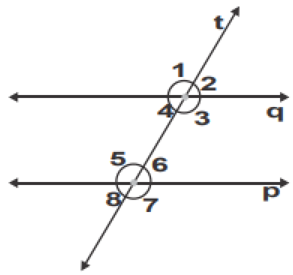


Polygons that are not regular

8. **Angle sum property of triangle**: The sum of the measures of the three angles of a triangle is 180°.
9. **Angle sum property of quadrilateral**: The sum of measures of the four angles of a quadrilateral is360^0.
10. **Complementary angles**: Ifthesumoftwoanglesis90^0 ,then the angles are called as complementary angles to each other
11. **Supplementary angles**: If the sum of two angles is 180^0 , then the angles are called as supplementary angles to each other.
12. **Conjugate angles**: If the sum of two angles is 360^0 , then the angles are called as conjugate angles to each other.
13. **Linear pair of angles**: “A pair of adjacent angles whose sum is 1800 are called linear pair of angles.

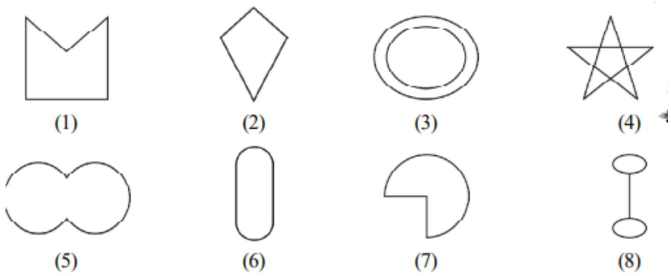
14.

Interior angles	∠3, ∠4, ∠5, ∠6
Exterior angles	∠1, ∠2, ∠7, ∠8
Corresponding angles are equal	$\angle 1 = 5, \angle 2 = \angle 6,$ $\angle 3 = \angle 7, \angle 4 = \angle 8$
Alternate interior angles are equal	$\angle 3 = \angle 5, \angle 4 = \angle 6$
Alternate exterior angles are equal	$\angle 1 = \angle 7, \angle 2 = \angle 8$
Interior angles on same side of transversal(co-interior angles) are supplementary	$\angle 3 + \angle 6 = 180^0$ $\angle 4 + \angle 5 = 180^0$
Exterior angles on same side of transversal(co-exterior angles) are supplementary	$\angle 1 + \angle 8 = 180°$ $\angle 2 + \angle 7 = 180°$



EXERCISE 3.1

1. Given here are some figures.



Classify each of them on the basis of the following.

(a) Simple curve: (1), (2),(5),(6) and (7)

(b) Simple closed curve: (1), (2),(5),(6) and (7)

(c) Polygon: (1),(2)

(d) Convex polygon: (2)

(e) Concave polygon:(1)

5. What is a regular polygon? State the name of a regular polygon of

(i) 3 sides→ Equilateral triangle

(ii) 4 sides → Square

(iii) 6 sides→ Regular Hexagon

Sum of the Measures of the Exterior Angles of a Polygon

* The sum of the measures of the external angles of any polygon is 360°

Example 1: Find measure x

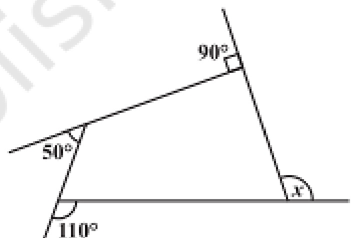


Fig 3.9

Sol: The sum of the external angles of any polygon = 360°

$x + 90^0 + 50^0 + 110^0 = 360^0$

$x + 250^0 = 360^0$

$x = 360^0 - 250^0$

$x = 110^0$

TRY THESE

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?

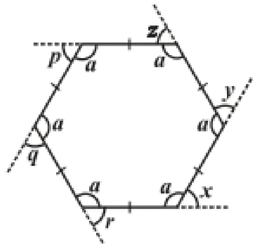
Sol: The sum of the measures of the external angles of any polygon is 360°

So, $x + y + z + p + q + r = 360^0$

Sum of the all interior angles of a hexagon$=(6-2)\times 180^0$

$$= 4 \times 180^0 = 720^0$$

$6a = 720^0 \Rightarrow a = \frac{720^0}{6} = 120^0$



(or)

Each interior angle of a regular hexagon $= \dfrac{(n-2)\times 180^0}{6}$

$= \dfrac{(6-2)\times 180^0}{6} = 4 \times 30^0 = 120^0$

$\therefore a = 120^0$

2. Is $x = y = z = p = q = r$? Why?

Sol: $x + a = 180^0$ (Linear pair)

$\Rightarrow x + 120^0 = 180^0$

$\Rightarrow x = 180^0 - 120^0$

$\Rightarrow x = 60^0$

Similarly

$y = 180^0 - 120^0 = 60^0$

$z = 180^0 - 120^0 = 60^0$

$p = 180^0 - 120^0 = 60^0$

$q = 180^0 - 120^0 = 60^0$

$r = 180^0 - 120^0 = 60^0$

Yes, $x = y = z = p = q = r$

3. **What is the measure of each?**

(i) Exterior angle$=60^0$ (ii) Interior angle$= 120^0$

Each external angle of a regular polygon with n sides $= \dfrac{360°}{n}$

Each interior angle of a regular polygon with 'n' sides $= \dfrac{(n-2)\times 180^0}{n}$

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Sol: Total measure of all exterior angles = 360°

Measure of each exterior angle = 45°

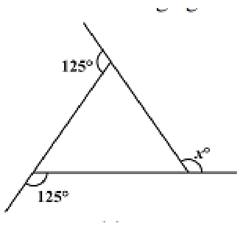
$\text{The number of exterior angles} = \dfrac{360°}{45°} = 8$

The polygon has 8 sides.

EXERCISE 3.2

1. Find x in the following figures.

(a) Total measure of all exterior angles $= 360°$



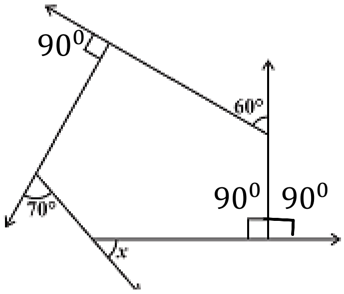
$x^0 + 125^0 + 125^0 = 360^0$

$x^0 + 250^0 = 360^0$

$x^0 = 360^0 - 250^0$

$x^0 = 110^0$

(b)) Total measure of all exterior angles $= 360°$



$x^0 + 90^0 + 60^0 + 90^0 + 70^0 = 360^0$

$x^0 + 310^0 = 360^0$

$x^0 = 360^0 - 310^0$

$x^0 = 50^0$

2. Find the measure of each exterior angle of a regular polygon of

(i) 9 sides

Sol: Total measure of all exterior angles $= 360°$

Each exterior angle of a regular polygon of 9 sides $= \dfrac{360°}{9} = 40°$

(ii) 15 sides

Sol: Total measure of all exterior angles $= 360°$

Each exterior angle of a regular polygon of 15 sides $= \dfrac{360°}{15} = 24°$

3. **How many sides does a regular polygon have if the measure of an exterior angle is 24°?**

Sol: Total measure of all exterior angles $= 360°$

Measure of each exterior angle $= 24°$

The number of exterior angles $= \dfrac{360°}{24°} = 15$

The polygon has 15 sides.

4. **How many sides does a regular polygon have if each of its interior angles is 165°?**

Sol: Total measure of all exterior angles $= 360°$

Measure of each interior angle $= 165°$

Measure of each exterior angle $= 180° - 165° = 15°$

The number of exterior angles $= \dfrac{360°}{15°} = 24$

The polygon has 24 sides.

5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?

Sol: Measure of each exterior angle as $22°$

Total measure of all exterior angles $= 360°$

The number of exterior angles $= \dfrac{360°}{22°}$ it is not a natural number

So, we cannot have regular polygon with each exterior angle $= 22°$

(b) Can it be an interior angle of a regular polygon? Why?

Sol: Measure of each interior angle as 22°

Measure of each exterior angle $= 180^0 - 22° = 158°$

Total measure of all exterior angles $= 360°$

The number of exterior angles $= \dfrac{360°}{158°}$ *it is not a natural number*

So, we cannot have regular polygon with each interior angle $= 22°$

6. (a) What is the minimum interior angle possible for a regular polygon? Why?

Sol: Equilateral triangle with 3 sides is the least regular polygon.

The interior angle of equilateral triangle $= \dfrac{180°}{3} = 60°$

Thus, minimum interior angle possible for a regular polygon $= 60°$

(b) What is the maximum exterior angle possible for a regular polygon?

Sol: Equilateral triangle is regular polygon with 3 sides has maximum exterior angle.

The interior angle of equilateral triangle $= 60°$

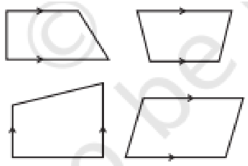
The exterior angle of equilateral triangle $= 180° - 60° = 120°$

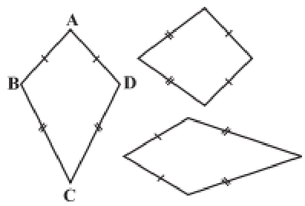
Thus, the maximum exterior angle possible for a regular polygon is 120°.

Kinds of Quadrilaterals

1. **Trapezium**: Trapezium is a quadrilateral with a pair of parallel sides.

2. **Kite**: A kite is a quadrilateral that has 2 pairs of equal-length sides and these sides are adjacent to each other.

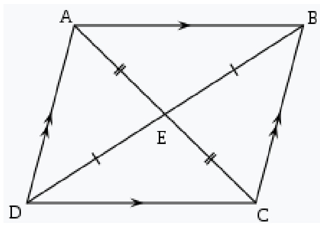

These are trapeziums



These are kites

3. **Parallelogram**: A parallelogram is a quadrilateral whose opposite sides are parallel.

Properties:



(i) Opposite sides are equal and parallel

AB=DC ,BC=AD and $AB \parallel DC$, $BC \parallel AD$

(ii) Opposite angles are equal.

$\angle A = \angle C$ and $\angle B = \angle D$

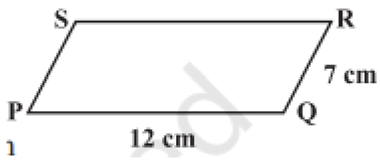
(iii) Diagonal are bisect each other.

AE=EC and BE=ED

(iv) The adjacent angles in a parallelogram are supplementary.

$\angle A+\angle B=180^0$; $\angle B+\angle C=180^0$; $\angle C+\angle D=180^0$; $\angle D+\angle A=180^0$

Example 3: Find the perimeter of the parallelogram PQRS



Sol: In a parallelogram opposite sides are equal

PQ=RS=12cm and QR=PS=7cm

So, Perimeter = PQ + QR + RS + SP

= 12 cm + 7 cm + 12 cm + 7 cm = 38 cm

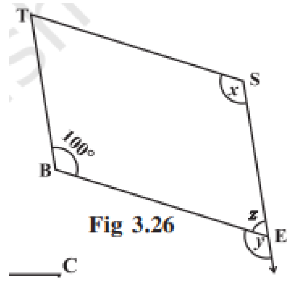


Fig 3.26

Example 4: BEST is a parallelogram. Find the values x, y and z.

Sol: $x = 100^0$ (In a parallelogram opposite angles are equal)

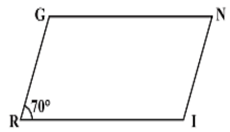
$y = 100^0$ (Interior alternate angles)

$z + y = 180^0$(Linear pair)

$z + 100^0 = 180^0$

$z = 180^0 - 100^0 = 80^0$

$\therefore x = 100^0, y = 100^0, z = 80^0$



Example 5: In a parallelogram RING if m∠R = 70°, find all the other angles

Sol: m∠R = 70°

m∠R+ m∠I=180°(adjacent angles in a parallelogram are supplementary)

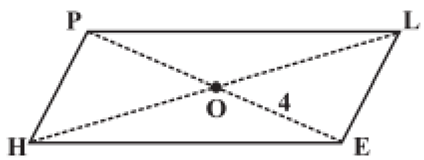
m∠I=180°− 70°=110°

m∠N=m∠R = 70° (Opposite angles are equal)

m∠I= m∠G= 110°(Opposite angles are equal)

Example 6: In Fig 3.31 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

Sol: Given that OE = 4 cm



OE=OP=4 cm (Diagonals are bisect each other)

PE=4+4=8 cm

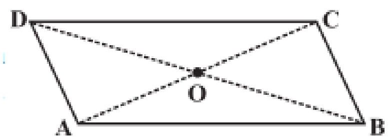
Given HL is 5 more than PE

HL=PE+5=8+5=13 cm

$$OH = OL = \frac{1}{2} \times HL = \frac{1}{2} \times 13 = 6.5 \text{ cm}$$

EXERCISE 3.3

1. Given a parallelogram ABCD. Complete each statement along with the definition or property used



(i) AD = BC (In a parallelogram opposites sides are equal)

(ii) ∠ DCB = ∠ DAB (In a parallelogram opposites angles are equal

(iii) OC = OA (Diagonals are bisect each other)

(iv) m ∠DAB + m ∠CDA = 180^0(Adjacent angles in a parallelogram are supplementary)

2. **Consider the following parallelograms. Find the values of the unknowns x, y, z.**

(i)

Sol: Adjacent angles in a parallelogram are supplementary

$x + 100^0 = 180^0$

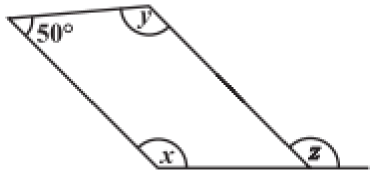
$x = 180^0 - 100^0$

$x = 80^0$

In parallelogram opposite angles are equal

$x = z = 80^0$ and $y = 100^0$

(ii)



Sol: Adjacent angles in a parallelogram are supplementary

$x + 50^0 = 180^0$

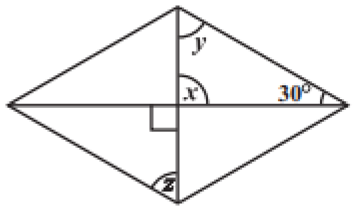
$x = 180^0 - 50^0$

$x = 130^0$

$y = x = 130^0$ (Opposite angles are equal)

$z = x = 130^0$(Corresponding angles)

(iii)



Sol: $x = 90^0$(Vertically opposite angles)

$x + y + 30^0 = 180^0$(Angle sum property of triangle)

$90^0 + y + 30^0 = 180^0$

$y + 120^0 = 180^0$

$y = 180^0 - 120^0 = 60^0$

$z = y = 60^0$(Alternate interior angles)

(iv)

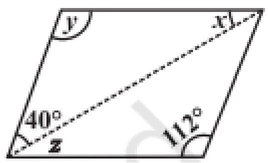
Sol: $x + 80^0 = 180^0$(Adjacent angles are supplementary)

$x = 180^0 - 80^0 = 100^0$

$y = 80^0$ (Opposite angles are equal)

$z = 80^0$(Corresponding angles)

(v)



Sol: $y = 112^0$(Opposite angles are equal)

$x + y + 40^0 = 180^0$(Angle sum property of triangle)

$x + 112^0 + 40^0 = 180^0$

$x + 152^0 = 180^0$

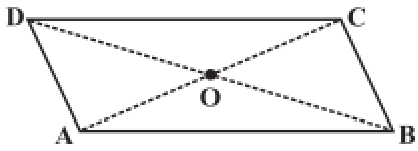
$x = 180^0 - 152^0 = 28^0$

$z = x = 28^0$(Alternate interior angles)

$\therefore x = 28^0, y = 112^0, z = 28^0$

3. Can a quadrilateral ABCD be a parallelogram if (i) ∠D + ∠B = 180°? (ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm? (iii) ∠A = 70° and ∠C = 65°?

(i) ∠D + ∠B = 180°?



Sol: Need not be a parallelogram.

(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

Sol: Here AD≠BC ⇒ one pair of opposite sides are not equal

So, ABCD is not a parallelogram.

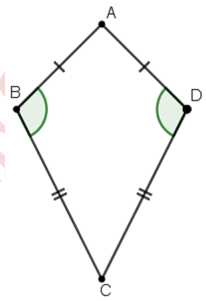
(iii) ∠A = 70° and ∠C = 65°?

Sol: Here ∠A≠∠C⇒ opposite angles are not equal

So, ABCD is not a parallelogram

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Sol:



In ABCD, m∠B=m∠D

ABCD is not a parallelogram.

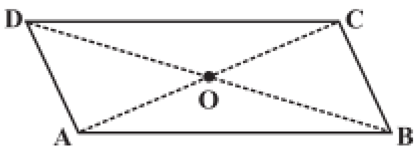
5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

Sol: The ratio of the measures of two adjacent angles of a parallelogram=3:2

Let the angles be $3x$ *and* $2x$

$3x + 2x = 180^0$ (Adjacent angles in a parallelogram are supplementary)

$5x = 180^0$



$$x = \frac{180^0}{5} = 36^0$$

$\angle A = \angle C = 3x = 3 \times 36^0 = 108^0$

$\angle B = \angle D = 2x = 2 \times 36^0 = 72^0$

6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Sol: Let the two adjacent angles be x, x

$x + x = 180^0$

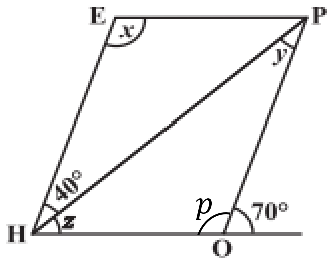
$2x = 180^0$

$$x = \frac{180^0}{2} = 90^0$$

$\angle A = \angle C = 90^0$

$\angle B = \angle D = 180^0 - 90^0 = 90^0$

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Sol: $p + 70^0 = 180^0$(Linear pair)

$\Rightarrow p = 180^0 - 70^0 = 110^0$

$x = p = 110^0$ (Opposite angles are equal)

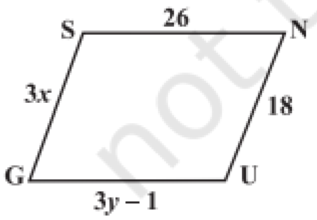
$y = 40^0$ (Alternate interior angles)

$z + 40^0 = 70^0$(Corresponding angles)

$\Rightarrow z = 70^0 - 40^0 = 30^0$

$\therefore x = 110^0, y = 40^0, z = 30^0$

8. **The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)**



(i) Sol: $3x = 18$(Opposite sides are equal)

$\Rightarrow x = \frac{18}{3} = 6$

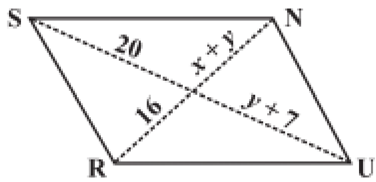
$3y - 1 = 26$ (Opposite sides are equal)

$\Rightarrow 3y = 26 + 1 = 27$

$\Rightarrow y = \frac{27}{3} = 9$

$\therefore x = 6 , y = 9$

(ii)



Sol: $y + 7 = 20$ (Diagonals are bisect each other)

$\Rightarrow y = 20 - 7 = 13$

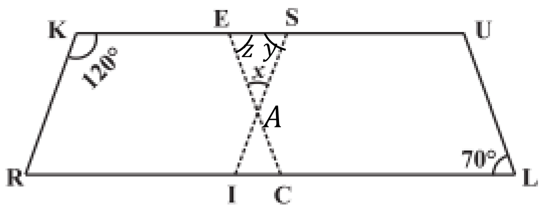
$x + y = 16$ (Diagonals are bisect each other)

$x + 13 = 16$

$x = 16 - 13 = 3.$

$\therefore x = 3 , y = 13$

9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

Sol: Let ∠ISK= y , ∠CEU=z

In parallelogram RISK

$y + 120^0 = 180^0$(Adjacent angles in a parallelogram are supplementary)

$y = 180^0 - 120^0 = 60^0$

$z = 70^0$ (Opposite angles are equal)

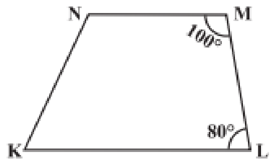
$x + y + z = 180^0$ (Angle sum property of a triangle)

$x + 60^0 + 70^0 = 180^0$

$x + 130^0 = 180^0$

$x = 180^0 - 130^0 = 50^0$

10. Explain how this figure is a trapezium. Which of its two sides are parallel?



Sol: (i) $\angle$L+$\angle$M=100^0+80^0=180^0.

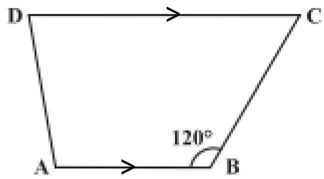
⇒ Interior angles are on the same side of the transversal LN are supplementary.

⇒ $KL \parallel NM$

In KLMN one pair of opposite sides are parallel.

So, KLMN is a trapezium.

11. Find m$\angle$C in if $\overline{AB} \parallel \overline{DC}$



Sol: $\overline{AB} \parallel \overline{DC}$

$\angle$B+$\angle$C=180^0(Co-interior angles are supplementary)

120^0+$\angle$C=180^0

$\angle$C=180^0−120^0=60^0

12. Find the measure of $\angle$P and $\angle$S if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.34. (If you find m$\angle$R, is there more than one method to find m$\angle$P?)

Sol: $\overline{SP} \parallel \overline{RQ}$, $\angle$R=90^0

$\angle$S+$\angle$R=180^0(Co-interior angles are supplementary)

$\angle$S+90^0=180^0

$\angle$S=180^0−90^0=90^0

$\angle$P+$\angle$Q+$\angle$R+$\angle$S=360^0(Angle sum property of quadrilateral)

$\angle$P+130^0+90^0+90^0=360^0

$\angle$P+310^0=360^0

$\angle$P=360^0−310^0=50^0

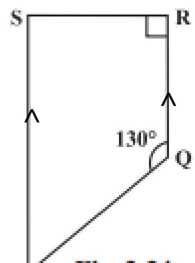
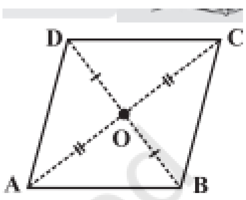


Fig 3.34

Some Special Parallelograms

Rhombus: A rhombus is a quadrilateral whose four sides of equal length.



Properties:

(i) All four sides are equal

AB=BC=CD=DA

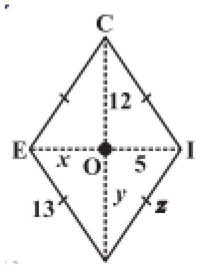
(ii) Opposite angles are equal

$\angle$A=$\angle$C and $\angle$B=$\angle$D

(iii) Diagonals are perpendicular bisector of one another.

$\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^0$ and OA=OC, OB=OD

Example 7: RICE is a rhombus (Fig 3.36). Find x, y, z. Justify your findings



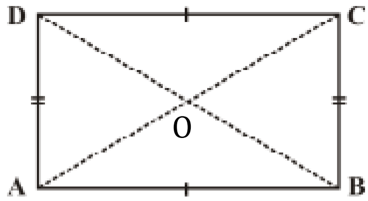
Sol: In a rhombus diagonals are perpendicularly bisect each other

OE=OI and OR=OC

$x = 5$ and $y = 12$

$z = 13$ (In a rhombus all sides are equal)

A rectangle:



A rectangle is a parallelogram with equal angles (right angle).

Properties:

(i) Opposite sides are parallel and equal lengths.

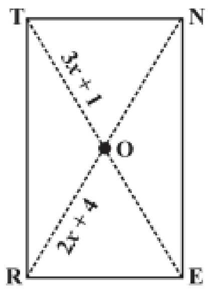
AB=CD and BC=AD

(ii) Diagonals are equal and bisect each other.

AC=BD , OA=OC and OB=OD

(iii) Every angle is right angle (90^0).

Example 8: RENT is a rectangle (Fig 3.41). Its diagonals meet at O. Find x, if OR = 2x + 4 and OT = 3x + 1.



Sol: In a rectangle diagonals are equal and bisect each other.

$\Rightarrow$ All four parts are equal

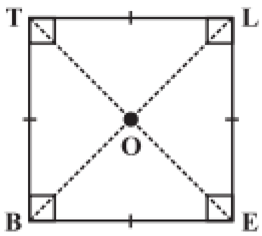
$\Rightarrow$ OT=OE=ON=OR

$\Rightarrow 3x + 1 = 2x + 4$

$\Rightarrow 3x - 2x = 4 - 1$

$\Rightarrow x = 3$

A square:



A square is a rectangle with equal sides.

Properties:

(i) All sides are equal.(BE=EL=LT=TB)

(ii) All angles are right angles.($\angle B = \angle E = \angle L = \angle T = 90^0$)

(iii) Diagonals are equal and perpendicularly bisect each other.

(BL=ET and $\overline{BL} \perp \overline{ET}$, OB=OL and OE=OT)

EXERCISE 3.4

1. State whether True or False.

(a) All rectangles are squares

Sol: False . A rectangle need not have all sides equal. So, it is not a square.

(b) All rhombuses are parallelograms

Sol: True

(c) All squares are rhombuses and also rectangles

Sol: True

(d) All squares are not parallelograms.

Sol: False. All squares are parallelograms.

(e) All kites are rhombuses.

Sol: False .A kite does not have all sides of the same length.

(f) All rhombuses are kites.

Sol: True

(g) All parallelograms are trapeziums.

Sol: True

(h) All squares are trapeziums.

Sol: True.

2. Identify all the quadrilaterals that have.

(a) Four sides of equal length

Sol: Square, Rhombus.

(b) Four right angles

Sol: Square, Rectangle.

3. Explain how a square is.

(i) A quadrilateral

Sol: A square has four sides . So, square is a quadrilateral.

(ii) A parallelogram

Sol: Square is a parallelogram because it's opposite sides are parallel.

(iii) A rhombus

Sol: A square is a rhombus because it's four sides is equal length.

(iv) A rectangle

Sol: A square is a rectangle because it's each angle is right angle.

4. Name the quadrilaterals whose diagonals.

(i) Bisect each other

Sol: Square, Rhombus, Rectangle and parallelogram.

(ii) Are perpendicular bisectors of each other.

Sol: Square, Rhombus.

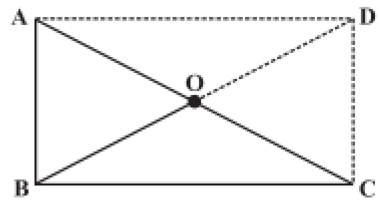
(iii) are equal

Sol: Square, Rectangle.

5. Explain why a rectangle is a convex quadrilateral.

Sol: A Convex polygon is defined as a polygon with no portions of their diagonals in their exteriors. In rectangle both of its diagonals are lie in its interior .So, a rectangle is a convex quadrilateral

6. ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle. Explain why O is equidistant from A, B and C.



Sol: Draw $AD \parallel BC\ and\ CD \parallel BA$

ABCD is a rectangle.

In rectangle diagonals are equal and bisect each other.

OA=OC=OB=OD

So, O is equidistant from A, B and C

THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

Sol:

(i) By measuring opposite sides (Opposite sides of rectangle are equal)

(ii) By measuring diagonals(Diagonals of a rectangle are equal)

(iii) By measuring each angle(Each angle of a rectangle is 90^0)

2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.

Sol: Yes, we define a square is a rhombus with equal angles.

Because rhombus has four equal sides and if all angles are equal then each angle is 90^0.so, it becomes a square.

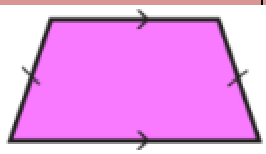
3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

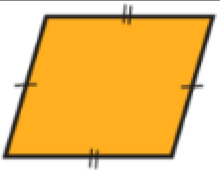
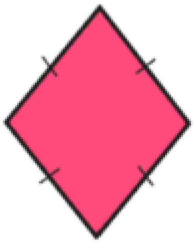
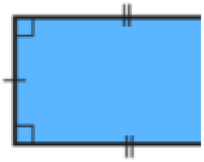
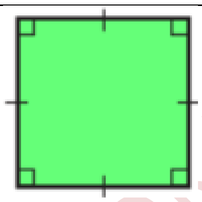
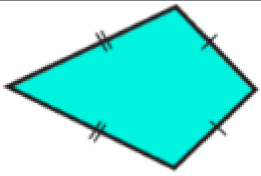
Sol: A trapezium is a quadrilateral with one pair of parallel sides.

A trapezium cannot have all angles equals and all sides equal.

If a trapezium has all angles are equal then it becomes a square or a rectangle

If a trapezium has all sides are equal then it becomes a square or a rhombus.

Quadrilateral	Figure	Properties
Trapezium A quadrilateral with a pair of parallel sides.		1. One pair of parallel lines

Parallelogram: A quadrilateral with each pair of opposite sides parallel		1. Opposite sides are equal. 2. Opposite angles are equal. 3. Diagonals not equal and bisect one another. 4. Adjacent angles are supplementary
Rhombus: A parallelogram with sides of equal length.		1. All sides are equal. 2. Opposite angles are equal 3. Diagonals are not equal and perpendicularly bisect one another. 4. Adjacent angles are supplementary
Rectangle: A parallelogram with a right angle		1. Opposite sides are equal 2. All angles are equal(right angle=90^0). 3. Diagonals are equal and bisect one another.
Square: A rectangle with sides of equal length.		1. All sides are equal. 2. Each of the angles is a right angle. 3. Diagonals are equal and perpendicularly bisect one another.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides		1. The diagonals are perpendicular to one another. 2. One of the diagonals bisects the other.

1) A simple closed curve made up of only line segments is called a polygon.
2) A diagonal of a polygon is a line segment connecting two non-consecutive vertices.
3) The number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}$
4) A convex polygon is a polygon in which no portion of its any diagonal is in its exterior.
5) A quadrilateral is a polygon having only four sides.
6) A regular polygon is a polygon whose all sides are equal and also all angles are equal.
7) The sum of interior angles of a polygon of n sides is (n-2) straight angles=(n-2)×180^0.
8) The sum of interior angles of a quadrilateral is 360°.
9) The sum of exterior angles, taken in an order, of a polygon is 360°.
10) Trapezium is a quadrilateral in which a pair of opposite sides is parallel.
11) Kite is a quadrilateral which has two pairs of equal consecutive sides.
12) A parallelogram is a quadrilateral in which each pair of opposite sides is parallel.

13) In a parallelogram, opposite sides are equal, opposite angles are equal and diagonals bisect each other.
14) A rhombus is a parallelogram in which adjacent sides are equal.
15) In a rhombus diagonals intersect at right angles
16) A rectangle is a parallelogram in which one angle is of 90^0.
17) In a rectangle diagonals are equal.
18) A square is a parallelogram in which adjacent sides are equal and one angle is of 90^0.
19) If diagonals of a quadrilateral bisect at right angles it is a Rhombus (or square).

DATA HANDLING (Notes)

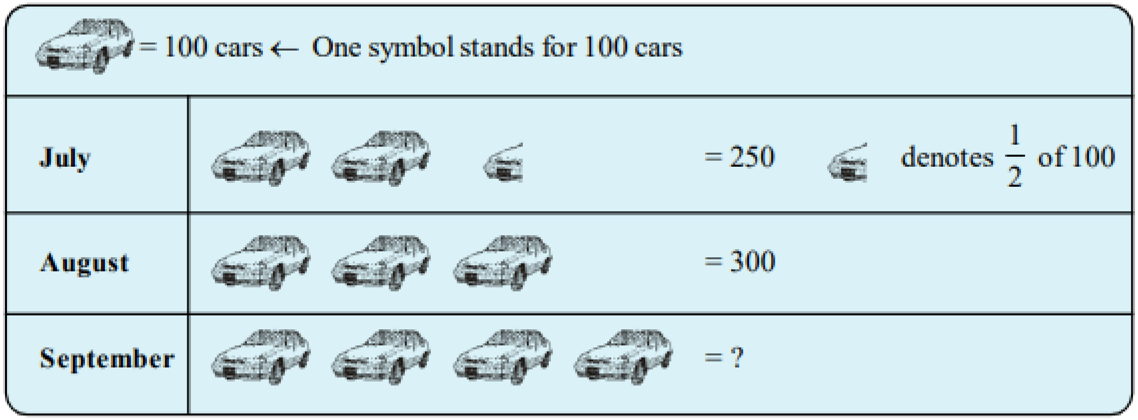
PREPARED BY : BALABHADRA SURESH-9866845885

1. **Primary data:** The data collected directly through personal experiences, interviews, direct observations, physical testing etc.
2. **Secondary data :** Secondary data is the information which has been collected in the past by someone else but used by the investigator for his own purpose
3. Measures of Central Tendency :
(i) Average (or) Arithmetic Mean
(ii) Mode
(iii) Median
4. Arithmetic Mean $= \frac{\text{Sumof observations}}{\text{Number of observations}}$
5. Arithmetic Mean' of given data always lies between the highest and lowest observations of the data.
6. Range = Maximum value – Minimum value
7. The observation which occurs most frequently in the given data is called 'Mode' of the data.
8. Data having only one mode is known as 'Unimodal Data'
9. Data having two modes is known as 'Bimodal Data'.
10. The middle most value of the data, when the observations are arranged in either ascending or descending order is called 'Median'.
11. If the number of observations (n) is odd then median $= \left(\frac{n+1}{2}\right)^{th}$ observation.
12. If number of observations(n) is even then the median

$$= \frac{\left(\frac{n}{2}\right)^{th}\text{observation}+\left(\frac{n}{2}+1\right)^{th}\text{observation}}{2}$$

13. A bar graph: A display of information using bars of uniform width, their heights being proportional to the respective values.
14. Double Bar Graph: A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data
15. 'Pie chart' is the visual representation of the numerical data by sectors of the circle such that angle of each sector (area of sector) is proportional to value of the data that it represents.
16. Frequency gives the number of times that a particular entry occurs.
17. Angle of sector $= \frac{\text{Valueof the item}}{\text{Sumof the values of all items}} \times 360^0$

1.A Pictograph: Pictorial representation of data using symbols



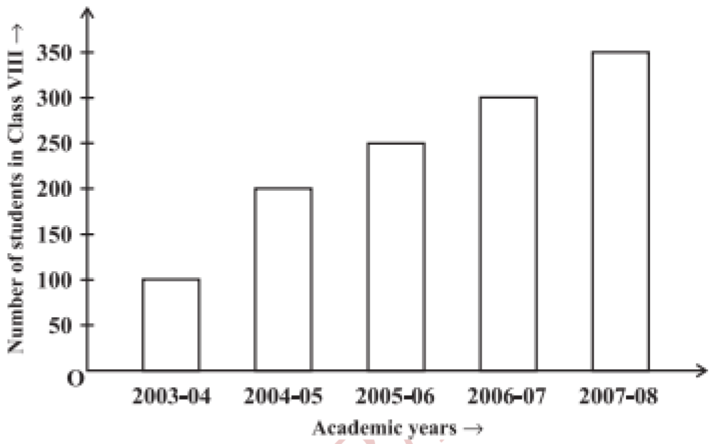
(i) How many cars were produced in the month of July?

Sol: 250

(ii) In which month was maximum number of cars produced?

Sol: September (400 cars)

2. A bar graph: A display of information using bars of uniform width, their heights being proportional to the respective values.



(i) What is the information given by the bar graph?

Sol: The information given by the bar graph is the number of students in class VIII in various academic years .

(ii) In which year is the increase in the number of students maximum?

Sol: 2004-05

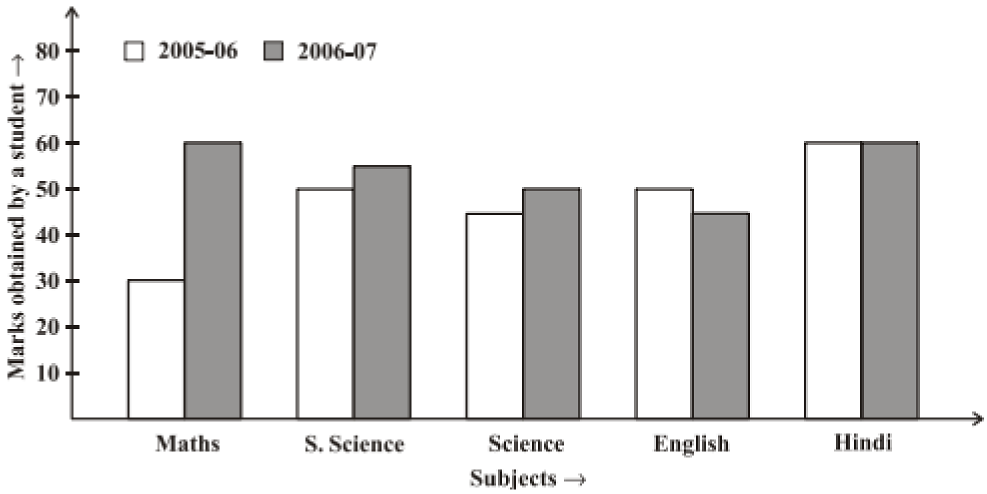
(iii) In which year is the number of students maximum?

Sol: 2007-08

(iv) State whether true or false: 'The number of students during 2005-06 is twice that of 2003-04".

Sol: False. The number of students during 2005-06=250 and during 2003-04=100

3. Double Bar Graph: A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.



(i) What is the information given by the double bar graph?

Sol: The graph gives the information of comparison of marks obtained by a student in the academic years 2005-06 and 2006-07 in varies subjects.

(ii) In which subject has the performance improved the most?

Sol: In Maths.

(iii) In which subject has the performance deteriorated?

Sol: In English.

(iv) In which subject is the performance at par?

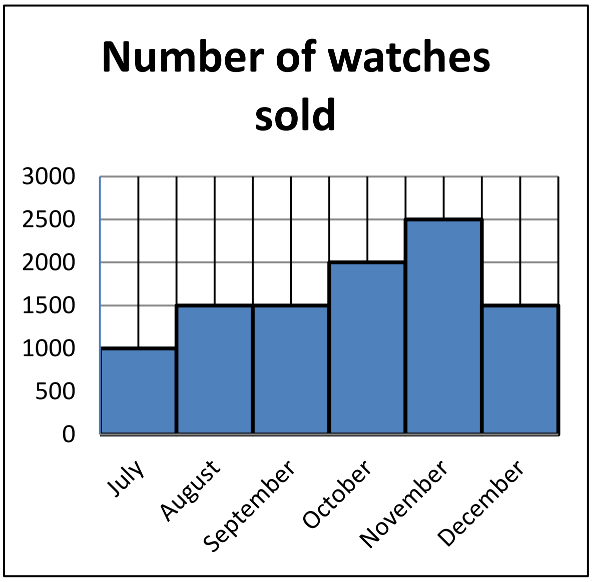
Sol: In Hindi.

TRY THESE

Draw an appropriate graph to represent the given information.

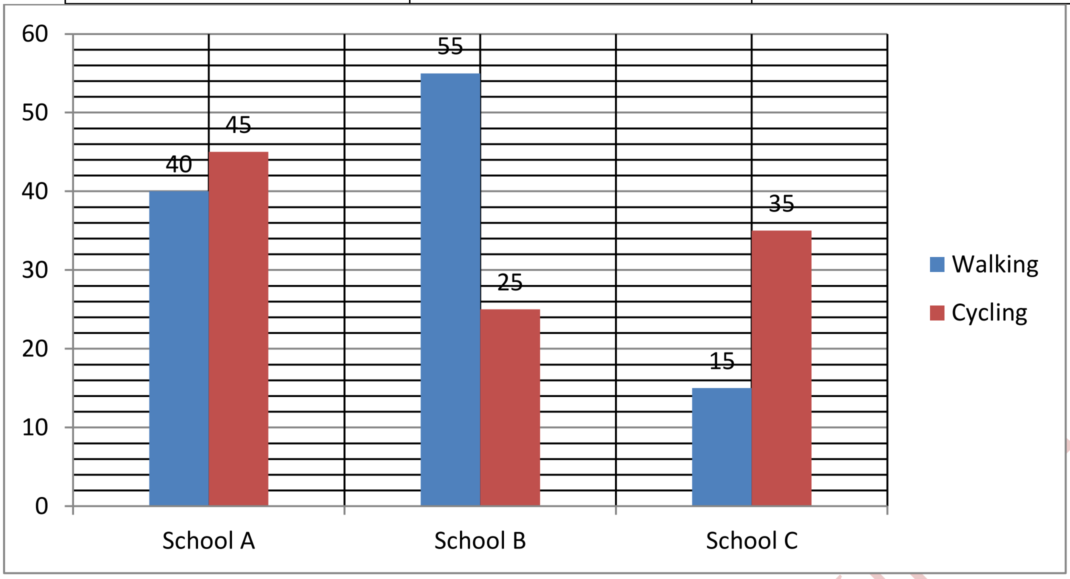
Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

Bar Graph



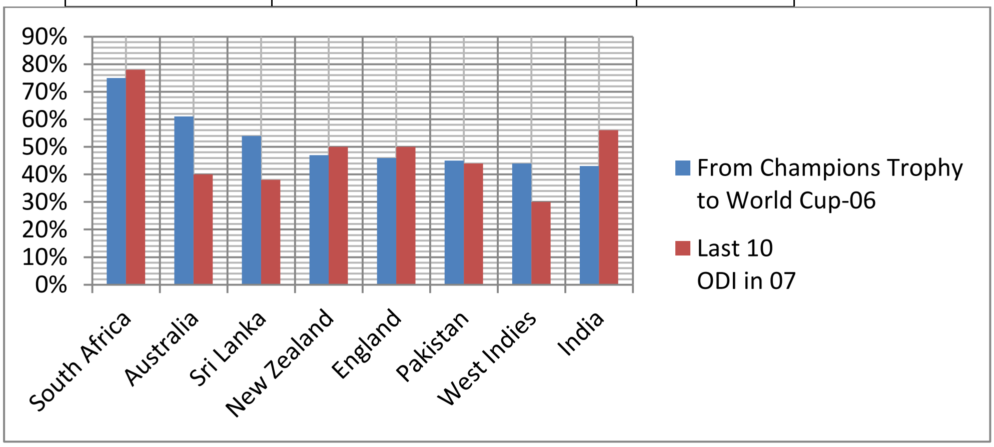
2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35



3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

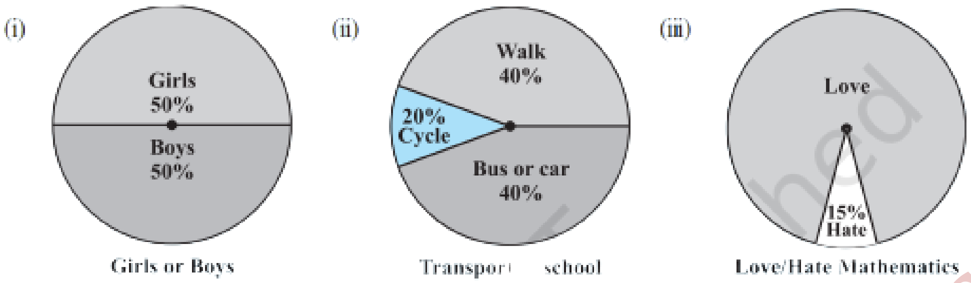


Circle Graph or Pie Chart:

A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents. A circle graph is also called a pie chart.

TRY THESE

1. Each of the following pie charts (Fig 5.5) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.



Sol:

(i)Girls $= 50\% = \frac{50}{100} = \frac{1}{2}$

Boys $= 50\% = \frac{50}{100} = \frac{1}{2}$

(ii) Walk $= 40\% = \frac{40}{100} = \frac{2}{5}$

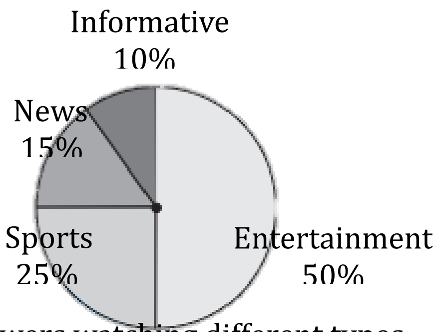
Cycle $= 20\% = \frac{20}{100} = \frac{1}{5}$

Bus or car $= 40\% = \frac{40}{100} = \frac{2}{5}$

(iii) Love mathematics $= 85\% = \frac{85}{100} = \frac{17}{20}$

Hate mathematics $= 15\% = \frac{15}{100} = \frac{3}{20}$

2. **Answer the following questions based on the pie chart given (Fig 5.6).**



Viewers watching different types
Of channels on TV

(i) Which type of programmes are viewed the most?

Sol: Entertain ment

(ii) Which two types of programmes have number of viewers equal to those watching sports channels?

Sol: Informative and News

4.2.1 Drawing pie charts

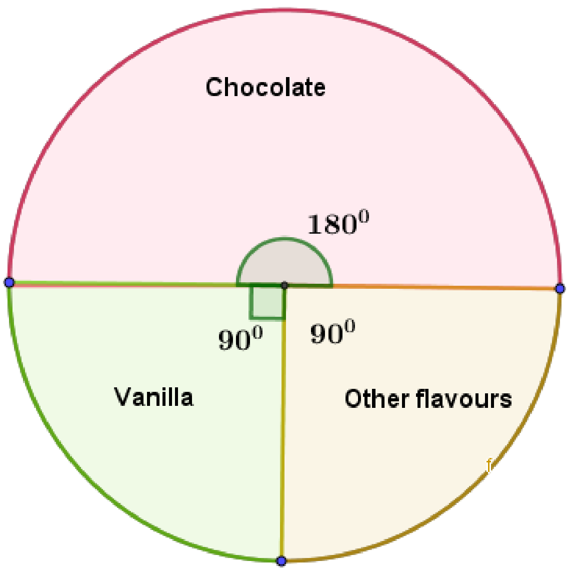
Represent the following data in pie chart.

Flavours	Percentage of students Preferring the flavours

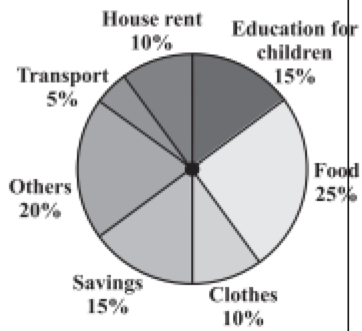
Chocolate	50%
Vanilla	25%
Other flavours	25%

Sol:

Flavours	Percentage of students Preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100} = \frac{1}{2}$	$\frac{1}{2} \times 360^0 = 180^0$
Vanilla	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4} \times 360^0 = 90^0$
Other flavours	25%	$\frac{25}{100} = \frac{1}{4}$	$\frac{1}{4} \times 360^0 = 90^0$



Example 1: Adjoining pie chart (Fig 4.4) gives the expenditure (in percentage) on various items and savings of a family during a month.



(i) On which item, the expenditure was maximum?

Sol: On food

(ii) Expenditure on which item is equal to the total savings of the family?

Sol: On Education (15%)

(iii) If the monthly savings of the family is ₹ 3000, what is the monthly expenditure on clothes?

Sol: 15% represents ₹3000

10% represents $\frac{₹3000}{15} \times 10 = ₹2000$

The monthly expenditure on clothes=₹2000

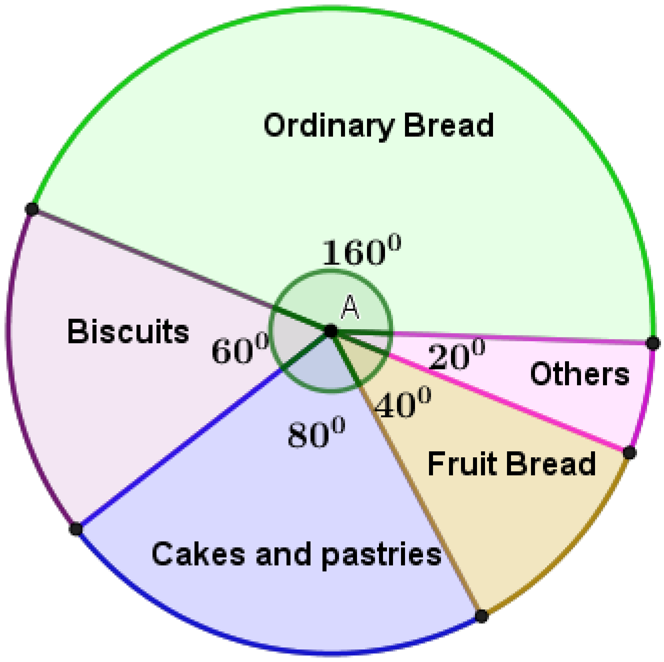
Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below. Draw a pie chart for this data.

ordinary bread	: 320
fruit bread	: 80
cakes and pastries	: 160
biscuits	: 120
others	: 40
Total	**: 720**

Sol:

Item	Sales (in ₹)	Central Angle
Ordinary Bread	320	$\frac{\cancel{320}^{160}}{\cancel{720}_2}\times\cancel{360}=160^0$
Biscuits	120	$\frac{\cancel{120}^{60}}{\cancel{720}_2}\times\cancel{360}=60^0$
Cakes and pastries	160	$\frac{\cancel{160}^{80}}{\cancel{720}_2}\times\cancel{360}=80^0$
Fruit Bread	80	$\frac{\cancel{80}^{40}}{\cancel{720}_2}\times\cancel{360}=40^0$
Others	40	$\frac{\cancel{40}^{20}}{\cancel{720}_2}\times\cancel{360}=20^0$
Total	720	

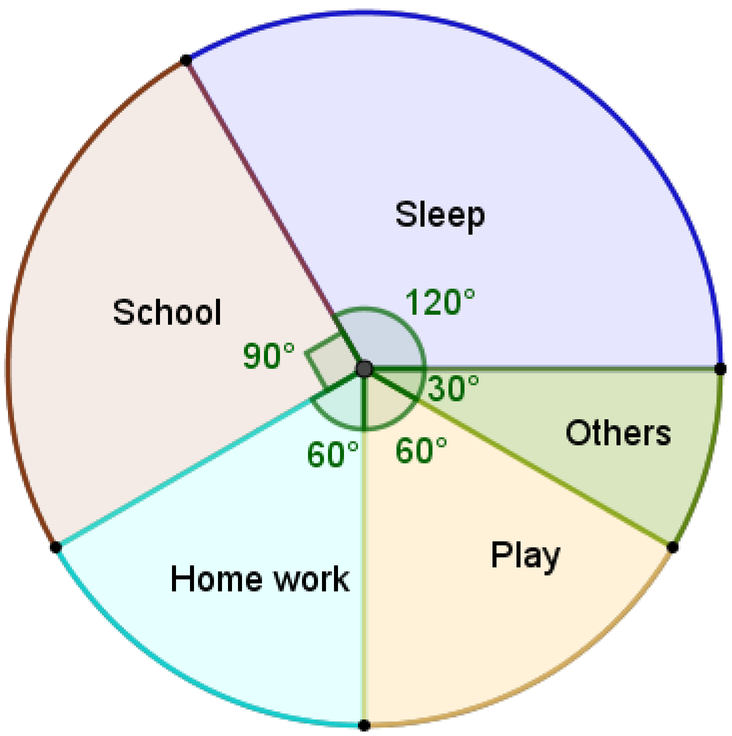
Draw a pie chart of the data given below. The time spent by a child during a day.



Draw a pie chart of the data given below. The time spent by a child during a day

Sleep — 8 hours; School — 6 hours ; Home work — 4 hours ;Play — 4 hours ;Others — 2 hours

Type of spent	Time spent by a child	In Fraction	Central Angle
Sleep	8	$\frac{8}{24}$	$\frac{8}{\cancel{24}} \times \cancel{360^0}^{15} = 8 \times 15^5 = 120^0$
School	6	$\frac{6}{24}$	$\frac{6}{24} \times 360^0 = 6 \times 15^0 = 90^0$
Home work	4	$\frac{4}{24}$	$\frac{4}{24} \times 360^0 = 4 \times 15^0 = 60^0$
Play	4	$\frac{4}{24}$	$\frac{4}{24} \times 360^0 = 4 \times 15^0 = 60^0$
Others	2	$\frac{2}{24}$	$\frac{2}{24} \times 360^0 = 2 \times 15^0 = 30^0$
Total	24		



THINK, DISCUSS AND WRITE

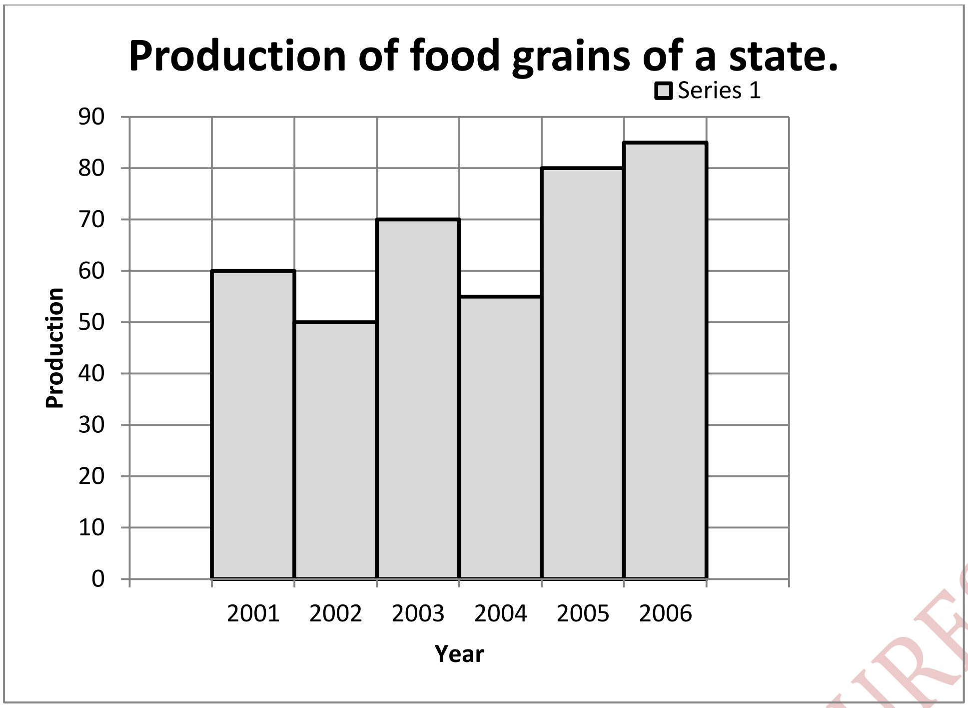
Which form of graph would be appropriate to display the following data.

1. Production of food grains of a state

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons	60	50	70	55	80	85

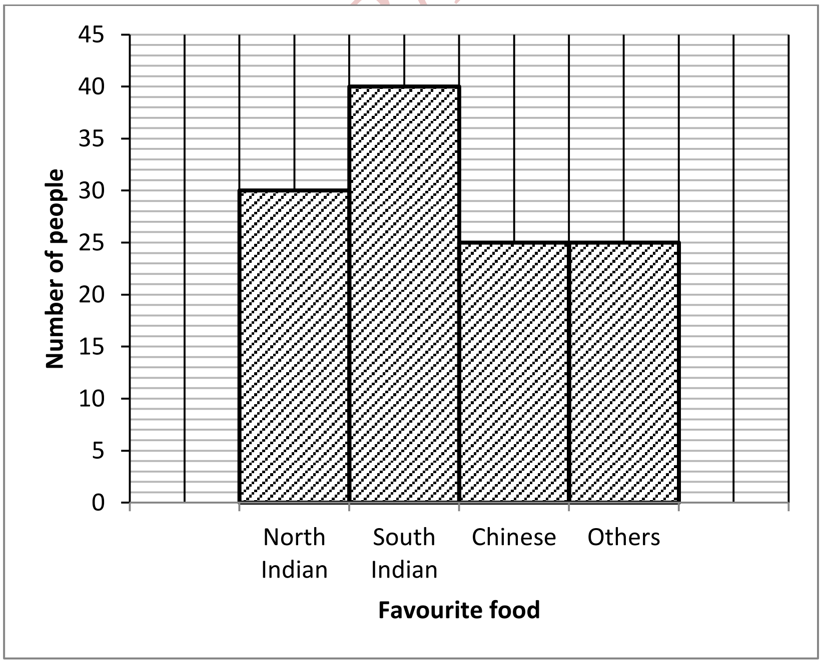
Sol:

Bar graph is appropriate to display the given data



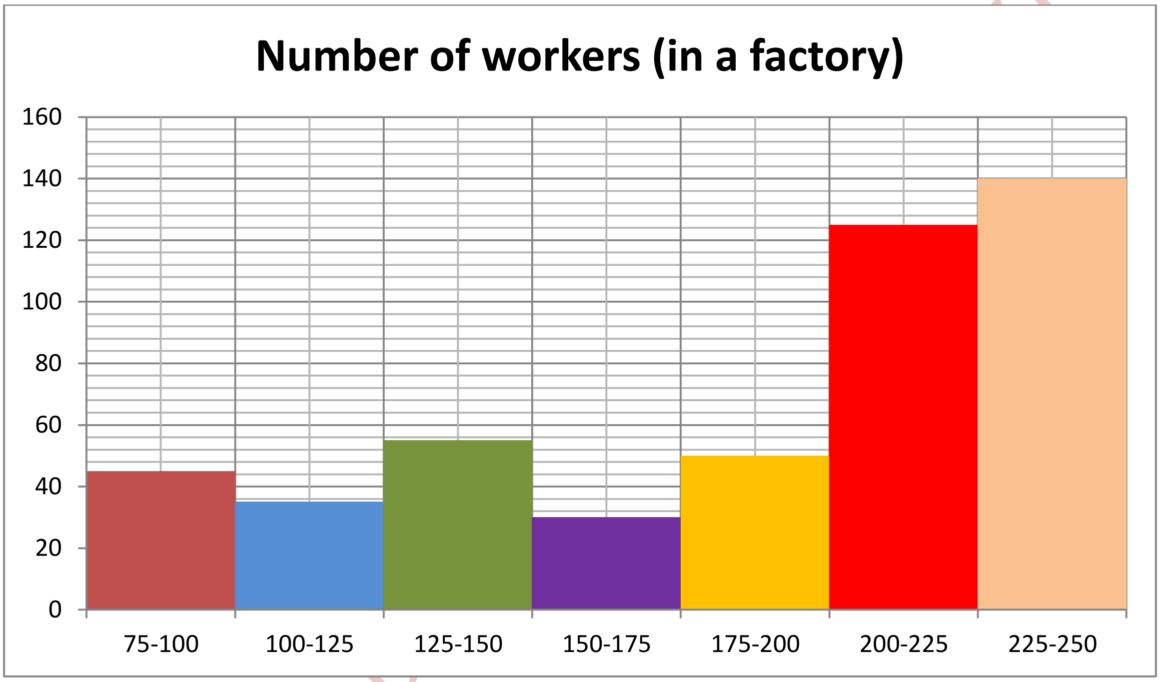
2. Choice of food for a group of people

Favourite food	Number of people
North Indian	30
South Indian	40
Chinese	25
Others	25
Total	120



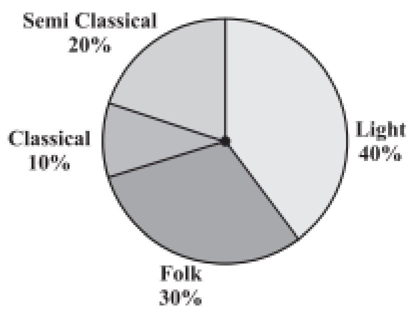
3. The daily income of a group of a factory workers.

Daily Income (in Rupees)	Number of workers (in a factory)
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	480



EXERCISE 4.1

1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey. From this pie chart answer the following:



(i) **If 20 people liked classical music, how many young people were surveyed?**

Sol: Number young people were surveyed$=20 \times \frac{100}{10} = 200$

(ii) Which type of music is liked by the maximum number of people?

Sol: the maximum number of people liked the Light music(40%)

(iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

Sol: Total number of CD's=1000

Number of classical music CD's=10% $of\ 1000 = \frac{10}{100} \times 1000 = 100$

Number of semi classical music CD's=20% $of\ 1000 = \frac{20}{100} \times 1000 = 200$

Number of folk music CD's=30% $of\ 1000 = \frac{30}{100} \times 1000 = 300$

Number of light music CD's=40% $of\ 1000 = \frac{40}{100} \times 1000 = 400$

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.

Season	No. of votes
Summer	90
Rainy	120
Winter	150

(i) Which season got the most votes?

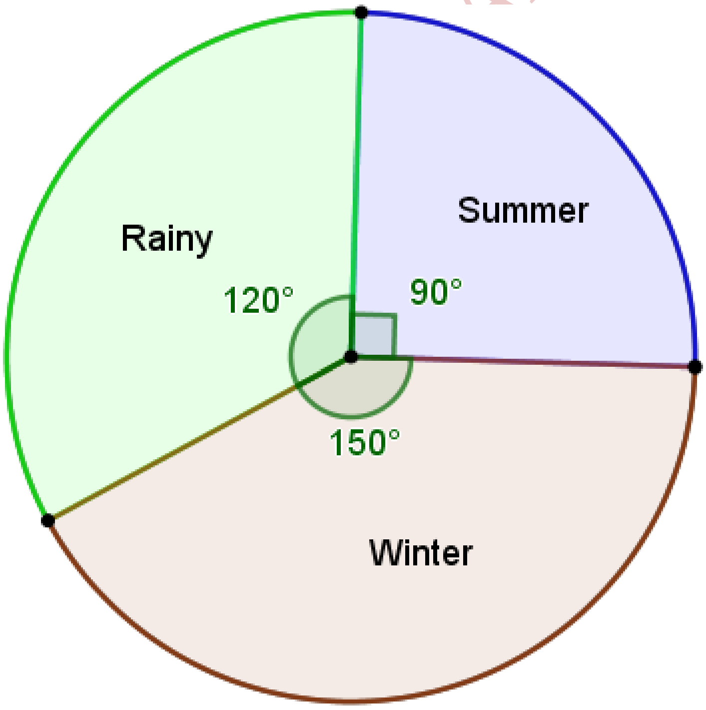
Sol: Rainy (120)

(ii) Find the central angle of each sector.

Sol:

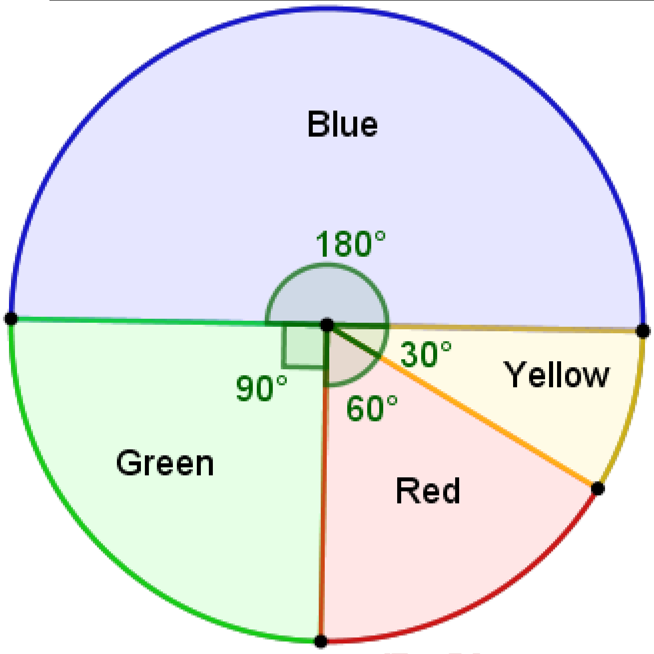
Season	No. of votes	Central angle
Summer	90	$\frac{90}{360} \times 360^0 = 90^0$
Rainy	120	$\frac{120}{360} \times 360^0 = 120^0$
Winter	150	$\frac{150}{360} \times 360^0 = 150^0$
Total	360	

(iii) Draw a pie chart to show this information.

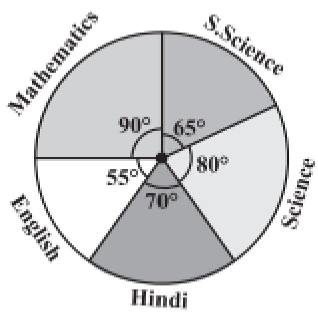


3. **Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.**

Colours	Number of people	Fraction	Central angle
Blue	18	$\frac{18}{36}$	$\frac{18}{\cancel{36}} \times \cancel{360^0}^{10} = 18 \times 10^0 = 180^0$
Green	9	$\frac{9}{36}$	$\frac{9}{36} \times 360^0 = 9 \times 10^0 = 90^0$
Red	6	$\frac{6}{36}$	$\frac{6}{36} \times 360^0 = 6 \times 10^0 = 60^0$
Yellow	3	$\frac{3}{36}$	$\frac{3}{36} \times 360^0 = 3 \times 10^0 = 30^0$
Total	36		



4. **The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.**



(i) **In which subject did the student score 105 marks?**

(Hint: for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)

Sol: For 105 marks the central angle$=\frac{105}{540} \times 360^0 = 70^0$

Marks obtained in Hindi=105

(ii) How many more marks were obtained by the student in Mathematics than in Hindi?

Sol: Marks obtained in Mathematics $= \frac{90}{360} \times 540 = 135$

Marks obtained in Mathematics more than Hindi=135-105=30

(iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.

Sol: Central angle of Social Science and Mathematics=$65^0+90^0=155^0$

Central angle of Science and Hindi =$80^0+70^0=150^0$

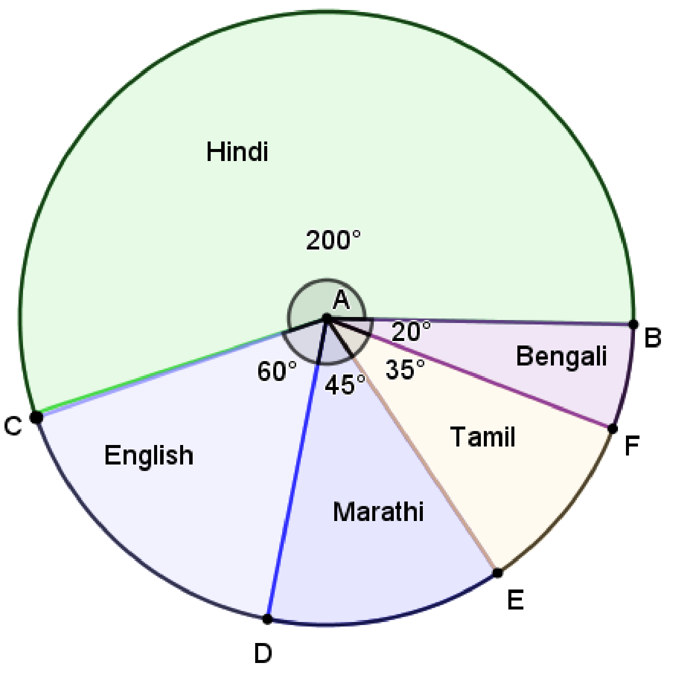
So, Sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.

5. **The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.**

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of student	40	12	9	7	4	72

Sol:

Language	Number of student	Fraction	Central angle
Hindi	40	$\frac{40}{72}$	$\frac{40}{72} \times 360^0 = 200^0$
English	12	$\frac{12}{72}$	$\frac{12}{72} \times 360^0 = 60^0$
Marathi	9	$\frac{9}{72}$	$\frac{9}{72} \times 360^0 = 45^0$
Tamil	7	$\frac{7}{72}$	$\frac{7}{72} \times 360^0 = 35^0$
Bengali	4	$\frac{40}{72}$	$\frac{4}{72} \times 360^0 = 20^0$
Total	72		



Chance and Probability

1. There are certain experiments whose outcomes have an equal chance of occurring.
2. A random experiment is one whose outcome cannot be predicted exactly in advance.
3. When a coin is tossed Head or Tail are the two outcomes of this experiment.
4. One or more outcomes of an experiment make an event.
5. Probability of an event = Number of outcomes that make an event/Total number of outcomes of the experiment , when the outcomes are equally likely

TRY THESE

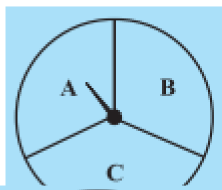
1. If you try to start a scooter, what are the possible outcomes?

Sol: The scooter starts or does not starts.

2. When a die is thrown, what are the six possible outcomes?

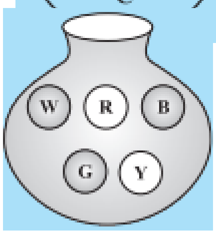
Sol:1,2,3,4,5 and 6.

3. When you spin the wheel shown, what are the possible outcomes? List them.



Sol: When you spin the wheel the possible outcomes are A,B and C

4. **You have a bag with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get.**



Sol: The required outcomes are R,B,G,W and Y

THINK, DISCUSS AND WRITE

In throwing a die:

1. **Does the first player have a greater chance of getting a six?**

Sol: No, the first player does not have a greater chance of getting a six.

2. Would the player who played after him have a lesser chance of getting a six?

Sol: No, the player who played after him does not have a lesser chance of getting a six

3. Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

Sol: No,

Equally likely outcomes:

Equally likely implies that the all of the outcomes of a random experiment are the same chance of occurring.

1. When a coin is tossed Head and Tail are equally likely out comes.
2. When a die is tossed 1,2,3,4,5 and 6 are equally likely outcomes.

Event

Each outcome of an experiment or a collection of outcomes make an event.

Ex: In the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Sol: Red balls=4, Yellow balls=2

Total outcomes=4+2=6

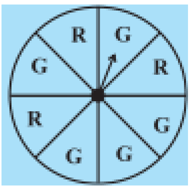
The probability of getting a red ball $= \dfrac{\text{Favourable}}{\text{Total possible}} = \dfrac{4}{6} = \dfrac{2}{3}$

The probability of getting a yellow ball $= \dfrac{\text{Favourable}}{\text{Total possible}} = \dfrac{2}{6} = \dfrac{1}{3}$

The probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel



(i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel

Sol: Number of outcomes getting a green sector=5

Number of outcomes not getting a green sector=3

(ii) Find the probability of getting a green sector.

Sol: Probability of getting a green sector $= \dfrac{\text{Favourable outcomes}}{\text{Total number of outcomes of the experiment}} = \dfrac{5}{8}$

(iii) Find the probability of not getting a green sector

Sol: The probability of not getting a green sector $= \frac{3}{8}$

Chance and probability related to real life

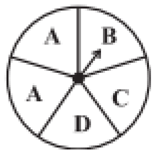
1. To find characteristics of a large group by using a small part of the group.

2. Metrological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 4.2

1. **List the outcomes you can see in these experiments.**

(a) Spinning a wheel



Sol: On spinning the wheel the outcomes are A,B,C and D

(b) Find the Probability of the pointer stopping on D.

Sol: The Probability of the pointer stopping on D$=\frac{1}{5}$

(c) Tossing two coins together

Sol: When two coins are tossed together, the outcomes are

HH,HT,TH and TT (Where H- Head and T- Tail)

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number

Sol: Outcomes for prime number are 2,3 and 5

(b) not a prime number.

Sol: Outcomes for not a prime number are 1,4 and 6

(ii) (a) a number greater than 5

Sol: Outcomes for a number greater than 5 is 6

(b) a number not greater than 5.

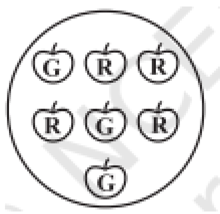
Sol: outcomes for a number not greater than 5 are 1,2,3,4 and 5.

3. Find

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

Sol: Number ace cards in deck=4

Probability of getting an ace $= \frac{4}{52} = \frac{1}{13}$



(c) Probability of getting a red apple

Sol: Total number of apples=7

Number of red apples=4

Probability of getting a red apple $= \frac{\text{Number of red apples}}{\text{Total number of apples}} = \frac{4}{7}$

4. **Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed**

well. One slip is chosen from the box without looking into it. What is the probability of

(i) Getting a number 6?

Sample space={1,2,3,4,5,6,7,8,9,10} , Total all possible outcomes=10

$$\text{Probability of getting a number 6} = \frac{1}{10}$$

(ii) Getting a number less than 6?

$$\text{probability of getting a number less than 6} = \frac{5}{10} = \frac{1}{2}$$

(iii) Getting a number greater than 6?

$$\text{Probability of getting a number greater than 6} = \frac{4}{10} = \frac{2}{5}$$

(iv) Getting a 1-digit number?

$$\text{Probability of getting a 1} - \text{digit number} = \frac{9}{10}$$

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

Sol: Green sectors=3, blue sectors=1, red sectors=1

Total number of sectors=3+1+1=5

$$\text{Probability of getting a green sector} = \frac{\text{Number of green sectors}}{\text{Total number of sectors}} = \frac{3}{5}$$

$$\text{Probability of getting a non blue sector} = \frac{\text{Number of non blue sectors}}{\text{Total number of sectors}} = \frac{4}{5}$$

6. When a die is thrown find the probabilities of the events of getting

Sol: When a die is thrown sample space={1,2,3,4,5,6}

i) (a) a prime number

Sol: Outcomes for prime number are 2,3 and 5

$$\text{Probability of getting a prime number} = \frac{\text{Number of primenumbers}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(b) Not a prime number.

Sol: Outcomes for not a prime number are 1,4 and 6

$$\text{Probability of getting not a prime number} = \frac{\text{Number of non primenumbers}}{\text{Total number of outcomes}} = \frac{3}{6} = \frac{1}{2}$$

(ii) (a) a number greater than 5

Sol: Outcomes for a number greater than 5 is 6

$$\text{Probability of getting a number greater than 5} = \frac{1}{6}$$

(b) a number not greater than 5.

Sol: outcomes for a number not greater than 5 are 1,2,3,4 and 5.

Probability of getting a number not greater than 5 $= \frac{5}{6}$

1. If a natural number m can be expressed as n^2, where n is also a natural number, then m is a square number.
2. $a^2 = a \times a$

Number(n)	Square(n^2)	Number(n)	Square(n^2)	Number(n)	Square(n^2)
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	279	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

3. The numbers 1, 4, 9, 16,.....are square numbers. These numbers are also called **Perfect squares**
4. All square numbers end with 0, 1, 4, 5, 6 or 9 at units place.
5. Square numbers can only have even number of zeros at the end.
6. The square number does not end with 2, 3, 7 or 8 at unit's place.

TRY THESE

1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60

Sol: (i) 36 (ii) There is no perfect square between 50 and 60

TRY THESE

1. Can we say whether the following numbers are perfect squares? How do we know?

   The square number does not end with 2,3,7 or 8 at unit's place

(i) 1057

Sol: Unit digit is 7 . So, it is not a perfect square

(ii) 23453

Sol: Unit digit is 3 . So, it is not a perfect square

(iii) 7928

Sol: Unit digit is 8 . So, it is not a perfect square

(iv) 222222

Sol: Unit digit is 2 . So, it is not a perfect square

(v) 1069

Sol: Unit digit is 9 . We don't say 1069 is a perfect square are not

(vi) 2061

Sol: Unit digit is 1 . We don't say 2061 is a perfect square are not

TRY THESE

Which of 123^2 , 77^2 , 82^2 , 161^2 , 109^2 would end with digit 1?

Sol: 123^2 is end with digit 9

77^2 is end with digit 9

82^2 is end with digit 4

161^2 is end with digit 1

109^2 is end with digit 1

So, 161^2 , 109^2 would end with digit 1.

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2 (iv) 36^2 (v) 34^2

Sol: (ii) 24^2 (iii) 26^2 (iv) 36^2 (v) 34^2

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) The unit digit in the square of 1234 is 6

(ii) The unit digit in the square of 26387 is 9

(iii) The unit digit in the square of 52698 is 4

(iv) The unit digit in the square of 99880 is 0

(v) The unit digit in the square of 21222 is 4

(vi) The unit digit in the square of 9106 is 6

TRY THESE

1. The square of which of the following numbers would be an odd number/an even number? Why?

   The square of an even number is an even number and the square of an odd number is an odd

   (i) 727^2 is an odd number

   (ii) 158^2 is an even number

   (iii) 269^2 is an odd number

(iv) 1980^2 is an even number

Numbers between square numbers

i) Between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

ii) There are $2n$ non perfect square numbers between the squares of the numbers n and (n + 1)

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2 ?

Sol: The number of natural numbers between 9^2 and 10^2 is $2 \times 9 = 18$

The number of natural numbers between 11^2 and 12^2 is $2 \times 11 = 22$

2. How many non-square numbers lie between the following pairs of numbers

(i) Number of non square numbers lie between 100^2 and 101^2 is $2 \times 100 = 200$

(ii) Number of non square numbers lie between 90^2 and 91^2 is $2 \times 90 = 180$

(iii) Number of non square numbers lie between 1000^2 and 1001^2 is $2 \times 1000 = 2000$

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 is a perfect square $(121 = 11^2)$

(ii) 55 is not a perfect squarer

(iii) 81 is a perfect square$(81 = 9^2)$

(iv) 49 is a perfect square$(49 = 7^2)$

(v) 69 is not a perfect squarer

A sum of consecutive natural numbers

1) we can express the square of any odd number as the sum of two consecutive positive integers.

2) n is an odd number . $n^2 = \frac{n^2-1}{2} + \frac{n^2+1}{2}$

Ex: $9^2 = 81 = 40 + 41$

$11^2 = 121 = 60 + 61$

$15^2 = 225 = 112 + 113$

TRY THESE

1. Express the following as the sum of two consecutive integers.

(i) $21^2 = 441 = 220 + 221$

(ii) $13^2 = 169 = 84 + 85$

(ii) $11^2 = 121 = 60 + 61$

(ii) $19^2 = 361 = 180 + 181$

EXERCISE 6.1

1. What will be the unit digit of the squares of the following numbers?

Number	Unit digit of the square	Number	Unit digit of the square
(i) 81	1	(vi)26387	9
(ii) 272	4	(vii)52698	4
(iii) 799	1	(viii)99880	0
(iv) 3853	9	(ix) 12796	6
(v)1234	6	(x) 55555	5

2. The following numbers are obviously not perfect squares. Give reason.

Perfect squares are does not end with 2,3,7 or 8 at unit's place. Perfect square numbers can only have even number of zeros at the end.

(i) 1057 is end with 7 .

So, 1057 is not a perfect square .

(ii) 23453 is end with 3 .

So, 23453 is not a perfect square

(iii) 7928 is end with 8 .

So, 7928 is not a perfect square

(iv) 222222 is end with 2 .

So, 222222 is not a perfect square

(v) 64000 has odd zeroes at the end .

So, 64000 is not a perfect square

(vi) 89722 is end with 2 .

So, 89722 is not a perfect square

(vii) 222000 has odd zeroes at the end.

So, 222000 is not a perfect square

(viii) 505050 has odd zeroes at the end.

So, 505050 is not a perfect square

3. The squares of which of the following would be odd numbers?

The square of an even number is an even number and the square of an odd number is an odd

(i) The square of 431 is an odd number.

(ii) The square of 2826 is an even number.

(iii) The square of 7779 is an odd number.

(iv) The square of 82004 is an odd number.

4. Observe the following pattern and find the missing digits

11^2	121
101^2	10201
1001^2	1002001
100001^2	10000200001
10000001^2	100000020000001

5. Observe the following pattern and supply the missing numbers.

11^2	121
101^2	10201
10101^2	102030201
1010101^2	1020304030201
101010101^2	10203040504030201

6. Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + 20^2 = 21^2$

$5^2 + 6^2 + 30^2 = 31^2$

$6^2 + 7^2 + 42^2 = 43^2$

$$\boxed{n^2 + (n+1)^2 + [n(n+1)]^2 = [n(n+1)+1]^2}$$

7. Without adding, find the sum.

$\boxed{\text{The sum of first n odd numbers} = n^2}$

(i) $1+3+5+7+9 = 5^2 = 25$

(ii) $1+3+5+7+9+11+13+15+17+19 = 10^2 = 100$

(i) $1+3+5+7+9+11+13+15+17+19+21+23 = 12^2 = 144$

8. (i) Express 49 as the sum of 7 odd numbers.

Sol: 49=1+3+5+7+9+11+13

(ii) Express 121 as the sum of 11 odd numbers.

Sol: 121=1+3+5+7+9+11+13+15+17+19+21

9. How many numbers lie between squares of the following numbers?

There are '2n' numbers lie between n^2 and $(n+1)^2$.

(i) 12 and 13

Sol: $2 \times 12 = 24$ numbers lie between 12^2 and 13^2

(ii) 25 and 26

Sol: $2 \times 25 = 50$ numbers lie between 25^2 and 26^2

(iii) 99 and 100

Sol: $2 \times 99 = 198$ numbers lie between 99^2 and 100^2

FINDING THE SQUARE OF A NUMBER CONTAINING 5 IN UNIT'S PLACE.

$(a5)^2 = a(a+1)hundreds + 25$

$15^2 = (1 \times 2)hundreds + 25 = 200 + 25 = 225$

$25^2 = (2 \times 3)hundreds + 25 = 600 + 25 = 625$

$35^2 = (3 \times 4)hundreds + 25 = 1200 + 25 = 1225$

$45^2 = (4 \times 5)hundreds + 25 = 2000 + 25 = 2025$

$55^2 = (5 \times 6)hundreds + 25 = 3000 + 25 = 3025$

$65^2 = (6 \times 7)hundreds + 25 = 4200 + 25 = 4225$

$75^2 = (7 \times 8)hundreds + 25 = 5600 + 25 = 5625$

$85^2 = (8 \times 9)hundreds + 25 = 7200 + 25 = 7225$

$95^2 = (9 \times 10)hundreds + 25 = 9000 + 25 = 9025$

$105^2 = (10 \times 11)hundreds + 25 = 11000 + 25 = 11025$

$205^2 = (20 \times 21)hundreds + 25 = 42000 + 25 = 42025$

Pythagorean triplets

a, b, c *are positive integers . If* $\boldsymbol{a^2} + \boldsymbol{b^2} = \boldsymbol{c^2}$ *then* (a, b, c) *are said tobe* pythagorean triplet

Example: (i) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$(3,4,5)$ *is a* pythagorean triplet

(ii) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

$(5,12,13)$ *is a* pythagorean triplet

Fore any natural number m>1 , we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$.

So, $2m, m^2 - 1$ *and* $m^2 + 1$ *form a Pythagorean triplet*.

EXERCISE 6.2

1. Find the square of the following numbers.

(i) 32

Sol: $32^2 = (30 + 2)^2 = (30 + 2)(30 + 2)$

$= 30(30 + 2) + 2(30 + 2)$

$= 30^2 + 30 \times 2 + 2 \times 30 + 2^2$

$= 900 + 60 + 60 + 4$

$= 1024$

(ii) 35

Sol : $35^2 = (3 \times 4)hundreds + 25 = 1200 + 25 = 1225$

(iii) 86

Sol: $86^2 = (80+6)^2 = (80+6)(80+6)$

$= 80(80+6) + 6(80+6)$

$= 80^2 + 80 \times 6 + 6 \times 80 + 6^2$

$= 6400 + 480 + 480 + 36$

$= 7396$

Shortcut: $86^2 = (80+6)^2$

$= 80^2 + 2 \times 80 \times 6 + 6^2$

$= 6400 + 960 + 36$

$= 7396$

(iv) 93

Sol: $93^2 = (90+3)^2 = (90+3)(90+3)$

$= 90(90+3) + 6(90+3)$

$= 90^2 + 90 \times 3 + 3 \times 90 + 3^2$

$= 8100 + 270 + 270 + 9$

$= 8649$

(v) 71

Sol: $71^2 = (70+1)^2 = (70+1)(70+1)$

$= 70(70+1) + 1(70+1)$

$= 70^2 + 70 \times 1 + 1 \times 70 + 1^2$

$= 4900 + 70 + 70 + 1$

$= 5041$

(vi) 46

Sol: $46^2 = (40+6)^2 = (40+6)(40+6)$

$= 40(40+6) + 6(40+6)$

$= 40^2 + 40 \times 6 + 6 \times 40 + 6^2$

$= 1600 + 240 + 240 + 36$

$= 2116$

Shortcut: $46^2 = (40+6)^2$

$= 40^2 + 2 \times 40 \times 6 + 6^2$

$= 1600 + 480 + 36$

$= 2116$

2. Write a Pythagorean triplet whose one member is.

(i) 6

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 6 \Rightarrow m = 3$

We get $m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$

$m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

The triplet is 6, 8, 10

Let $m^2 - 1 = 6 \Rightarrow m^2 = 7 \Rightarrow$ *The value of m will not be an integer*

Let $m^2 + 1 = 6 \Rightarrow m^2 = 5 \Rightarrow$ *The value of m will not be an integer*

(ii) 14

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 14 \Rightarrow m = 7$

We get $m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$

$m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$

The triplet is 14, 48, 50

Let $m^2 - 1 = 14 \Rightarrow m^2 = 15 \Rightarrow \textit{The value of m will not be an integer}$

Let $m^2 + 1 = 14 \Rightarrow m^2 = 13 \Rightarrow \textit{The value of m will not be an integer}$

(iii) 16

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 16 \Rightarrow m = 8$

We get $m^2 - 1 = 8^2 - 1 = 64 - 1 = 63$

$m^2 + 1 = 8^2 + 1 = 64 + 1 = 65$

The triplet is 16, 63, 65

Let $m^2 - 1 = 16 \Rightarrow m^2 = 17 \Rightarrow \textit{The value of m will not be an integer}$

Let $m^2 + 1 = 16 \Rightarrow m^2 = 15 \Rightarrow \textit{The value of m will not be an integer}$

(vi) 18

Sol: We know that $2m, m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet

Let $2m = 18 \Rightarrow m = 9$

We get $m^2 - 1 = 9^2 - 1 = 81 - 1 = 80$

$m^2 + 1 = 9^2 + 1 = 81 + 1 = 82$

The triplet is 18, 80, 82

Let $m^2 - 1 = 14 \Rightarrow m^2 = 15 \Rightarrow \textit{The value of m will not be an integer}$

Let $m^2 + 1 = 14 \Rightarrow m^2 = 13 \Rightarrow \textit{The value of m will not be an integer}$

Square Roots

If a square number is expressed, as the product of two equal factors, then one the factors is called the square root of that square number.

Since $9^2 = 81$ *and* $(-9)^2 = 81$. *We say that the square root opf* 81 *are* 9 *and* -9

In this chapter, we shall take up only positive square root of a natural number.

Symbol used for square root is $\sqrt{\ }$.

Statement	Inference	Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$	$11^2 = 121$	$\sqrt{121} = 11$
$2^2 = 4$	$\sqrt{4} = 2$	$12^2 = 144$	$\sqrt{144} = 12$
$3^2 = 9$	$\sqrt{9} = 3$	$13^2 = 169$	$\sqrt{169} = 13$
$4^2 = 16$	$\sqrt{16} = 4$	$14^2 = 196$	$\sqrt{196} = 14$
$5^2 = 25$	$\sqrt{25} = 5$	$15^2 = 225$	$\sqrt{225} = 15$
$6^2 = 36$	$\sqrt{36} = 6$	$16^2 = 256$	$\sqrt{256} = 16$

$7^2 = 49$	$\sqrt{49} = 7$	$17^2 = 289$	$\sqrt{289} = 7$
$8^2 = 64$	$\sqrt{64} = 8$	$18^2 = 324$	$\sqrt{324} = 18$
$9^2 = 81$	$\sqrt{81} = 9$	$19^2 = 361$	$\sqrt{361} = 19$
$10^2 = 100$	$\sqrt{100} = 10$	$20^2 = 400$	$\sqrt{400} = 20$

Finding square root through repeated subtraction

TRY THESE (Page-100)

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121

Sol: Step 1: $121 - 1 = 120$

Step 2: $120 - 3 = 117$

Step 3: $117 - 5 = 112$

Step 4: $112 - 7 = 105$

Step 5: $105 - 9 = 96$

Step 6: $96 - 11 = 85$

Step 7: $85 - 13 = 72$

Step 8: $72 - 15 = 57$

Step 9: $57 - 17 = 40$

Step 10: $40 - 19 = 21$

Step 11: $21 - 21 = 0$

From 121 we have subtracted successive odd numbers starting from 1 and obtained 0 at 11^{th} step. Therefore $\sqrt{121} = 11$

(ii) 55

Sol: Step 1: $55 - 1 = 54$

Step 2: $54 - 3 = 51$

Step 3: $51 - 5 = 46$

Step 4: $46 - 7 = 39$

Step 5: $39 - 9 = 30$

Step 6: $30 - 11 = 19$

Step 7: $19 - 13 = 6$

The result is not zero 55 is not a perfect square.

(iii) 36

Sol: Step 1: $36 - 1 = 35$

Step 2: $35 - 3 = 32$

Step 3: $32 - 5 = 27$

Step 4: $27 - 7 = 20$

Step 5: $20 - 9 = 11$

Step 6: $11 - 11 = 0$

From 36 we have subtracted successive odd numbers starting from 1 and obtained 0 at 6th step. Therefore $\sqrt{36} = 6$

(iv) 49

Sol: Step 1: $49 - 1 = 48$

Step 2: $48 - 3 = 45$

Step 3: $45 - 5 = 40$

Step 4: $40 - 7 = 33$

Step 5: $33 - 9 = 24$

Step 6: $24 - 11 = 13$

Step 7: $13 - 13 = 0$

From 49 we have subtracted successive odd numbers starting from 1 and obtained 0 at 6th step. Therefore $\sqrt{49} = 7$

(v) 90

Sol: Step 1: $90 - 1 = 89$

Step 2: $89 - 3 = 86$

Step 3: $86 - 5 = 81$

Step 4: $81 - 7 = 74$

Step 5: $74 - 9 = 65$

Step 6: $65 - 11 = 54$

Step 7: $54 - 13 = 41$

Step 8: $41 - 15 = 26$

Step 9: $26 - 17 = 9$

The result is not zero 90 is not a perfect square.

Finding square root through prime factorisation

$$Prime\ numbers = \{2,3,5,7,11,13,17,19,23,29, \ldots.\}$$

Ex: Find the square root of 324.

2	324
2	162
3	81
3	27
3	9
	3

Sol: $324 = (2 \times 2) \times (3 \times 3) \times (3 \times 3)$

$\sqrt{324} = 2 \times 3 \times 3$

$\sqrt{324} = 18$

Example 4: Find the square root of 6400.

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Sol: $6400 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (5 \times 5)$

$\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5$

$\sqrt{6400} = 80$

EXERCISE 6.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?

Number	The possible 'one's' digits of the square root of the number
(i) 9801	1 or 9 (since $1^2 = 1$ *and* $9^2 = 1$)
(ii) 99856	4 or 6 (since $4^2 = 16$ *and* $6^2 = 36$)
(iii) 998001	1 or 9 (since $1^2 = 1$ *and* $9^2 = 1$)
(iv) 657666025	5 (since $5^2 = 5$)

2. Without doing any calculation, find the numbers which are surely not perfect squares.

(The perfect square number does not end with 2, 3, 7 or 8 at unit's place)

(i) 153 → Not a perfect square

(ii) 257 → Not a perfect square

(iii) 408 → Not a perfect square

(iv) $441 = 21^2 \rightarrow$ 441 is a perfect square

3. Find the square roots of 100 and 169 by the method of repeated subtraction.

(i) Step 1: $100 - 1 = 99$

Step 2: $99 - 3 = 96$

Step 3: $96 - 5 = 91$

Step 4: $91 - 7 = 84$

Step 5: $84 - 9 = 75$

Step 6: $75 - 11 = 64$

Step 7: $64 - 13 = 51$

Step 8: $51 - 15 = 36$

Step 9: $36 - 17 = 19$

Step 10: $19 - 19 = 0$

From 100 we have subtracted successive odd numbers starting from 1 and obtained 0 at 10^{th} step. Therefore $\sqrt{100} = 10$

(ii) Step 1: $169 - 1 = 168$

Step 2: $168 - 3 = 165$

Step 3: $165 - 5 = 160$

Step 4: $160 - 7 = 153$

Step 5: $153 - 9 = 144$

Step 6: $144 - 11 = 133$

Step 7: $133 - 13 = 120$

Step 8: $120 - 15 = 105$

Step 9: $105 - 17 = 88$

Step 10: $88 - 19 = 69$

Step 11: $69 - 21 = 48$

Step 12: $48 - 23 = 25$

Step 13: $25 - 25 = 0$

From 169 we have subtracted successive odd numbers starting from 1 and obtained 0 at 13^{th} step. Therefore $\sqrt{169} = 13$

4. Find the square roots of the following numbers by the Prime Factorisation Method.

(i) 729

Sol: $729=\underline{3\times3}\times\underline{3\times3}\times\underline{3\times3}$

$\sqrt{729} = 3 \times 3 \times 3$

$\sqrt{729} = 27$

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

(ii) 400

Sol: $400=\underline{2\times2}\times\underline{2\times2}\times\underline{5\times5}$

$\sqrt{400} = 2 \times 2 \times 5$

$\sqrt{400} = 20$

$$\begin{array}{r|l} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

(iii) 1764

Sol: $1764=\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

$\sqrt{1764} = 2 \times 3 \times 7$

$\sqrt{1764} = 42$

$$\begin{array}{r|l} 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(iv) 4096

Sol: $4096=\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}$

$\sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt{4096} = 64$

$$\begin{array}{r|l} 2 & 4096 \\ \hline 2 & 2048 \\ \hline 2 & 1024 \\ \hline 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

(v) 7744

Sol: $7744=\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{11\times11}$

$\sqrt{7744} = 2 \times 2 \times 2 \times 11$

$\sqrt{7744} = 88$

$$\begin{array}{r|l} 2 & 7744 \\ \hline 2 & 3872 \\ \hline 2 & 1936 \\ \hline 2 & 968 \\ \hline 2 & 484 \\ \hline 2 & 242 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

(vi) 9604

Sol: $9604=\underline{2\times2}\times\underline{7\times7}\times\underline{7\times7}$

$$\begin{array}{r|l} 2 & 9604 \\ \hline 2 & 4802 \\ \hline 7 & 2401 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$\sqrt{9604} = 2 \times 7 \times 7$

$\sqrt{9604} = 98$

(vii) 5929

$$\begin{array}{r|l} 7 & 5929 \\ \hline 7 & 847 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

Sol: $5929 = 7 \times 7 \times 11 \times 11$

$\sqrt{5929} = 7 \times 11$

$\sqrt{5929} = 77$

(viii) 9216

$$\begin{array}{r|l} 2 & 9216 \\ \hline 2 & 4608 \\ \hline 2 & 2304 \\ \hline 2 & 1152 \\ \hline 2 & 576 \\ \hline 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Sol: $9216=\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}$

$\sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\sqrt{9216} = 96$

(ix) 529

$$\begin{array}{r|l} 23 & 529 \\ \hline & 23 \end{array}$$

Sol: $529 = 23 \times 23$

$\sqrt{529} = 23$

(x) 8100

$$\begin{array}{r|l} 2 & 8100 \\ \hline 2 & 4050 \\ \hline 3 & 2025 \\ \hline 3 & 675 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $8100=\underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}\times\underline{5\times5}$

$\sqrt{8100} = 2 \times 3 \times 3 \times 5$

$\sqrt{8100} = 90$

5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(i) 252

$$\begin{array}{r|l} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Sol: $252 = 2 \times 2 \times 3 \times 3 \times 7$

The prime factor 7 has no pair.

So, we multiply 252 by 7 to get a perfect square.

$252 \times 7 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$1724=\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

$\sqrt{1724} = 2 \times 3 \times 7$

$\sqrt{1724} = 42$

(ii) 180

$$\begin{array}{r|l} 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Sol: $180 = 2 \times 2 \times 3 \times 3 \times 5$

The prime factor 5 has no pair.

So, we multiply 180 by 5 to get a perfect square.

$180 \times 5 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$900=\underline{2\times2}\times\underline{3\times3}\times\underline{5\times5}$

$\sqrt{900} = 2 \times 3 \times 5 = 30$

(iii) 1008

$$\begin{array}{r|l} 2 & 1008 \\ \hline 2 & 504 \\ \hline 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Sol: $1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

The prime factor 7 has no pair.

So, we multiply 1008 by 7 to get a perfect square

$1008 \times 7 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$7056=\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}$

$\sqrt{7056} = 2 \times 2 \times 3 \times 7$

$\sqrt{7056} = 84$

(iv) 2028

$$\begin{array}{r|l} 2 & 2028 \\ \hline 2 & 1014 \\ \hline 3 & 507 \\ \hline 13 & 169 \\ \hline & 13 \end{array}$$

Sol: $2028=\underline{2\times2}\times3\times\underline{13\times13}$

The prime factor 3 has no pair.

So, we multiply 2028 by 3 to get a perfect square

$2028 \times 3 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$

$6084=\underline{2\times2}\times\underline{3\times3}\times\underline{13\times13}$

$\sqrt{6084} = 2 \times 3 \times 13$

$\sqrt{6084} = 78$

(v) 1458

$$\begin{array}{r|l} 2 & 1458 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Sol: $1458=2\times\underline{3\times3}\times\underline{3\times3}\times\underline{3\times3}$

The prime factor 2 has no pair.

So, we multiply 1458 by 2 to get a perfect square.

$1458 \times 2 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$2916=\underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}\times\underline{3\times3}$

$\sqrt{2916} = 2 \times 3 \times 3 \times 3$

$\sqrt{2916} = 54$

(vi) 768

$$\begin{array}{r|l} 2 & 768 \\ \hline 2 & 384 \\ \hline 2 & 192 \\ \hline 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

Sol: $768=\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times3$

The prime factor 3 has no pair.

So, we multiply 768 by 3 to get a perfect square.

$768 \times 3 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$2304=\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}$

$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3$

$\sqrt{2304} = 48$

6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

(i) 252

$$\begin{array}{r|l} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Sol: $252 = (2 \times 2) \times (3 \times 3) \times 7$

The prime factor 7 has no pair.

So, we divided 252 by 7 to get a perfect square.

$252 \div 7 = 2 \times 2 \times 3 \times 3$

$36=\underline{2\times2}\times\underline{3\times3}$

$\sqrt{36} = 2 \times 3$

$\sqrt{36} = 6$

(ii) 2925

$$\begin{array}{r|l} 3 & 2925 \\ \hline 3 & 975 \\ \hline 5 & 325 \\ \hline 5 & 65 \\ \hline & 13 \end{array}$$

Sol: $2925=\underline{3\times3}\times\underline{5\times5}\times13$

The prime factor 13 has no pair.

So, we divided 2925 by 13 to get a perfect square.

$2925 \div 13 = 3 \times 3 \times 5 \times 5 \times 13 \div 13$

$225 = 3 \times 3 \times 5 \times 5$

$\sqrt{225} = 3 \times 5$

$\sqrt{225} = 15$

(iii) 396

$$\begin{array}{r|l} 2 & 396 \\ \hline 2 & 198 \\ \hline 3 & 99 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

Sol: $396 = 2 \times 2 \times 3 \times 3 \times 11$

The prime factor 11 has no pair.

So, we divided 396 by 11 to get a perfect square.

$396 \div 11 = 2 \times 2 \times 3 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$\sqrt{36} = 2 \times 3$

$\sqrt{36} = 6$

(iv) 2645

$$\begin{array}{r|l} 5 & 2645 \\ \hline 23 & 529 \\ \hline & 23 \end{array}$$

Sol: $2645 = 5 \times 23 \times 23$

The prime factor 5 has no pair.

So, we divided 2645 by 5 to get a perfect square.

$2645 \div 5 = 23 \times 23$

$529 = 23 \times 23$

$$\sqrt{529} = 23$$

(v) 2800

2	2800
2	700
2	1400
2	350
5	175
5	35
	7

Sol: $2800 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5} \times 7$

The prime factor 7 has no pair

So, we divided 2800 by 7 to get a perfect square.

$2800 \div 7 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

$400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

$\sqrt{400} = 2 \times 2 \times 5$

$\sqrt{400} = 20$

(vi) 1620

2	1620
2	810
3	405
3	135
3	45
3	15
	5

Sol: $1620 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \times 5$

The prime factor 5 has no pair

So, we divided 1620 by 5 to get a perfect square.

$1620 \div 5 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$\sqrt{324} = 2 \times 3 \times 3$

$\sqrt{324} = 18$

7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class

Sol: Let the total number of students $= x$

Amount donated by each student $= x$

Total donation $= 2401$

$x \times x = 2401$

$x^2 = 2401$

7	2401
7	343
7	49
	7

$x = \sqrt{2401}$

$x = \sqrt{\underline{7 \times 7} \times \underline{7 \times 7}}$

$x = 7 \times 7 = 49$

The number of students in the class $= 49$

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Sol: Let the number of rows $= x$

The number of plants in each row $= x$

Total plants $= x \times x = x^2$

Given total plants are planted=2025

$x^2 = 2025$

$x = \sqrt{2025}$

$x = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}}$

$x = 3 \times 3 \times 5$

$x = 45$

∴ The number of rows=45 and

The number of plants in each row=45

$$\begin{array}{r|l} 3 & 2025 \\ \hline 3 & 675 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10

Sol: The smallest square number divisible by 4,9 and 10=least multiple of LCM(4,9,10)

LCM(4,9,10)=$2 \times 2 \times 9 \times 5 = 2 \times 2 \times 3 \times 3 \times 5 = 180$

$$\begin{array}{r|l} 2 & 4,9,10 \\ \hline & 2,9,5 \end{array}$$

The prime factor 5 has no pair

So, we multiply 180 by 5 to get a perfect square

Required number=180$\times$ $5 = 900$

10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

Sol: The smallest square number divisible by 8,15 and 20=Multiple of LCM(8,15,20)

LCM(8,15,20)=$2 \times 2 \times 2 \times 3 \times 5 = 120$

The prime factors 2,3, 5 have no pairs

So, we multiply 120 by $2 \times 3 \times 5 = 30$ to get a perfect square

Required number=120$\times$ $30 = 3600$

$$\begin{array}{r|l} 2 & 8,15,20 \\ \hline 2 & 4,15,10 \\ \hline 5 & 2,15,\ 5 \\ \hline & 2,3,\ 1 \end{array}$$

Finding square root by division method

Number	Square	
10	100	Which is the smallest 3-digit perfect square
31	961	Which is the greatest 3-digit perfect square
32	1024	Which is the smallest 4-digit perfect square
99	9801	Which is the greatest 3-digit perfect square

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers

(i) 25600

Sol: The number of digits in $\sqrt{25600}$ *is* $\frac{5+1}{2} = \frac{6}{2} = 3$

(ii) 100000000

Sol: The number of digits in $\sqrt{100000000}$ $is\ \frac{9+1}{2} = \frac{10}{2} = 5$

(iii) 36864

Sol: The number of digits in $\sqrt{36864}$ $is\ \frac{5+1}{2} = \frac{6}{2} = 3$

EXERCISE 6.4

1. Find the square root of each of the following numbers by Division method.

(i) 2304

$\sqrt{2304} = 48$

$$\begin{array}{r|l} & 48 \\ \hline 4 & \overline{23}\ \overline{04} \\ & -16 \\ \hline 88 & 704 \\ & 704 \\ \hline & 0 \end{array}$$

(i) 4489

$\sqrt{4489}$ =67

$$\begin{array}{r|l} & 67 \\ \hline 6 & \overline{44}\ \overline{89} \\ & -36 \\ \hline 127 & 889 \\ & 889 \\ \hline & 0 \end{array}$$

(ii) 3481

$\sqrt{3481} = 59$

$$\begin{array}{r|l} & 59 \\ \hline 5 & \overline{34}\ \overline{81} \\ & -25 \\ \hline 109 & 981 \\ & 981 \\ \hline & 0 \end{array}$$

(iii) 529

$\sqrt{529} = 23$

$$\begin{array}{r|l} & 23 \\ \hline 2 & \overline{5}\ \overline{29} \\ & -4 \\ \hline 43 & 129 \\ & -129 \\ \hline & 0 \end{array}$$

(iv) 3249

$\sqrt{3249} = 57$

$$\begin{array}{r|l} & 57 \\ \hline 5 & \overline{32}\ \overline{49} \\ & -25 \\ \hline 107 & 749 \\ & -749 \\ \hline & 0 \end{array}$$

(v) 1369

$\sqrt{1369} = 37$

$$\begin{array}{r|l} & 37 \\ \hline 3 & \overline{13}\ \overline{69} \\ & -9 \\ \hline 67 & 469 \\ & -469 \\ \hline & 0 \end{array}$$

(vi) 5776

$\sqrt{5776} = 76$

$$\begin{array}{r|l} & 76 \\ \hline 7 & \overline{57}\ \overline{76} \\ & -49 \\ \hline 146 & 876 \\ & -876 \\ \hline & 0 \end{array}$$

(vii) 7921

$\sqrt{7921} = 89$

$$\begin{array}{r|l} & 89 \\ \hline 8 & \overline{79}\ \overline{21} \\ & -64 \\ \hline 169 & 15\ 21 \\ & -15\ 21 \\ \hline & 0 \end{array}$$

(viii) 576

$\sqrt{576} = 24$

$$\begin{array}{r|l} & 24 \\ \hline 2 & \overline{5}\ \overline{76} \\ & -4 \\ \hline 44 & 176 \\ & -176 \\ \hline & 0 \end{array}$$

(ix) 1024

$\sqrt{1024} = 32$

$$\begin{array}{r|l} & 32 \\ \hline 3 & \overline{10}\ \overline{24} \\ & -9 \\ \hline 62 & 124 \\ & -124 \\ \hline & 0 \end{array}$$

(x) 3136

$\sqrt{3136} = 56$

$$\begin{array}{r|l} & 56 \\ \hline 5 & \overline{31}\ \overline{36} \\ & -25 \\ \hline 106 & 636 \\ & 636 \\ \hline & 0 \end{array}$$

(xi) 900

$\sqrt{900} = 30$

$$\begin{array}{r|l} & 30 \\ \hline 3 & \overline{9}\ \overline{00} \\ & -9 \\ \hline & 0 \end{array}$$

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

If a perfect square is of n-digits, then its square root will have

$(a)\ \frac{n}{2}\ digits\ if\ n\ is\ even \quad (b)\ \frac{n+1}{2}\ digits\ \ if\ n\ is\ odd$

(i) 64

Number of digits in 64=2 (ie) n=2 which is even.

$$\frac{n}{2}=\frac{2}{2}=1$$

The number of digits in the square root of 64=1

(ii) 144

Number of digits in 144=3 (ie) n=3 which is odd.

$$\frac{n}{2}=\frac{2}{2}=1$$

The number of digits in the square root of 64=1

(iii) 4489

Number of digits in 4489=4 (ie) n=4 which is even.

$$\frac{n}{2}=\frac{4}{2}=2$$

The number of digits in the square root of 4489=2

(iv) 27225

Number of digits in 27225=5 (ie) n=5 which is odd.

$$\frac{n+1}{2}=\frac{5+1}{2}=\frac{6}{2}=3$$

The number of digits in the square root of 27225=3

(v) 390625

Number of digits in 390625=6 (ie) n=6 which is even.

$$\frac{n}{2}=\frac{6}{2}=3$$

The number of digits in the square root of 390625=3

3. Find the square root of the following decimal numbers.

(i) 2.56

$\sqrt{2.56}=1.6$

$$\begin{array}{r|l} & \quad 1.6 \\ \hline 1 & \overline{2}.\ \overline{56} \\ & -1 \\ \hline 26 & \ 156 \\ & -156 \\ \hline & \quad 0 \end{array}$$

(ii) 7.29

$\sqrt{7.29}=2.7$

$$\begin{array}{r|l} & \quad 2.7 \\ \hline 2 & \overline{7}.\overline{29} \\ & -4 \\ \hline 47 & \ 329 \\ & -329 \\ \hline & \quad 0 \end{array}$$

(iii) 51.84

$\sqrt{51.84}=7.$

$$\begin{array}{r|l} & \quad 7.2 \\ \hline 7 & \overline{51}.\ \overline{84} \\ & -49 \\ \hline 142 & \ 284 \\ & -284 \\ \hline & \quad 0 \end{array}$$

(iv) 42.25

$\sqrt{42.25} = 6.5$

$$\begin{array}{r|l} & \ \ 6.5 \\ \hline 6 & \overline{42}\,.\,\overline{25} \\ & -36 \\ \hline 125 & \ \ 625 \\ & -625 \\ \hline & \ \ \ 0 \end{array}$$

(v) 31.36

$\sqrt{31.36} = 5.6$

$$\begin{array}{r|l} & \ \ 5.6 \\ \hline 5 & \overline{31}.\ \overline{36} \\ & -25 \\ \hline 106 & \ \ 636 \\ & -636 \\ \hline & \ \ \ 0 \end{array}$$

4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402

Sol:The remainder=2

(If we subtract the remainder 2 from 402, we get a perfect square).

The required perfect square=402-2=400

$\sqrt{400} = 20$

$$\begin{array}{r|l} & \ \ 20 \\ \hline 2 & \overline{4}\ \overline{02} \\ & -4 \\ \hline 40 & \ \ 002 \\ & -000 \\ \hline & \ \ \ 2 \end{array}$$

(ii) 1989

Sol: The remainder=53

(If we subtract the remainder 53 from 1989, we get a perfect square).

The required perfect square=1989-53=1936

$\sqrt{1936} = 44$

$$\begin{array}{r|l} & \ \ 44 \\ \hline 4 & \overline{19}\ \overline{89} \\ & -16 \\ \hline 84 & \ \ 389 \\ & \ \ 336 \\ \hline & \ \ \ 53 \end{array}$$

(iii) 3250

Sol: The remainder=1

(If we subtract the remainder 1 from 3250, we get a perfect square).

The required perfect square=3250-1=3249

$\sqrt{3249} = 57$

$$\begin{array}{r|l} & \ \ 57 \\ \hline 5 & \overline{32}\ \overline{50} \\ & -25 \\ \hline 107 & \ \ 750 \\ & \ \ 749 \\ \hline & \ \ \ 1 \end{array}$$

(iv) 825

Sol: The remainder=41

(If we subtract the remainder 41 from 825, we get a perfect square).

The required perfect square=825-41=784

$\sqrt{784} = 28$

$$\begin{array}{r|l} & \ \ 28 \\ \hline 2 & \overline{8}\ \overline{25} \\ & -4 \\ \hline 48 & \ \ 425 \\ & -384 \\ \hline & \ \ \ 41 \end{array}$$

(v) 4000

Sol:The remainder=31

(If we subtract 31 from 4000, we get a perfect square).

$$\begin{array}{r|l} & \ \ 63 \\ \hline 6 & \overline{40}\ \overline{00} \\ & -36 \\ \hline 123 & \ \ 400 \\ & -369 \\ \hline & \ \ \ 31 \end{array}$$

The require least number=31

The required perfect square= $4000-31=3969$

$\sqrt{3969}=63$

5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525

Sol: The remainder=41

$$\begin{array}{r|l} & 22 \\ \hline 2 & \overline{5}\ \overline{25} \\ & -4 \\ \hline 42 & 125 \\ & -84 \\ \hline & 41 \end{array}$$

$22^2<525$

The next perfect square number=$23^2=529$

The number to be added=$529-525$=4

The perfect square obtained=529 and $\sqrt{529}=23$

Alternate method:

$$\begin{array}{r|l} & 23 \\ \hline 2 & \overline{5}\ \overline{25} \\ & -4 \\ \hline 43 & 125 \\ & -129 \\ \hline & -4 \end{array}$$

Remainder=-4

If we add 4 to 525, we get a perfect square

The required least number=4

The perfect square=525+4=529 and $\sqrt{529}=23$

(ii) 1750

Sol: Remainder=-14

$$\begin{array}{r|l} & 42 \\ \hline 4 & \overline{17}\ \overline{50} \\ & -16 \\ \hline 42 & 150 \\ & -164 \\ \hline & -14 \end{array}$$

If we add 14 to 1750, we get a perfect square

The required least number to be added =14

The perfect square=1750+14=1764 and $\sqrt{1764}=42$

(iii) 252

Sol: Remainder=-4

$$\begin{array}{r|l} & 16 \\ \hline 1 & \overline{2}\ \overline{52} \\ & -1 \\ \hline 26 & 152 \\ & -156 \\ \hline & -4 \end{array}$$

If we add 4 to 252, we get a perfect square

The required least number to be added =4

The perfect square=252+4=256 and $\sqrt{256}=16$

(iv) 1825

Sol: Remainder=-24

$$\begin{array}{r|l} & 43 \\ \hline 4 & \overline{18}\ \overline{25} \\ & -16 \\ \hline 83 & 225 \\ & -249 \\ \hline & -24 \end{array}$$

If we add 24 to 1825, we get a perfect square

The required least number to be added =24

The perfect square=1825+24=1849 and $\sqrt{1849}=43$

(v) 6412

Sol: Remainder=-149

$$\begin{array}{r|l} & 81 \\ \hline 8 & \overline{64}\ \overline{12} \\ & -64 \\ \hline 161 & 12 \\ & -161 \\ \hline & -149 \end{array}$$

If we add 149 to 6412, we get a perfect square

The required least number to be added =149

The perfect square=6412+149=6561 and $\sqrt{6561} = 81$

6. Find the length of the side of a square whose area is 441 m².

Sol: Let side of the square=x

Area of the square=$side \times side = x \times x = x^2$

Given area of the square=441 m².

$$
\begin{array}{r|r}
 & 21 \\
\hline
2 & \overline{4}\ \overline{41} \\
 & -4 \\
\hline
41 & 41 \\
 & -41 \\
\hline
 & 0
\end{array}
$$

$x^2 = 441$

$x = \sqrt{441} = 21$

The length of the side of the square=21 m

7. In a right triangle ABC, ∠B = 90°. (a) If AB = 6 cm, BC = 8 cm, find AC (b) If AC = 13 cm, BC = 5 cm, find AB

Sol: (a) $In\ \Delta ABC\ , \angle B = 90^0$

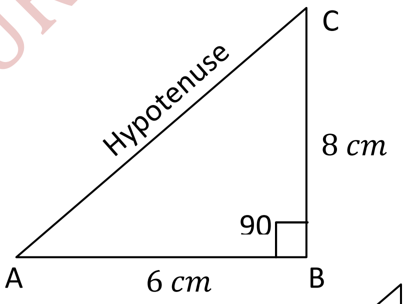
From Pythagoras theorem

$(Hypotenuse)^2 = (side)^2 + (side)^2$

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$

$AC = \sqrt{100} = 10\ cm$



(b) $In\ \Delta ABC\ , \angle B = 90^0$

From Pythagoras theorem

$(Hypotenuse)^2 = (side)^2 + (side)^2$

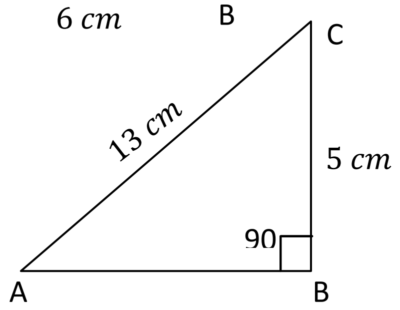
$AC^2 = AB^2 + BC^2$

$13^2 = AB^2 + 5^2$

$169 = AB^2 + 25$

$AB^2 = 169 - 25 = 144$

$AB = \sqrt{144} = 12\ cm$



CHAPTER 6

AP VIII CLASS-CBSE (2023-24)

Cubes and Cube Roots (Notes)

PREPARED BY : BALABHADRA SURESH-9866845885

1. $a^3 = a \times a \times a$

   $x^3 = x \times x \times x$

$1^3 = 1$	$8^3 = 512$	$15^3 = 3375$
$2^3 = 8$	$9^3 = 729$	$16^3 = 4096$
$3^3 = 27$	$10^3 = 1000$	$17^3 = 4913$
$4^3 = 64$	$11^3 = 1331$	$18^3 = 5832$
$5^3 = 125$	$12^3 = 1728$	$19^3 = 6859$
$6^3 = 216$	$13^3 = 2197$	$20^3 = 8000$
$7^3 = 343$	$14^3 = 2744$	

2. 1, 8, 27,64,125,216,343,……are called perfect cubes.
3. How many perfect cubes are there from 1 to 100?

Sol: Four {1,8,27,64}

4. How many perfect cubes are there from 1 to 1000?

Sol: Ten{1,8,27,64,125,216,343,512,729,1000}

The one's digit of the number	The one's digit of cube of the number	The one's digit of the number	The one's digit of cube of the number
1	1	6	6
2	8	7	3
3	7	8	2
4	4	9	9
5	5	0	0

Hardy – Ramanujan Number: 1729

1729 is the smallest number that can be expressed as a sum of two cubes in two different ways:

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

Some this type of numbers:

1). $4104 = 8 + 4096 = 2^3 + 16^3$; $4104 = 729 + 3375 = 9^3 + 15^3$

2). $13832 = 8 + 13824 = 2^3 + 24^3$; $13832 = 5832 + 8000 = 18^3 + 20^3$

TRY THESE

Find the one's digit of the cube of each of the following numbers.

Number	The one's digit of the cube of number	Number	The one's digit of the cube of number
(i) 3331	1	(v) 1024	4
(ii) 8888	2	(vi) 77	3
(iii) 149	9	(vii) 5022	8
(iv) 1005	5	(viii) 53	7

Some interesting patterns:

$1 = 1 = 1^3$

$3 + 5 = 8 = 2^3$

$7 + 9 + 11 = 27 = 3^3$

$13 + 15 + 17 + 19 = 64 = 4^3$

$21 + 23 + 25 + 27 + 29 = 125 = 5^3$

TRY THESE

Express the following numbers as the sum of consecutive odd numbers pattern?

$(a)\ 6^3 = 216 = 31 + 33 + 35 + 37 + 39 + 41$

$(b)\ 8^3 = 512 = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71$

$(c)\ 7^3 = 343 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following.

$(i)\ 7^3 - 6^3 = 1 + 7 \times 6 \times 3$

$n^3 - (n-1)^3 = 1 + n \times (n-1) \times 3$

$(ii)\ 12^3 - 11^3 = 1 + 12 \times 11 \times 3$

$(iii)\ 20^3 - 19^3 = 1 + 20 \times 19 \times 3$

$(vi)\ 51^3 - 50^3 = 1 + 51 \times 50 \times 3$

Cubes and their prime factors:

If a number can be expressed as a product of three equal factors then it is said to be a perfect cube or

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 9 \\ \hline 3 & 27 \end{array}$$

cubic number.

Example 1: Is 243 a perfect cube?

Sol: 243=$\underline{3\times3\times3}\times3\times3$

After grouping 3×3 remains.

Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400

$$\begin{array}{r|l} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

Here, 2 and 5 do not appear in groups of three.

Hence 400 is not a perfect cube.

2. 3375

$$\begin{array}{r|l} 3 & 3375 \\ \hline 3 & 1125 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

The prime factors can be grouped in triples.

Hence 3375 is a perfect cube.

$3375 = (3 \times 5)^3 = 15^3$

3. 8000

$$\begin{array}{r|l} 2 & 8000 \\ \hline 2 & 4000 \\ \hline 2 & 2000 \\ \hline 2 & 1000 \\ \hline 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $8000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

The prime factors can be grouped in triples

Hence 8000 is a perfect cube.

$8000 = (2 \times 2 \times 5)^3 = 20^3$

4. 15625

$$\begin{array}{r|l} 5 & 15625 \\ \hline 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: 15625=$\underline{5\times5\times5}\times5\times5$

Here, 5 do not appear in groups of three.

Hence 15625 is not a perfect cube.

5. 9000

$$\begin{array}{r|l} 2 & 9000 \\ \hline 2 & 4500 \\ \hline 2 & 2250 \\ \hline 3 & 1125 \\ \hline 5 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: 9000=$\underline{2\times2\times2}\times3\times\underline{5\times5\times5}\times5$

Here, after grouping 3 and 5 do not appear in groups of three.

Hence 9000 is not a perfect cube.

6. 6859

$$\begin{array}{r|l} 19 & 6859 \\ \hline 19 & 361 \\ \hline & 19 \end{array}$$

Sol: 6859=$\underline{19\times19\times19}$

The prime factors can be grouped in triples

Hence 6859 is a perfect cube.

$6859 = (19)^3$

7. 2025

$$\begin{array}{r|l} 5 & 2025 \\ \hline 5 & 405 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\begin{array}{r|l} 2 & 10648 \\ \hline 2 & 5324 \\ \hline 2 & 2662 \\ \hline 11 & 1331 \\ \hline 11 & 121 \end{array}$$

Sol: 2025=$\underline{3\times3\times3}\times3\times5\times5$

After grouping 3 and 5 do not appear in groups of three.

Hence 2025 is not a perfect cube.

8. 10648

Sol: 10648=$\underline{2\times2\times2}\times\underline{11\times11\times11}$

$10648 = (2 \times 11)^3 = 22^3$

The prime factors can be grouped in triples

Hence 10648 is a perfect cube.

Example 2: Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube

$$\begin{array}{r|l} 2 & 392 \\ \hline 2 & 196 \\ \hline 7 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Sol: $392 = (2 \times 2 \times 2) \times 7 \times 7$

The prime factor 7 does not appear in a group of three.

Hence, the smallest number by which 392 should be multiplied to make it a perfect cube is 7.

Example 3: Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?

$$\begin{array}{r|l} 2 & 53240 \\ \hline 2 & 26620 \\ \hline 2 & 1331 \\ \hline 5 & 2662 \\ \hline 11 & 1331 \\ \hline 11 & 121 \\ \hline & 11 \end{array}$$

Sol: $53240 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11} \times 5$

After grouping 5 remains

Hence the smallest number by which 53240 should be divided by 5 to make it a perfect cube .

$53240 \div 5 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

$10648 = 22^3$

Example 4: Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?

$$\begin{array}{r|l} 2 & 1188 \\ \hline 2 & 594 \\ \hline 3 & 297 \\ \hline 3 & 99 \\ \hline 3 & 33 \\ \hline & 11 \end{array}$$

Sol: $1188 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 11$

After grouping remaining $2 \times 2 \times 11 = 44$

1188 should be divided by 44 to make it a perfect cube.

$1188 \div 44 = (3 \times 3 \times 3)$

$27 = 3^3$

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

$$\begin{array}{r|l} 2 & 68600 \\ \hline 2 & 34300 \\ \hline 2 & 17150 \\ \hline 5 & 8575 \\ \hline 5 & 1715 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Sol: $68600 = \underline{2 \times 2 \times 2} \times 5 \times 5 \times \underline{7 \times 7 \times 7}$.

After grouping remaining 5×5

Required smallest number=5

$68600 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$.

$343000 = (2 \times 5 \times 7)^3 = 70^3$

THINK, DISCUSS AND WRITE

Checks which of the following are perfect cubes.

(i) **2700** –Not a perfect cube

(ii) **16000** –Not a perfect cube

(iii) **64000** - perfect cube

(iv) **900** –Not a perfect cube

(v) **125000**- perfect cube

(vi) **36000**–Not a perfect cube

(vii) **21600** –Not a perfect cube

(viii) **10,000** –Not a perfect cube

(ix) **27000000** - perfect cube

(x) **1000** - Perfect cube

What pattern do you observe in these perfect cubes?

We observe that perfect cube have the number of zeroes in multiple of 3.

EXERCISE 7.1

1. Which of the following numbers are not perfect cubes?

(i) **216**

Sol: $216 = \underline{2\times2\times2}\times\underline{3\times3\times3}$

The prime factors can be grouped in triples

Hence 216 is a perfect cube.

$216 = (2 \times 3)^3 = 6^3$

2	216
2	108
2	54
3	27
3	9
	3

(ii) **128**

Sol: $128 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times2$

128 is not a perfect cube

2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 1000

Sol: $1000 = \underline{2\times2\times2}\times\underline{5\times5\times5}$

The prime factors can be grouped in triples.

Hence 1000 is a perfect cube.

$1000 = (2 \times 5)^3 = 10^3$

2	1000
2	500
2	250
5	125
5	25
	5

(iv) **100**

Sol: $100 = 2 \times 2 \times 5 \times 5$

100 is not a perfect cube

2	100
2	50
5	25
	5

(v) **46656**

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

Sol: $46656 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{3\times3\times3}$

The prime factors can be grouped in triples

Hence 1000 is a perfect cube.

$46656 = (2 \times 2 \times 3 \times 3)^3 = 36^3$

2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 9 \\ \hline 3 & 27 \\ \hline & 3 \end{array}$$

Sol: $243 = \underline{3\times3\times3}\times3\times3$

Here, after grouping remaining is 3×3

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is 3.

(ii) 256

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

Sol: $256 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times2\times2$

After grouping remaining is 2×2

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is 2.

(iii) 72

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Sol: $72 = \underline{2\times2\times2}\times3\times3$

After grouping remaining is 3×3

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is 3

iv) 675

$$\begin{array}{r|l} 3 & 675 \\ \hline 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

Sol: $675 = \underline{3\times3\times3}\times5\times5$

Here, after grouping remaining is 5×5

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is 5

(v) 100

Sol: $100 = 2 \times 2 \times 5 \times 5$

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is $2\times5=10$

3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) 81

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Sol: $81 = \underline{3\times3\times3}\times3$

After grouping remaining is 3

Hence, the smallest number by which it is to be divided to make it a perfect cube is 3

(ii) 128

$$\begin{array}{r|l} 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

Sol: $128 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times$

After grouping remaining is 2 .

Hence, the smallest number by which it is to be divided to make it a perfect cube is 2.

(iii) 135

3	135
3	45
3	15
	5

Sol: $135 = \underline{3 \times 3 \times 3} \times 5$

After grouping remaining is 5 .

Hence, the smallest number by which 135 it is to be divided to make it a perfect cube is 5.

(iv) 192

2	192
2	96
2	48
2	24
2	12
2	6
	3

Sol: $192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

After grouping remaining is 3 .

Hence, the smallest number by which it is to be divided to make it a perfect cube is 3.

(v) 704

2	704
2	352
2	176
2	88
2	44
2	22
	11

Sol: $704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$

Here, after grouping remaining is 11 .

Hence, the smallest number by which it is to be divided to make it a perfect cube is 11.

4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Sol: Volume of cuboid $= 5\text{cm} \times 2\text{cm} \times 5\text{cm}$

To make cube we multiply with $2 \times 2 \times 5 = 20$

Number of required cuboids $= 20$

Cube Roots

Cubes	Cube roots	Cubes	Cube roots
$1^3 = 1$	$\sqrt[3]{1} = 1$	$11^3 = 1331$	$\sqrt[3]{1331} = 11$
$2^3 = 8$	$\sqrt[3]{8} = 2$	$12^3 = 1728$	$\sqrt[3]{1728} = 12$
$3^3 = 27$	$\sqrt[3]{27} = 3$	$13^3 = 2197$	$\sqrt[3]{2197} = 13$
$4^3 = 64$	$\sqrt[3]{64} = 4$	$14^3 = 2744$	$\sqrt[3]{2744} = 14$
$5^3 = 125$	$\sqrt[3]{125} = 5$	$15^3 = 3375$	$\sqrt[3]{3375} = 15$
$6^3 = 216$	$\sqrt[3]{216} = 6$	$16^3 = 4096$	$\sqrt[3]{4096} = 16$
$7^3 = 343$	$\sqrt[3]{343} = 7$	$17^3 = 4913$	$\sqrt[3]{4913} = 17$
$8^3 = 512$	$\sqrt[3]{512} = 8$	$18^3 = 5832$	$\sqrt[3]{5832} = 18$
$9^3 = 729$	$\sqrt[3]{729} = 9$	$19^3 = 6859$	$\sqrt[3]{6859} = 19$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$	$20^3 = 8000$	$\sqrt[3]{8000} = 20$

Example 6: Find the cube root of 8000.

2	8000
2	4000
2	2000
2	1000
2	500

2	13824
2	6912
2	3456
2	1728
2	864

Solution: $8000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

(or)

$8000 = 8 \times 1000 = 2^3 \times 10^3$

$\sqrt[3]{8000} = 2 \times 10 = 20$

Example 7: Find the cube root of 13824 by prime factorisation method.

Sol: $13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$.

$\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

THINK, DISCUSS AND WRITE

State true or false: for any integer m, $m^2 < m^3$. Why?

Sol: False

$For\ m = -2;$

$m^2 = (-2)^2 = 4\ \ and\ \ m^3 = (-2)^3 = -8$

$m^2 > m^3$

EXERCISE 6.2

1. Find the cube root of each of the following numbers by prime factorisation method.

(i) 64

Sol: $64 = \underline{2\times2\times2}\times\underline{2\times2\times2}$

$\sqrt[3]{64} = 2 \times 2 = 4$

2	64
2	32
2	16
2	8
2	4
	2

(ii) 512

Sol: $512 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}$

$\sqrt[3]{512} = 2 \times 2 \times 2 = 8$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(iii) 10648

Sol: $10648 = \underline{2\times2\times2}\times\underline{11\times11\times11}$

$\sqrt[3]{10648} = 2 \times 11 = 22$

2	10648
2	5324
2	2662
11	1331
11	121
	11

(iv) 27000

Sol: $27000 = \underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{5\times5\times5}$

$\sqrt[3]{27000} = 2 \times 3 \times 5 = 30$

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
	5

(v) 15625

Sol: $15625 = \underline{5\times5\times5}\times\underline{5\times5\times5}$

$\sqrt[3]{15625} = 5 \times 5 = 25$

5	15625
5	3105
5	625
5	125
5	25
	5

(vi) 13824

Sol: $13824 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

$\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

(vii) 110592

Sol: $110592 = 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3$

$\sqrt[3]{110592} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

(viii) 46656

Sol: $46656 = 2^3 \times 2^3 \times 3^3 \times 3^3$

$\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
	3

(ix) 175616

Sol: $175616 = 2^3 \times 2^3 \times 2^3 \times 7^3$

$\sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56$

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
	7

(x) 91125

Sol: $91125 = 5^3 \times 3^3 \times 3^3$

$\sqrt[3]{91125} = 5 \times 3 \times 3 = 45$

5	91125
5	18225
5	3645
3	729
3	243
3	81
3	9
3	27
	3

2. State true or false.

(i) Cube of any odd number is even.

Sol: False

Cube of any odd number is odd and

Cube of any even number is even.

(ii) A perfect cube does not end with two zeros.

Sol: True

A perfect cube end with multiple of three zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

Sol: False

$15^2 = 225$ ends with 5 but

$15^3 = 3375$ does not ends with25

(iv) There is no perfect cube which ends with 8.

Sol: False

$12^3 = 1728 \rightarrow$ End with 8

Unit digit place is 2 its cube end with 8.

(v) The cube of a two digit number may be a three digit number.

Sol: False

The cube of a two digit number may have 4 digits to 6 digits.

(vi) The cube of a two digit number may have seven or more digits.

Sol: False

The cube of a two digit number may have 4 digits to 6 digits.

(vii) The cube of a single digit number may be a single digit number.

Sol: True

The cube of a single digit number may have 1 digit to 3 digits.

Comparing Quantities (Notes)

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1. Comparing two quantities of the same kind by division is called 'Ratio' of those quantities.
2. The ratio of two numbers $'a' and\ 'b' is\ a \div b = \frac{a}{b} = a:b$
3. $\boldsymbol{a}:\boldsymbol{b}$ *is read as* "a is to b" . $\boldsymbol{a}$ *is called Antecedent* , $\boldsymbol{b}$ *is called Consequent*.
4. **Proportion**: The equality of ratios is called proportion. If two ratios $a:b$ *and* $c:d$ are equal, then we represent it as $a:b :: c:d$ [We read 'a' is to 'b' is as 'c' is to 'd']
5. $a:b :: c:d$ *is also written as* $a:b = c:d$
6. If $a:b = c:d$ then a, b, c, d are in proportion. a, d are extremes and b, c are means.
7. $If\ a:b = c:d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = b \times c$

   (ie) The product of extremes= the product of Means
8. For any two ratios $a:b$ *and* $c:d$, the **compound ratio** is $\boldsymbol{a \times c : b \times d}$.
9. Direct proportion: In two quantities when one quantity increases, then the other also increases or if one quantity decreases, then the other also decreases in the same proportion, then the two quantities are said to be in direct proportion.
10. *When x and y are in direct proportion, then*

$$\frac{x}{y} = k \text{ or } x = k \times y \ \ (k \text{ is called constant of proportion})$$

11. Inverse Proportion :

If in two quantities, when one quantity increases, then the other quantity decreases in the same proportion or vice versa, then the two quantities are said to be in inverse proportion.

12. When x and y are in Inverse proportion, then

$$x \times y = k \quad or \quad x = \frac{k}{y} \quad or \quad y = \frac{k}{x}$$

13. Percentage: The word "percent" means "out of hundred". The symbol "%" is used to represent 'Percentage'.
14. $20\% = \frac{20}{100} = \frac{1}{5}[Simplified\ form] = 1:5[Ratio] = 0.2[Decimal\ form]$

PROFIT OR LOSS:

15. The price at which you sell is known as the 'Selling Price'. It is written in short as SP.
16. The buying price of any item is known as its 'Cost Price'. It is written in short as CP
17. If selling price is higher than Cost Price, then we get profit.(S.P>C.P)

$$Profit = S.P - C.P$$

18. If cost price is higher than selling price, then we get loss. (C.P>S.P)

$$Loss = C.P - S.P$$

19. Cost price is equal to Selling price, then we get neither profit nor loss

20. Profit Percentage $= \frac{\text{Profit}}{\text{Cost price}} \times 100\%$

21. Loss Percentage $= \frac{\text{Loss}}{\text{Cost price}} \times 100\%$

Discount

22. The price shown on the item is called the 'Marked price'(M.P)

23. The "Discount" is always calculated on 'Marked price'

24. Discount= Marked price –Selling price=M.P−S.P

25. Discount percentage $= \frac{\text{Discount}}{\text{Marked price}} \times 100\%$

Simple Interest:

26. The excess amount we paid on lending amount is called interest and the lending amount is called principal.

27. Simple Interest(I) $= \frac{\text{P}\times\text{T}\times\text{R}}{100}$

P= Principal amount , R= Rate of interest, T= time

28. Total amount$(A) = P\left(1 + \frac{TR}{100}\right)$

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number. The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

Sol: Let the total number of students=x

Girls=18

$60\%\ of\ x = 18$

$\frac{60}{100} \times x = 18$

$x = \frac{18 \times 100}{60} = 30$

Number of students=30

So, the number of boys = 30 – 18 = 12.

Method II:

$60\% \rightarrow 18$

$100\% \rightarrow x$

$x = \frac{100}{60} \times 18 = 30$

1. The ratio of the number of girls to the number of boys in the class=18:12=3:2.

2. The cost per head if two teachers are also going with the class?

Transportation charge = Distance both ways × Rate

= ₹ (55 × 2) × 12 = ₹ 110 × 12 = ₹ 1320

Total expenses = Refreshment charge + Transportation charge = ₹ 4280 + ₹ 1320 = ₹ 5600

Total number of persons =18 girls + 12 boys + 2 teachers = 32 persons

The amount spent for 1 person $= \frac{5600}{32} = ₹175$

3. **If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?**

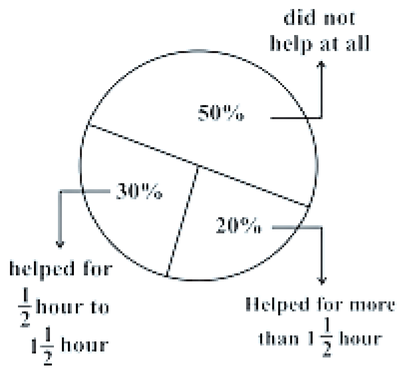
Sol: Total distance=55 km, Distance for fist stop=22km

Percentage of distance covered for first stop $= \frac{22}{55} \times 100\% = 2 \times 20\% = 40\%$

Therefore, the percent distance left to be travelled = 100% – 40% = 60%.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for more than $1\frac{1}{2}$ hours per day; 30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$hours; 50% did not help at all. Using this, answer the following: (i) How many parents were surveyed? (ii) How many said that they did not help? (iii) How many said that they helped for more than $1\frac{1}{2}$ hours?



Sol: Let number of parents=x

(i)Given number of parents helped for $\frac{1}{2}$ $hour\ to\ 1\frac{1}{2}\ hour = 90$

$30\%\ of\ x = 90$

$\frac{30}{100} \times x = 90$

$x = \frac{90 \times 100}{30} = 300$

Alternate method:

30% $\rightarrow$ 90

100% $\rightarrow$?

$= \frac{100}{30} \times 90 = 300$

(ii) Number of parents did not help=50% of 300

$= \frac{50}{\cancel{100}} \times \cancel{300} = 50 \times 3 = 150$

(ii) Number of parents helped for more than $1\frac{1}{2}\ hours = 20\%\ of\ 300$

$= \frac{20}{\cancel{100}} \times \cancel{300} = 20 \times 3 = 60$

EXERCISE 7.1

1. **Find the ratio of the following**

a) **Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.**

Sol: Ratio of speeds=15 km per hour : 30 km per hour

=15:30=1:2

b) 5 m to 10 km

Sol: Ratio=5 m : 10 km

=5m:10000m

=5:10000

=1:2000

1 km=1000 m

c) **50 paise to ₹ 5**

Sol: Ratio= 50 paise : ₹ 5

=50 paise : 500 paise

=50:500=1:10

₹1=100 paise

2. **Convert the following ratios to percentages.**

a) **3:4**

Sol: $3:4 = \frac{3}{4} = \frac{3}{4} \times 100\% = 3 \times 25\% = 75\%$

b) **2:3**

Sol: $2:3 = \frac{2}{3} = \frac{2}{3} \times 100\% = \frac{200}{3}\% = 66\frac{1}{6}\%$

3. **72% of 25 students are interested in mathematics. How many are not interested in mathematics?**

Sol: Total number of students= 25

Number of students interested in mathematics =72% of 25

$= \frac{72}{100} \times 25 = \frac{72}{4} = 18$

Number of students not interested in mathematics=25−18=7

Alternate method:

Number of students not interested in mathematics=(100−72)% of 25

$= 28\%\ of\ 25 = \frac{28}{100} \times 25 = \frac{28}{4} = 7$

4. **A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?**

Sol: Let the total number of matched played=x

Wined matches=10 and win percentage=40

$40\%\ of\ x = 10$

$\frac{40}{100} \times x = 10$

$x = \frac{10 \times 100}{40} = 25$

40% $\rightarrow$ 10
100% $\rightarrow$?
$= \frac{100}{40} \times 10 = 25$

5. **If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?**

Sol: Let the money beginning at Chameli=₹x

Spending=75%   left= 100-75=25%

$25\%\ of\ x = 600$

$$\frac{25}{100} \times x = 600$$

$$x = \frac{600 \times 100}{25} = 6004 = 2400$$

Hence Chameli had ₹ 2400

$25\% \rightarrow 600$

$100\% \rightarrow ?$

$$= \frac{100}{25} \times 600 = 2400$$

6. **If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.**

Sol: Cricket=60% , football=30%

The other games= (100-60-30)%=10%

The total number of people=50,00,000

Number of people who like cricket=60% of 50,00,000

$$= \frac{60}{1\cancel{00}} \times 50,00,0\cancel{00} = 60 \times 50,000 = 30,00,000$$

Number of people who like football=30% of 50,00,000

$$= \frac{30}{100} \times 50,00,000 = 30 \times 50,000 = 15,00,000$$

Number of people who like other games=10% of 50,00,000

$$= \frac{10}{100} \times 50,00,000 = 10 \times 50,000 = 5,00,000$$

Discounts

(i) Discount is a reduction given on the Marked Price (MP) of the article

(ii) Discount = Marked price – Sale price=MP-SP

(iii)Discount percent $= \frac{\text{Discount}}{\text{Marked price}} \times 100\%$

Example 2: An item marked at ₹840 is sold for ₹ 714. What is the discount and discount %?

Sol: Marked price (M.P)= ₹840

Sale price (S.P)= ₹714

Discount = Marked Price – Sale Price

= ₹ 840 – ₹ 714 = ₹ 126

$$\text{Discount percent} = \frac{\text{Discount}}{\text{Marked price}} \times 100\%$$

$$= \frac{126}{840} \times 100\% = 15\%$$

MP is 840 the discount is 126

MP is 100 the discount is ?

$$= \frac{126}{840} \times 100 = 15$$

Discount%=15%

Example 3: The list price of a frock is ₹220. A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.

Sol: The list price of a frock is $= 220$

Discount %= 20%

Discount $= 20\%\ of\ 220 = \frac{20}{100} \times 220 = ₹44$

Sale price $= 80\%\ of\ 220 = \frac{80}{100} \times 220 = ₹176$

Sale price=List price−Discount =220−44=176

TRY THESE

1. A shop gives 20% discount. What would the sale price of each of these be?

(a) A dress marked at ₹ 120

Sol: MP of dress=₹120

Discount =20% of 120

$= \frac{2\cancel{0}}{10\cancel{0}} \times 12\cancel{0} = 2 \times 12 = ₹24$

Sale price=Marked price-Discount

$= 120 - 24 = ₹96$

Alternate Method: If MP is ₹100 then SP is ₹80 When MP is ₹120 then SP is ? $SP = \frac{8\cancel{0}}{10\cancel{0}} \times 12\cancel{0} = 8 \times 12 = ₹96$

(b) A pair of shoes marked at ₹ 750

Sol: MP of shoes =₹750

Discount =20% of 750

$= \frac{20}{100} \times 750 = 2 \times 75 = ₹150$

Sale price=Marked price−Discount

$= 750 - 150 = ₹600$

Alternate Method: If MP is ₹100 then SP is ₹80 If MP is ₹ 750 then SP is ? $SP = \frac{80}{100} \times 750 = 8 \times 75 = ₹600$

(c) A bag marked at ₹ 250

Sol: MP of a bag =₹250

Discount =20% of 250

$= \frac{20}{100} \times 250 = 2 \times 25 = ₹50$

Sale price=Marked price−Discount

$= 250 - 50 = ₹200$

Alternate Method: If MP is ₹100 then SP is ₹80 If MP is ₹ 250 then SP is ? $SP = \frac{80}{100} \times 250 = 8 \times 25 = ₹200$

2. A table marked at ₹15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.

Sol: MP of a table=₹15000

SP of table=₹14400

Discount=MP−SP

=15000−14400=₹600

$$\text{Discount percente} = \frac{\text{Discount}}{\text{MP}} \times 100\%$$

$$= \frac{60\cancel{0}}{150\cancel{00}} \times 1\cancel{00} = \frac{60}{15} = 4\ \%$$

3. An Amirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

Sol: SP of Amirah =₹5225

Discount percent=5%

If SP is ₹95 then MP is ₹100

If SP is ₹5225 then MP is ?

$$\text{MP of almirah} = \frac{100}{95} \times 5225 = 100 \times 55 = ₹5500$$

Sales Tax/Value Added Tax/Goods and Services Tax

(i) Sales tax is charged on the sale of an item by the government and is added to the Bill Amount.

Sales tax = Tax% of Bill Amount

(ii) GST stands for Goods and Services Tax and is levied on supply of goods or services or both

(iii) There is another type of tax which is included in the prices known as Value Added Tax (VAT).

Example 4: (Finding Sales Tax) The cost of a pair of roller skates at a shop was ₹ 450. The sales tax charged was 5%. Find the bill amount.

Sol: CP of a pair of roller skates=₹450

Sales tax=5%

$$\text{Sales tax on roller skates} = 5\%\ of\ 450 = \frac{5}{100} \times 450 = ₹22.50$$

$$\text{Bill amount} = \text{CP} + \text{sales tax} = ₹450 + ₹22.50 = ₹472.50$$

Example 5: (Value Added Tax (VAT)) Waheeda bought an air cooler for ₹ 3300 including a tax of 10%. Find the price of the air cooler before VAT was added.

Sol: If the price without VAT is ₹100 then price including VAT is ₹ 110.

Now, when price including VAT is ₹ 110, original price is ₹ 100

Hence when price including tax is ₹ 3300, the original price $= \frac{100}{110} \times 3300 = ₹3000$

Example 6: Salim bought an article for ₹784 which included GST of 12% . What is the price of the article before GST was added?

Sol: GST=12%

When the selling price is ₹ 112 then original price = ₹ 100.

When the selling price is ₹ 784, then original price $= \frac{100}{112} \times 784 = ₹700$

The price of the article before GST= ₹700

THINK, DISCUSS AND WRITE

1. **Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?**

Sol: let the number be 100

If the number is 100% increased the new number=100+100=200=Two times the number.

If we take half the number the new number=50

The number decrease in 50%

2. **By what per cent is ₹2,000 less than ₹2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?**

Sol: The percent is₹ 2000 less than ₹2400 $= \frac{2400-2000}{2400} \times 100$

$$= \frac{400}{2400} \times 100 = \frac{100}{6} = 16\frac{2}{3}\% \ \ or\ 16.66\%$$

The percent by which₹2400 is more than₹2000 $= \frac{2400-2000}{2000} \times 100$

$$= \frac{400}{2000} \times 100$$

$$= 20\%$$

Therefore the given percent are not same

EXERCISE 7.2

1. **During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?**

Sol: MP of a jean=1450, MP of a shirt=₹850

Total MP of pair of jeans and two shirts= $1450 + 2 \times 850 = 1450 + 1700 = ₹3150$

MP=₹3150

Discount percent=10%

Discount =10% of 3150

$$= \frac{10}{100} \times 3150 = ₹315$$

$SP = MP - Discount = 3150 - 315 = 2835$

The customer would have to pay ₹2835

(OR)

MP=1450+2×850=1450+1700=₹3150

Discount percent=10%

If MP is ₹100 then SP is ₹90

When MP is ₹3150 then SP $= \frac{90}{100} \times 3150 = 9 \times 315 = ₹2835$

2. The price of a TV is ₹13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.

Sol: The price of a TV is ₹ 13,000

Sales tax percentage=12%

Tax amount=12% of 13000

$= \frac{12}{100} \times 13000 = 12 \times 130 = ₹1560$

Bill amount=Price of TV+ tax amount

=13000+1560=₹14560

Vinod will have to pay 14560 if he buys it

(OR)

The price of a TV is ₹ 13,000

Sales tax percentage=12%

If price is ₹100 then bill amount is ₹112

When price is ₹13000 then bill amount $= \frac{112}{100} \times 13000$

$= 112 \times 130 = ₹14560$

3. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.

Sol: SP of a pair of skates = 1600

Discount percent=20%

If SP is ₹80 then MP is ₹100

When SP is ₹1600 then MP $= \frac{100}{80} \times 1600 = 100 \times 20 = ₹2000$

Marked price=₹2000

4. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.

Sol: Amount paid=₹5,400

VAT percent=8%

If bill amount is ₹108 then the price before VAT is ₹100

When amount paid is ₹5400 then the price before VAT $= \frac{100}{108} \times 5400$

$= 100 \times 50 = ₹5000$

The original price of hair dryer=₹5000

5. An article was purchased for ₹1239 including GST of 18%. Find the price of the article before GST was added?

Sol: Purchased amount of article=₹1239

GST percent=18%

If GST added amount is₹ 118 then the before GST is ₹100

When GST added amount is ₹1239 then

The price of article before GST $= \frac{100}{118} \times 1239 = \frac{50}{59} \times 1239 = 50 \times 21 =$ ₹1050

Original price=₹1050

Compound Interest

(i) Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money

(ii) The interest is calculated on the amount of the previous year. This is known as interest compounded or Compound Interest (C.I.)

(iii) Amount when interest is compounded annually

$$A = P\left(1 + \frac{R}{100}\right)^n$$

A=amount, P = principal, R = rate of interest, n = time period (number of years)

(iv) Amount when interest is compounded half yearly

$$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

(v) Amount when interest is compounded quarterly

$$A = P\left(1 + \frac{R}{400}\right)^{4n}$$

Example 8: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Sol: Principal (P) = ₹ 12600, Rate (R) = 10, Number of years (n) = 2

$$A = P\left(1 + \frac{R}{100}\right)^n$$

$$= 12600\left(1 + \frac{10}{100}\right)^2$$

$$= 12600\left(1 + \frac{1}{10}\right)^2$$

$$= 12600\left(\frac{11}{10}\right)^2$$

$$= 12600 \times \frac{11}{10} \times \frac{11}{10} = 126 \times 121 = ₹15246$$

CI = A – P = ₹15246 – ₹12600 = ₹ 2646

TRY THESE

1. **Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.**

Sol: Principal (P) = ₹8000

Rate of interest(R) =5

Time period (n) =2

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{5}{100}\right)^2$$

$$= 8000\left(1+\frac{1}{20}\right)^2$$

$$= 8000\left(\frac{21}{20}\right)^2$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= 20 \times 21 \times 21 = ₹8820$$

$$\text{CI} = \text{A} - \text{P} = ₹8820 - ₹8000 = ₹820$$

Applications of Compound Interest Formula

(i) Increase (or decrease) in population.

(ii) The growth of a bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 9: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

Sol: Population (P) =20,000

Rate of increase(R) =5%

Time (n) =2 years

$$\text{Population after 3 years} = P\left(1+\frac{R}{100}\right)^n$$

$$= 20000\left(1+\frac{5}{100}\right)^3$$

$$= 20000\left(1+\frac{1}{20}\right)^3 = 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 23152.5$$

The population at the end of the year 2000 =23153

Example 10: A TV was bought at a price of ₹ 21,000. After one year the value of the TV was depreciated by 5% (Depreciation means reduction of value due to use and age of the item). Find the value of the TV after one year.

Sol: Price of TV(P)= ₹21,000

Depreciate=5% i.e. R=−5%

Time period (n)=1

Value of the TV at the end of 1 year $= 21000\left(1-\frac{5}{100}\right) = 21000 \times \frac{95}{100}$

$= 210 \times 95 =$ ₹19,950

TRY THESE

1. **A machinery worth ₹ 10,500 depreciated by 5%. Find its value after one year.**

Sol: Principal= ₹10500

Reduction for one year=5% of 10500

$$= \frac{5}{100} \times 10500 = 5 \times 105 = ₹525$$

Value after one year=10500−525= ₹9975

(OR)

Principal(P)= ₹10500

Reduction=5%

Period (n)=1

Valua after one year $= P\left(1-\frac{R}{100}\right)^n$

$$= 10500\left(1-\frac{5}{100}\right)^1$$

$$= 10500 \times \frac{95}{100} = 105 \times 95 = ₹9975$$

2. **Find the population of a city after 2 years, which is at present 12 lakh, if the rate of increase is 4%.**

Sol: Present population (P)= 12,00,000

Rate of increase(R) =4%

Period (n) =2

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 12,00,000\left(1+\frac{4}{100}\right)^2$$

$$= 12,00,000\left(\frac{104}{100}\right)^2$$

$$= 12,00,000 \times \frac{104}{100} \times \frac{104}{100}$$

$$= 120 \times 104 \times 104 = 12,97,920$$

The population after 2 years= 12,97,920

EXERCISE 7.3

1. **The population of a place increased to 54,000 in 2003 at a rate of 5% per annum (i) find the population in 2001. (ii) what would be its population in 2005?**

Sol: (i) Population (A) =54,000

Increased rate(R)=5%

n=2

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$54{,}000 = P\left(1+\frac{5}{100}\right)^2$$

$$54{,}000 = P\left(1+\frac{5}{100}\right)^2$$

$$54{,}000 = P\left(1+\frac{1}{20}\right)^2$$

$$54{,}000 = P\left(\frac{21}{20}\right)^2$$

$$P = 54000 \times \frac{20}{21} \times \frac{20}{21} = 48979.6$$

The population in 2001=48980

(ii) Population (P) =54,000

Increased rate(R)=5%

n=2 (Population after 2 years)

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 54000\left(1+\frac{5}{100}\right)^2$$

$$= 54000\left(1+\frac{1}{20}\right)^2$$

$$= 54000\left(\frac{21}{20}\right)^2$$

$$= 54000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= 135 \times 441 = 59535$$

The population in 2005 = 59535.

2. **In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5, 06,000.**

Sol: Initially count of bacteria (P)=5,06,000

Increasing rate per hour$(R) = 2.5\% = \frac{25}{10}\%$

n=2

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 506000\left(1+\frac{\frac{25}{10}}{100}\right)^2$$

$$= 506000\left(1+\frac{25}{1000}\right)^2$$

$$= 506000\left(1+\frac{1}{40}\right)^2$$

$$= 506000\left(\frac{41}{40}\right)^2$$

$$= 506000 \times \frac{41}{40} \times \frac{41}{40}$$

$$= \frac{1265 \times 1681}{4} = 531616.25$$

The bacteria at the end of 2 hours=531616

3. **A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.**

Sol: Cost price of scooter (P)=42,000

Rate of depreciation=8% $\therefore$ R=−8% , n=1

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 42{,}000 \times \left(1-\frac{8}{100}\right)^1$$

$$= 42000\left(1-\frac{2}{25}\right)$$

$$= 42000 \times \frac{23}{25}$$

$$= 1680 \times 23 = 38640$$

The value of the scooter after 1 year=38640

CHAPTER 8

VIII CLASS-NCERT (2023-24)

ALGEBRAIC EXPRESSIONS AND IDENTITIES (Notes)

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1. A variable can take various values and its value cannot be fixed. a, b, x, y, z etc. A constant has a fixed value. For example 6, 8, –10 etc., are some constants
2. Terms are formed as a product of constants and one or more variables.
3. Terms are added or subtracted to form an expression.

(OR)

Expression: An expression is a constant or a variable or combination of these two, using the mathematical operations (+, –, ×, ÷) i.e., terms are added to form expressions.

Examples of expressions are: $2x - 5, -4y + 2, 5x^2, -2xy + 2x + 3y + 7$ etc

4. If an expression has at least one algebraic term, then that expression is **Algebraic expression.**

5. The sum of all exponents of the variables in a monomial is the **degree** of the monomial
6. The highest degree among the degrees of the different terms of an algebraic expression is called the degree of that algebraic expression.
7. **Monomial**: Expression that contains only one term is called a monomial.
   $Ex: 4x^2, 5xy, -8z, 5xy^2, 10y, \ldots$
8. **Binomial**: Expression that contains two terms is called a binomial.
   $Exp: x + y, a + b, 4l + 5m, 5 - 3xy, ..$
9. **Trinomial**: An expression containing three terms is a trinomial.
   $Exp: a + b + c, 2x + 3y - 5, xy + x^2 + y^2, \ldots$
10. **Polynomial**: An expression containing, one or more terms with non-zero coefficient (with variables having non-negative integers [whole numbers]as exponents) is called a polynomial
    $Exp: a + b + c + d, 3xy, 2x + 5y, \ldots$
11. **Like and Unlike Terms**:
    The terms have same variable with same exponents (powers) are called like terms.
    $Exp: (i)\ 2x, 5x, -7x \quad (ii) - 3x^2y, 7x^2y, \frac{2}{3}x^2y$
    Like terms may not have same numerical coefficients.
12. A monomial multiplied by a monomial always gives a monomial.
13. In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them

Addition and Subtraction of Algebraic Expressions.

1. We can only combine like terms by adding or subtracting them with one another.
2. Unlike terms cannot combine by adding or subtracting.
3. Subtraction of a number is the same as addition of its additive inverse.

Example 1: Add: $7xy + 5yz - 3zx, 4yz + 9zx - 4y, -3xz + 5x - 2xy$.

Sol:

$$\begin{array}{llll} & 7xy + 5yz - 3zx, & & \\ + & \quad\;\; 4yz + 9zx - 4y & & \\ + & -2xy \quad\;\; -3xz & & +5x \\ \hline & 5xy + 9yz + 3zx - 4y + 5x & & \\ \hline \end{array}$$

Example 2: Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$.

Sol:

$$\begin{array}{ccccccc} 7x^2 & -4xy & +8y^2 & +5x & -3y. & & \\ 5x^2 & & -4y^2 & & +6y & -3 & \\ (-) & & (+) & & (-) & (+) & \\ \hline 2x^2 & -4xy & +12y^2 & +5x & -9y & +3. & \\ \hline \end{array}$$

EXERCISE 8.1

1. Add the following

(i) $\boldsymbol{ab - bc, bc - ca, ca - ab}$

Sol: $(ab - bc) + (bc - ca) + (ca - ab) = ab - ab - bc + bc - ca + ca = 0$

(ii) $\boldsymbol{a - b + ab, b - c + bc, c - a + ac}$

Sol: $a - b + ab + b - c + bc + c - a + ac$

$= a - a - b + b + ab - c + c + bc + ac = ab + bc + ca$

(iii) $\boldsymbol{2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2}$

Sol: $2p^2q^2 - 3pq + 4 + 5 + 7pq - 3p^2q^2$

$= 2p^2q^2 - 3p^2q^2 - 3pq + 7pq + 4 + 5$

$= -p^2q^2 + 4pq + 9$

(iv) $\boldsymbol{l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl}$

Sol: $l^2 + m^2 + m^2 + n^2 + n^2 + l^2 + 2lm + 2mn + 2nl$

$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

$= 2(l^2 + m^2 + n^2 + lm + mn + nl)$

2. (a) **Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$**

Sol: $(12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$

$$\boxed{\begin{array}{l} -(+) = - \\ -(-) = + \end{array}}$$

$= 12a - 9ab + 5b - 3 - 4a - 12$

$= 12a - 4a - 9ab + 7ab + 5b - 3b - 3 - 12$

$= 8a - 2ab + 2b - 15$

(or)

$$\begin{array}{rrrr} 12a & -\ 9ab & +\ 5b & -\ 3 \\ 4a & -\ 7ab & +\ 3b & +\ 12 \\ (-) & (+) & (-) & (-) \\ \hline 8a & -\ 2ab & +\ 2b & -\ 15 \\ \hline \end{array}$$

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

Sol: $(5xy - 2yz - 2zx + 10xyz) - (3xy + 5yz - 7zx)$

$= 5xy - 2yz - 2zx + 10xyz - 3xy - 5yz + 7zx$

$= 5xy - 3xy - 2yz - 5yz - 2zx + 7zx + 10xyz$

$= 2xy - 7yz + 5zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Sol: $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

$= 18 + 10 - 3p + 8p - 11q - 7q + 5pq + 3pq - 2pq^2 - 5pq^2 + 5p^2q - 4p^2q$

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

(or)

$$\begin{array}{l} 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\ -10 - 8p - 7q - 3pq + 5pq^2 + 4p^2q \\ (+)\ (+)\ \ (+)\ \ \ (+)\ \ \ \ (-)\ \ \ \ \ (-) \\ \hline 28 + 5p - 18q + 8pq - 7pq^2 + p^2q \\ \hline \end{array}$$

8.2 Multiplication of Algebraic Expressions: Introduction:

Multiplying two monomials:

(i) $x \times x = x^2$; $x \times x^2 = x^3$; $x^2 \times x^3 = x^5$

(ii) $4 \times 3x = 4 \times 3 \times x = 12x$

(iii) $5x \times 3y = 5 \times 3 \times x \times y = 15xy$

(iv) $(-7x) \times 5y = (-7) \times 5 \times x \times y = -35xy$

(v) $5x \times 4x^2 = 5 \times 4 \times x \times x^2 = 20x^3$

(vi) $6x \times (-7xyz) = 6 \times (-7) \times x \times xyz = -42x^2yz$

Multiplying three or more monomials:

(i) $2x \times 5y \times 7z = 2 \times 5 \times 7 \times x \times y \times z = 70xyz$

(ii) $4xy \times 5x^2y^2 \times 6x^3y^3 = 4 \times 5 \times 6 \times x \times x^2 \times x^3 \times y \times y^2 \times y^3 = 120x^6y^6$

TRY THESE

Find $4x \times 5y \times 7z$ First find $4x \times 5y$ and multiply it by $7z$; or first find $5y \times 7z$ and multiply it by $4x$. Is the result the same? What do you observe? Does the order in which you carry out the multiplication matter?

Sol: $4x \times 5y = 4 \times 5 \times x \times y = 20xy$

$(4x \times 5y) \times 7z = 20xy \times 7z = 20 \times 7 \times xy \times z = 140xyz$

$5y \times 7z = 5 \times 7 \times y \times z = 35yz$

$4x \times (5y \times 7z) = 4x \times 35yz = 4 \times 35 \times x \times yz = 140xyz$

We observe the order in which multiply the monomials does not matter.

Also the multiplication of monomials is associative.

Example 3: Complete the table for area of a rectangle with given length and breadth.

Sol:

length	**breadth**	**Area= length× breadth**
$3x$	$5y$	$3x \times 5y = 3 \times 5 \times x \times y = 15xy$
$9y$	$4y^2$	$9y \times 4y^2 = 9 \times 4 \times y \times y^2 = 36y^3$
$4ab$	$5bc$	$4ab \times 5bc = 4 \times 5 \times a \times b \times b \times c = 20ab^2c$
$2l^2m$	$3lm^2$	$2l^2m \times 3lm^2 = 2 \times 3 \times l^2 \times l \times m \times m^2 = 6l^3m^3$

Example 4: Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Sol: Volume = length × breadth × height

(i) Volume = $(2ax) \times (3by) \times (5cz) = 2 \times 3 \times 5 \times a \times b \times c \times x \times y \times z = 30abcxyz$

(ii) Volume = $(m^2n) \times (n^2p) \times (p^2m) = m^2 \times m \times n \times n^2 \times p \times p^2 = m^3n^3p^3$

(iii) Volume = $2q \times 4q^2 \times 8q^3 = 2 \times 4 \times 8 \times q \times q^2 \times q^3 = 64q^6$

EXERCISE 8.2

1. **Find the product of the following pairs of monomials.**

(i) 4, 7p

Sol: $4 \times 7p = 28p$

(ii) – 4p, 7p

Sol: $-4p \times 7p = (-4 \times 7) \times p \times p = -28p^2$

(iii) – 4p, 7pq

Sol: $-4p \times 7pq = (-4 \times 7) \times p \times p \times q = -28p^2q$

(*iv*) $4p^3$, – $3p$

Sol: $4p^3 \times (-3p) = (4 \times -3) \times p^3 \times p = -12p^4$

(*v*) $4p, 0$

Sol: $4p \times 0 = 0$

2. **Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.**

(i) (p,q) $\boldsymbol{Length(l) = p \ \ and \ \ breadth(b) = q}$

Sol: Area of rectangle = l × b = p × q = pq squnits

(ii) (10m,5n) $\boldsymbol{Length(l) = 10m \ \ and \ \ breadth(b) = 5n}$

Sol: Area of rectangle $= l \times b =$ 10m × 5n = 50mn squnits

(iii) $(\boldsymbol{20x^2, 5y^2})$ $Length(l) = 20x^2 \ \ and \ \ breadth(b) = 5y^2$

Sol: Area of rectangle $= l \times b = 20x^2 \times 5y^2 = 100x^2y^2$ squnits

(iv) $(\boldsymbol{4x, 3x^2})$ $\boldsymbol{Length(l) = 4x \ \ and \ \ breadth(b) = 3x^2}$

Sol: Area of rectangle $= l \times b = 4x \times 3x^2 = 12x^3$ squnits

(v) $(\boldsymbol{3mn, 4np})$ $\boldsymbol{Length(l) = 3mn \ \ and \ \ breadth(b) = 4np}$

Sol: Area of rectangle $= l \times b = 3mn \times 4np = 12mn^2p$ squnits

3. **Complete the table of products.**

First monomial→ / Second monomial↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-36x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-36x^4y^3$	$81x^4y^4$

4. **Obtain the volume of rectangular boxes with the following length, breadth and height respectively**

(i) $\boldsymbol{5a, 3a^2, 7a^4}$

sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h = 5a \times 3a^2 \times 7a^4$

$= 5 \times 3 \times 7 \times a \times a^2 \times a^4 = 105a^7\ cubic\ units$

(ii) $\boldsymbol{2p, 4q, 8r}$

sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h = 2p \times 4q \times 8r$

$= 2 \times 4 \times 8 \times p \times q \times r = 64pqr\ \ cubic\ units$

(iii) $\boldsymbol{xy, 2x^2y, 2xy^2}$

sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h = xy \times 2x^2y \times 2xy^2$

$= 2 \times 2 \times x \times x^2 \times x \times y \times y \times y^2 cubic\ units$

$= 4x^4y^4\ cubic\ units$

(iv) $\boldsymbol{a, 2b, 3c}$

sol: $Volume\ of\ rectangular\ box(cuboid) = l \times b \times h = a \times\ 2b \times 3c$

$= 2 \times 3 \times a \times b \times c = 6abc$ *cubic units*

5. **Obtain the product of**

(i) $\boldsymbol{xy, yz, zx}$

sol: Product $= xy \times yz \times zx = x^2y^2z^2$

(ii) $\boldsymbol{a, -a^2, a^3}$

Sol: Product $= a \times (-a^2) \times a^3 = -a^6$

(iii) $\boldsymbol{2, 4y, 8y^2, 16y^3}$

sol: Product $= 2 \times 4y \times 8y^2 \times 16y^3 = 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3 = 1024y^6$

(iv) $\boldsymbol{a, 2b, 3c, 6abc}$

sol: Product $= a \times 2b \times 3c \times 6abc = 2 \times 3 \times 6 \times a \times b \times c \times abc = 36a^2b^2c^2$

(v) $\boldsymbol{m, -mn, mnp}$

sol: Product $= m \times (-mn) \times mnp = -m^3n^2p$

Multiplying a monomial by a binomial, a trinomial:

Distributive law

$a(b+c) = a \times b + a \times c$

$a(b+c+d) = a \times b + a \times c + a \times d$

$a(b-c) = a \times b - a \times c$

Commutative law

$a \times b = b \times a$

TRY THESE

(i) **Find the product** $\boldsymbol{2x\,(3x + 5xy)}$

Sol: $2x\,(3x + 5xy) = (2x \times 3x) + (2x \times 5xy) = 6x^2 + 10x^2y$

(ii) ***Find the product*** $\boldsymbol{a^2\,(2ab - 5c)}$

sol: $a^2\,(2ab - 5c) = (a^2 \times 2ab) - (a^2 \times 5c) = 2a^3b - 5a^2c$

(iii) ***Find the product*** $\boldsymbol{(4p^2 + 5p + 7) \times 3p}$

Sol: $(4p^2 + 5p + 7) \times 3p = 3p \times (4p^2 + 5p + 7)$

$= (3p \times 4p^2) + (3p \times 5p) + (3p \times 7)$

$= 12p^3 + 15p^2 + 21p$

Example 5: Simplify the expressions and evaluate them as directed:

(i) $\boldsymbol{x\,(x - 3) + 2 \text{ for } x = 1}$

Sol: $x\,(x-3) + 2 = x \times x - x \times 3 + 2 = x^2 - 3x + 2$

For $x = 1$,

$x^2 - 3x + 2 = (1)^2 - 3(1) + 2$

$= 1 - 3 + 2 = 3 - 3 = 0$

(ii) $\boldsymbol{3y\,(2y - 7) - 3\,(y - 4) - 63 \text{ for } y = -2}$

Sol: $3y\,(2y - 7) - 3\,(y - 4) - 63$

$= 3y \times 2y - 3y \times 7 - 3 \times y - 3 \times (-4) - 63$

$= 6y^2 - 21y - 3y + 12 - 63$

$= 6y^2 - 24y - 51$

$For \quad y = -2,$

$6y^2 - 24y - 51 = 6(-2)^2 - 24(-2) - 51$

$= 6 \times 4 + 24 \times 2 - 51$

$= 24 + 48 - 51 = 72 - 51 = 21$

Example 6: Add

$(\boldsymbol{i})$ $\quad \boldsymbol{5m\,(3 - m) and\ 6m^2 - 13m}$

Sol: $5m\left(3 - m\right) = 5m \times 3 - 5m \times m = 15m - 5m^2$

$5m\left(3 - m\right) + 6m^2 - 13m$

$= 15m - 5m^2 + 6m^2 - 13m$

$= -5m^2 + 6m^2 + 15m - 13m$

$= m^2 + 2m$

$(\boldsymbol{ii})$ $\quad \boldsymbol{4y\,(3y^2 + 5y - 7) and\ 2\,(y^3 - 4y^2 + 5)}$

Sol: $4y\left(3y^2 + 5y - 7\right) = (4y \times 3y^2) + (4y \times 5y) - (4y \times 7)$

$= 12y^3 + 20y^2 - 28y$

$2\left(y^3 - 4y^2 + 5\right) = (2 \times y^3) - (2 \times 4y^2) + (2 \times 5)$

$= 2y^3 - 8y^2 + 10$

Sum$= 12y^3 + 20y^2 - 28y + 2y^3 - 8y^2 + 10$

$= 12y^3 + 2y^3 + 20y^2 - 8y^2 - 28y + 10$ (OR)

$= 14y^3 + 12y^2 - 28y + 10$

$$\begin{array}{rrrr} & 12y^3 + 20y^2 - 28y & \\ (+) & 2y^3 - 8y^2 & + 10 \\ \hline & 14y^3 - 12y^2 - 28y + 10 & \\ \hline \end{array}$$

Example 7: Subtract $\boldsymbol{3pq\,(p - q)}$ from $\boldsymbol{2pq\,(p + q)}$.

Sol: $3pq\left(p - q\right) = 3pq \times p - 3pq \times q = 3p^2q - 3pq^2$

$2pq\,(p + q) = 2pq \times p + 2pq \times q = 2p^2q + 2pq^2$

Subtracting,

$$\begin{array}{rr} & 2p^2q + 2pq^2 \\ & 3p^2q - 3pq^2 \\ (-) & (+) \\ \hline & -p^2q + 5pq^2 \\ \hline \end{array}$$

EXERCISE 8.3

1. Carry out the multiplication of the expressions in each of the following pairs.

$(\boldsymbol{i})$ $\quad \boldsymbol{4p, q + r}$

sol: $4p \times (q + r) = (4p \times q) \times (4p \times r) = 4pq + 4pr$

$(\boldsymbol{ii})$ $\quad \boldsymbol{ab, a - b}$

Sol: $ab \times (a - b) = (ab \times a) - (ab \times b) = a^2b - ab^2$

$(\boldsymbol{iii})$ $\boldsymbol{a + b, 7a^2b^2}$

sol: $7a^2b^2 \times (a + b) = (7a^2b^2 \times a) + (7a^2b^2 \times b) = 7a^3b^2 + 7a^2b^3$

$(\boldsymbol{iv})$ $\boldsymbol{a^2 - 9, 4a}$

Sol: $4a(a^2 - 9) = (4a \times a^2) - (4a \times 9) = 4a^3 - 36a$

$\boldsymbol{v)}$ $\boldsymbol{pq + qr + rp, 0}$

Sol: $0 \times (pq + qr + rp) = 0$

2. Complete the table

(i) $\boldsymbol{a \times (b + c + d)} = (a \times b) + (a \times c) + (a \times d)$

$= ab + ac + ad$

(ii) $\boldsymbol{5xy \times (x + y - 5)} = (5xy \times x) + (5xy \times y) - (5xy \times 5) = 5x^2y + 5xy^2 - 25xy$

(iii) $\boldsymbol{p \times (6p^2 - 7p + 5)} = (p \times 6p^2) - (p \times 7p) + (p \times 5) = 6p^3 - 7pq + 5p$

(iv) $\boldsymbol{4p^2q^2(p^2 - q^2)} = (4p^2q^2 \times p^2) - (4p^2q^2 \times q^2) = 4p^4q^2 - 4p^2q^4$

(v) $\boldsymbol{abc \times (a + b + c)} = (abc \times a) + (abc \times b) + (abc \times c) = a^2bc + ab^2c + abc^2$

3. Find the product

$(\boldsymbol{i})$ $\boldsymbol{(a^2) \times (2a^{22}) \times (4a^{26})}$

Sol: $(a^2) \times (2a^{22}) \times (4a^{26}) = (1 \times 2 \times 4) \times (a^2 \times a^{22} \times a^{26}) = 8a^{50}$

$(\boldsymbol{ii})$ $\boldsymbol{\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)}$

Sol: $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right) = \left(\frac{2}{3} \times \frac{-9}{10}\right) \times (x \times x^2 \times y \times y^2) = \frac{-3}{5}x^3y^3$

$(\boldsymbol{iii})$ $\boldsymbol{\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right) = \left(-\frac{10}{3} \times \frac{6}{5}\right) \times (p \times p^3) \times (q^3 \times q) = -4p^4q^4}$

(iv) $x \times x^2 \times x^3 \times x^4 = x^{1+2+3+4} = x^{10}$

4. (a)Simplify $3x(4x - 5) + 3$ *and find its values for* $(i)x = 3$ $(ii)x = \frac{1}{2}$.

sol: $3x(4x - 5) + 3 = (3x \times 4x) - (3x \times 5) + 3 = 12x^2 - 15x + 3$

(i) $For\ x = 3,$

$12x^2 - 15x + 3 = 12(3)^2 - 15(3) + 3$

$= 12 \times 9 - 45 + 3 = 108 - 42 = 66$

(ii) $For\ x = \frac{1}{2}$

$12x^2 - 15x + 3 = 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3$

$= 12 \times \frac{1}{4} - 15 \times \frac{1}{2} + 3 = 3 - \frac{15}{2} + 3 = 6 - \frac{15}{2} = \frac{12 - 15}{2} = \frac{-3}{2}$

$(\boldsymbol{b})$ ***Simplify*** $\boldsymbol{a(a^2 + a + 1) + 5}$ ***and find its value for*** $\boldsymbol{(i)a = 0, (ii)a = 1}$

$(iii)\ a = -1$

$Sol:\ a(a^2 + a + 1) + 5 = (a \times a^2) + (a \times a) + (a \times 1) + 5 = a^3 + a^2 + a + 5$

$(i)\ For\ a = 0$

$a^3 + a^2 + a + 5 = 0^3 + 0^2 + 0 + 5 = 5$

$(ii)\ For\ a = 1$

$a^3 + a^2 + a + 5 = 1^3 + 1^2 + 1 + 5 = 1 + 1 + 1 + 5 = 8$

$(iii)\ For\ a = 1$

$a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5 = -1 + 1 - 1 + 5 = 4$

5. (a) Add: $p(p-q), q(q-r)$ and $r(r-p)$

$Sol:\ p(p-q) = p \times p - p \times q = p^2 - pq$

$q(q-r) = q \times q - q \times r = q^2 - qr$

$r(r-p) = r \times r - r \times p = r^2 - rp$

$Sum = p^2 - pq + q^2 - qr + r^2 - rp = p^2 + q^2 + r^2 - pq - qr - rp$

(b) Add: $2x(z-x-y)$ and $2y(z-y-x)$

Sol: $2x(z-x-y) + 2y(z-y-x)$

$= (2x \times z) - (2x \times x) - (2x \times y) + (2y \times z) - (2y \times y) - (2y \times x)$

$= 2xz - 2x^2 - 2xy + 2yz - 2y^2 - 2xy$

$= -2x^2 - 2y^2 - 4xy + 2yz + 2xz$

(c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

$Sol:\ 4l(10n - 3m + 2l) = (4l \times 10n) - (4l \times 3m) + (4l \times 2l)$

$= 40ln - 12lm + 8l^2$

$3l(l - 4m + 5n) = (3l \times l) - (3l \times 4m) + (3l \times 5n)$

$= 3l^2 - 12lm + 15ln$

$Now\ (40ln - 12lm + 8l^2) - (3l^2 - 12lm + 15ln)$

$= 40ln - 12lm + 8l^2 - 3l^2 + 12lm - 15ln$

$= 40ln - 15ln - 12lm + 12lm + 8l^2 - 3l^2$

$= 25ln + 5l^2$

(d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

$Sol:\ 4c(-a + b + c) = (4c \times -a) + (4c \times b) + (4c \times c) = -4ac + 4bc + 4c^2$

$3a(a + b + c) - 2b(a - b + c) = (3a^2 + 3ab + 3ac) - (2ab - 2b^2 + 2bc)$

$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc = 3a^2 + 2b^2 + ab + 3ac - 2bc$

$Now(-4ac + 4bc + 4c^2) - (3a^2 + 2b^2 + ab + 3ac - 2bc)$

$= -4ac + 4bc + 4c^2 - 3a^2 - 2b^2 - ab - 3ac + 2bc$

$= -3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

Multiplying a binomial by a binomial:

$(a+b) \times (c+d) = a \times (c+d) + b \times (c+d) = a \times c + a \times d + b \times c + b \times d$

first term *second term*

$$(a+b) \times (c+d)$$

third term *fourth term*

(In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them).

Example 8: Multiply

(i) $(\boldsymbol{x - 4})$ ***and*** $(\boldsymbol{2x + 3})$

Sol: $(x - 4) \times (2x + 3)$

$= x \times (2x + 3) - 4 \times (2x + 3)$

$= (x \times 2x) + (x \times 3) - (4 \times 2x) - (4 \times 3)$

$= 2x^2 + 3x - 8x - 12$

$= 2x^2 - 5x - 12$

(ii) $(\boldsymbol{x - y})$ ***and*** $(\boldsymbol{3x + 5y})$

Sol: $(x - y) \times (3x + 5y) = x \times (3x + 5y) - y \times (3x + 5y)$

$= (x \times 3x) + (x \times 5y) - (y \times 3x) - (y \times 5y)$

$= 3x^2 + 5xy - 3xy - 5y^2$

$= 3x^2 + 2xy - 5y^2$

Example 9: Multiply

(i) $(\boldsymbol{a + 7})$***and*** $(\boldsymbol{b - 5})$

Sol: $(a + 7) \times (b - 5) = a \times (b - 5) + 7 \times (b - 5)$

$= a \times b - a \times 5 + 7 \times b - 7 \times 5$

$= ab - 5a + 7b - 35$

(ii) $(\boldsymbol{a^2 + 2b^2})$***and*** $(\boldsymbol{5a - 3b})$

Sol: $(a^2 + 2b^2) \times (5a - 3b) = a^2 \times (5a - 3b) + 2b^2 \times (5a - 3b)$

$= a^2 \times 5a - a^2 \times 3b + 2b^2 \times 5a - 2b^2 \times 3b$

$= 5a^3 - 3a^2b + 10ab^2 - 6b^3$

Multiplying a binomial by a trinomial

$(a + b) \times (p + q + r) = a \times p + a \times q + a \times r + b \times p + b \times q + b \times r$

Ex: $(a + 7) \times (a^2 + 3a + 5) = a \times (a^2 + 3a + 5) + 7 \times (a^2 + 3a + 5)$

$= a \times a^2 + a \times 3a + a \times 5 + 7 \times a^2 + 7 \times 3a + 7 \times 5$

$= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35$

$= a^3 + 3a^2 + 7a^2 + 5a + 21a + 35$

$= a^3 + 10a^2 + 26a + 35$

Example 10: Simplify $(\boldsymbol{a} + \boldsymbol{b})\,(\boldsymbol{2a} - \boldsymbol{3b} + \boldsymbol{c}) - (\boldsymbol{2a} - \boldsymbol{3b})\,\boldsymbol{c}$

Sol: $(a + b)(2a - 3b + c) = (a \times 2a) - (a \times 3b) + (a \times c) + (b \times 2a) - (b \times 3b) + (b \times c)$

$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc$

$= 2a^2 - ab + ac - 3b^2 + bc$

$(2a - 3b)c = c \times (2a - 3b) = c \times 2a - c \times 3b = 2ac - 3bc$

Now $(a + b)(2a - 3b + c) - (2a - 3b)c$

$= (2a^2 - ab + ac - 3b^2 + bc) - (2ac - 3bc)$

$= 2a^2 - ab + ac - 3b^2 + bc - 2ac + 3bc$

$= 2a^2 - ab + ac - 2ac - 3b^2 + bc + 3bc$

$= 2a^2 - ab - ac - 3b^2 + 4bc$

EXERCISE 8.4

1. Multiply the binomials

$(\boldsymbol{i})$ $\quad (\boldsymbol{2x} + \boldsymbol{5})\,\boldsymbol{and}\,(\boldsymbol{4x} - \boldsymbol{3})$

Sol: $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5 \times (4x - 3)$

$= (2x \times 4x) - (2x \times 3) + (5 \times 4x) + (5 \times 3)$

$= 8x^2 - 6x + 20x + 15$

$= 8x^2 + 14x + 15$

$(\boldsymbol{ii})$ $\quad (\boldsymbol{y} - \boldsymbol{8})\,\boldsymbol{and}\,(\boldsymbol{3y} - \boldsymbol{4})$

Sol: $(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$

$= (y \times 3y) - (y \times 4) - (8 \times 3y) - (8 \times -4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$

$(\boldsymbol{iii})$ $(\boldsymbol{2.5l} - \boldsymbol{0.5m})$ $\boldsymbol{and}$ $(\boldsymbol{2.5l} + \boldsymbol{0.5m})$

Sol: $(2.5l - 0.5m) \times (2.5l + 0.5m) = 2.5l \times (2.5l + 0.5m) - 0.5m \times (2.5l + 0.5m)$

$= (2.5l \times 2.5l) + (2.5l \times 0.5m) - (0.5m \times 2.5l) - (0.5m \times 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25m^2$

$(\boldsymbol{iv})$ $(\boldsymbol{a} + \boldsymbol{3b})$ $\boldsymbol{and}$ $(\boldsymbol{x} + \boldsymbol{5})$

Sol: $(a + 3b) \times (x + 5) = a \times (x + 5) + 3b \times (x + 5)$

$= (a \times x) + (a \times 5) + (3b \times x) + (3b \times 5) = ax + 5a + 3bx + 15b$

$(\boldsymbol{v})(\boldsymbol{2pq} + \boldsymbol{3q^2})\,\boldsymbol{and}\,(\boldsymbol{3pq} - \boldsymbol{2q^2})$

Sol: $(2pq + 3q^2) \times (3pq - 2q^2) = 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$

$= (2pq \times 3pq) - (2pq \times 2q^2) + (3q^2 \times 3pq) - (3q^2 \times 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6q^4$

$(\boldsymbol{vi})$ $\left(\frac{3}{4}\boldsymbol{a^2} + 3\boldsymbol{b^2}\right)$ $\boldsymbol{and}$ $4\left(\boldsymbol{a^2} - \frac{2}{3}\boldsymbol{b^2}\right)$

Sol: $\left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right)$

$= \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2 \times \left(4a^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \left(\frac{3}{4}a^2 \times 4a^2\right) - \left(\frac{3}{4}a^2 \times \frac{8}{3}b^2\right) + (3b^2 \times 4a^2) - \left(3b^2 \times \frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12a^2b^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4$

2. Find the product.

$(\boldsymbol{i})(\boldsymbol{5 - 2x})(\boldsymbol{3 + x})$

Sol: $(5 - 2x)(3 + x) = 5 \times (3 + x) - 2x \times (3 + x)$

$= (5 \times 3) + (5 \times x) - (2x \times 3) - (2x \times x)$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$

$(\boldsymbol{ii})(\boldsymbol{x + 7y})(\boldsymbol{7x - y})$

Sol: $(x + 7y)(7x - y) = x \times (7x - y) + 7y \times (7x - y)$

$= (x \times 7x) - (x \times y) + (7y \times 7x) - (7y \times y)$

$= 7x^2 - xy + 49xy - 7y^2$

$= 7x^2 + 48xy - 7y^2$

$(\boldsymbol{iii})(\boldsymbol{a^2 + b})(\boldsymbol{a + b^2})$

Sol: $(a^2 + b)(a + b^2) = a^2 \times (a + b^2) + b \times (a + b^2)$

$= (a^2 \times a) + (a^2 \times b^2) + (b \times a) + (b \times b^2)$

$= a^3 + a^2b^2 + ab + b^3$

$(\boldsymbol{iv})$ $(\boldsymbol{p^2 - q^2})$ $(\boldsymbol{2p + q})$

Sol: $(p^2 - q^2)(2p + q) = p^2 \times (2p + q) - q^2 \times (2p + q)$

$= (p^2 \times 2p) + (p^2 \times q) - (q^2 \times 2p) - (q^2 \times q)$

$= 2p^3 + p^2q - 2q^2p - q^3$

3. Simplify

$(\boldsymbol{i})(\boldsymbol{x^2 - 5})(\boldsymbol{x + 5}) + \boldsymbol{25}$

Sol: $(x^2 - 5)(x + 5) + 25 = x^2 \times (x + 5) - 5 \times (x + 5) + 25$

$= (x^2 \times x) + (x^2 \times 5) - (5 \times x) - (5 \times 5) + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

$\boldsymbol{(ii)(a^2 + 5)(b^3 + 3) + 5}$

$Sol: (a^2 + 5)(b^3 + 3) + 5 = a^2 \times (b^3 + 3) + 5 \times (b^3 + 3) + 5$

$= (a^2 \times b^3) + (a^2 \times 3) + (5 \times b^3) + (5 \times 3) + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

$\boldsymbol{(iii)\ (t + s^2)\ (t^2 - s)}$

$Sol: (t + s^2)(t^2 - s) = t \times (t^2 - s) + s^2 \times (t^2 - s)$

$= (t \times t^2) - (t \times s) + (s^2 \times t^2) - (s^2 \times s)$

$= t^3 - ts + s^2t^2 - s^3$

$\boldsymbol{(iv)(a + b)(c - d) + (a - b)(c + d) + 2\,(ac + bd)}$

$Sol: (a + b)(c - d) + (a - b)(c + d) + 2\,(ac + bd)$

$= a \times (c - d) + b \times (c - d) + a \times (c + d) - b \times (c + d) + 2\,(ac + bd)$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= 2ac - 2bd + 2ac + 2bd$

$= 4ac$

$\boldsymbol{(v)\ (x + y)(2x + y) + (x + 2y)(x - y)}$

$Sol: (x + y)(2x + y) + (x + 2y)(x - y)$

$= x \times (2x + y) + y \times (2x + y) + x \times (x - y) + 2y \times (x - y)$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= 3x^2 + 4xy - y^2$

$\boldsymbol{(vi)(x + y)(x^2 - xy + y^2)}$

$Sol: (x + y)(x^2 - xy + y^2) = x \times (x^2 - xy + y^2) + y \times (x^2 - xy + y^2)$

$= (x \times x^2) - (x \times xy) + (x \times y^2) + (y \times x^2) - (y \times xy) + (y \times y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 - y^3$

$\boldsymbol{(vii)(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y}$

$Sol: (1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x \times (1.5x + 4y + 3) - 4y \times (1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 - 16y^2$

$\boldsymbol{(viii)(a + b + c)(a + b - c)}$

$Sol: (a + b + c)(a + b - c)$

$= a \times (a + b - c) + b \times (a + b - c) + c \times (a + b - c)$

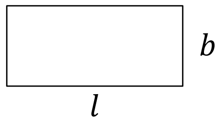
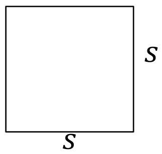
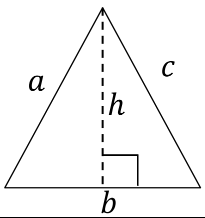
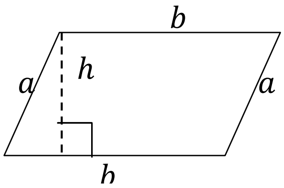
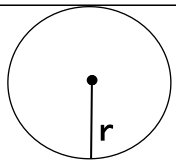
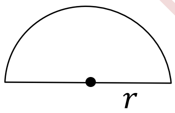
$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$

$= a^2 + b^2 - c^2 + 2ab$

CHAPTER 9

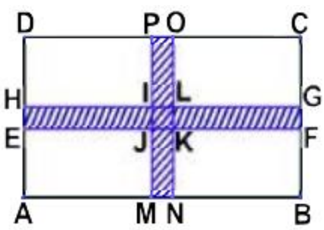
VIII CLASS-NCERT (2023-24)
MENSURATION (Notes)
PREPARED BY : BALABHADRA SURESH-9866845885
www.sureshmathsmaterial.com/

1.

Diagram	Shape	Area	Perimeter
	**RECTANGLE** Length=l Breadth=b	$l \times b$	$2(l+b)$
	**SQUARE** Side=s	$a \times a = a^2$	$4a$
	**TRIANGLE** Base=b , sides=a , c Height=h	$\frac{1}{2}bh$	$a+b+c$
	**PARALLELOGRAM** Base=b Corresponding height=h	$b \times h$	$2(a+b)$
	**CIRCLE** Radius=r $\pi = \frac{22}{7}$ or 3.14	πr^2	$2\pi r$
	**SEMI-CIRCLE** Radius=r	$\frac{1}{2}\pi r^2$	$\pi r + 2r = \frac{36}{7}r$

2. Area of shaded path

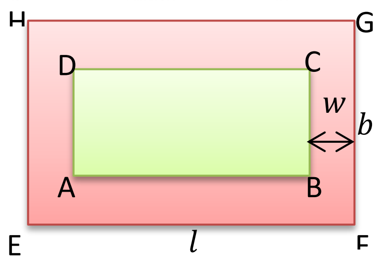
=Area of EFGH + Area of MNOP – (Area of IJKL)



3. Area of rectangular path

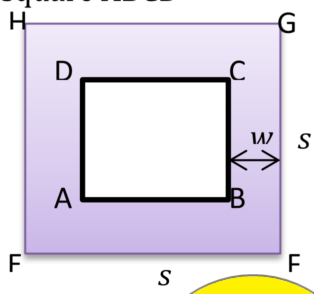
= Area of outer rectangle EFGH – Area of inner rectangle ABCD

$= l \times b - (l - 2w)(b - 2w)$



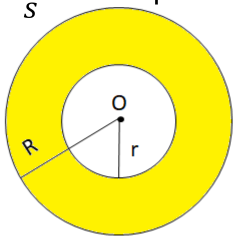
4. Area of square path= Area of outer square EFGH – Area of inner square ABCD

$$= s^2 - (s - 2w)^2$$
$$= (2s - 2w) \times 2w$$
$$= 4(s - w) \times w$$



5. The area of circular pathway = Area of outer circle – Area of inner circle

$$= \pi R^2 - \pi r^2$$
$$= \pi(R^2 - r^2)$$
$$= \pi(R + r)(R + r)\ sq.units$$



AREA OF TRAPEZIUM:

TRY THESE

1. **the area of trapezium WXYZ** $= \boldsymbol{h}\frac{(a+b)}{2}$

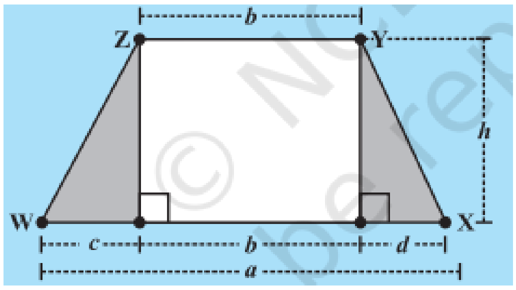
Sol: $Area\ of\ \Delta PZW = \frac{1}{2} \times c \times h$

$$Area\ of\ \Delta QXY = \frac{1}{2} \times d \times h$$

$$Area\ of\ rectangle\ PQYZ = b \times h$$

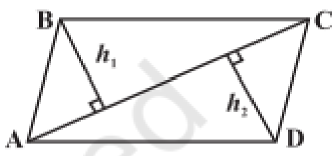
$The\ area\ of\ trapezium\ WXY$

$$= Area\ of\ \Delta PZW + Area\ of\ \Delta QXY$$
$$+ Area\ of\ rectangle\ PQYZ$$
$$= \frac{1}{2} \times c \times h + \frac{1}{2} \times d \times h + b \times h$$
$$= h \times \left(\frac{c}{2} + \frac{d}{2} + b\right)$$
$$= h\left(\frac{c + d + 2b}{2}\right)$$
$$= h\left(\frac{c + d + b + b}{2}\right)$$
$$= h\left(\frac{a + b}{2}\right)$$



Area of a General Quadrilateral

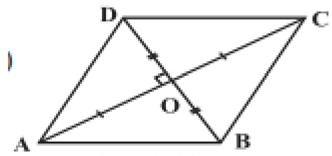
$$Area\ of\ quadrilateral\ ABCD = \frac{1}{2}d(h_1 + h_2)$$



Rhombus

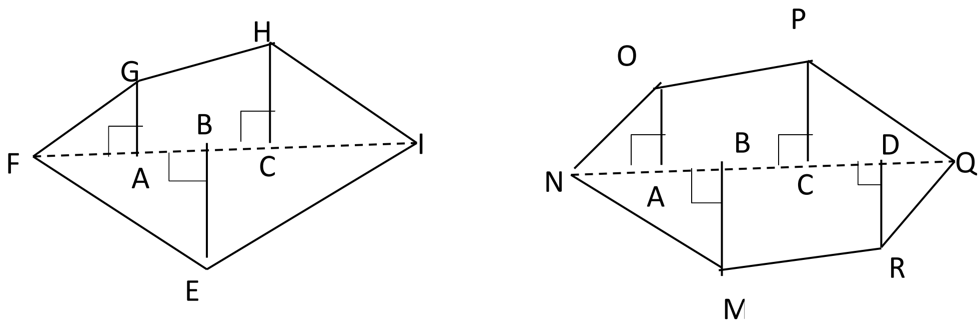
$Area\ of\ rhombus = \frac{1}{2} \times d_1 \times d_2$

Area of a rhombus is half the product of its diagonals



Area of a Polygon

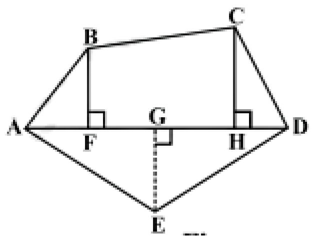
(i) Divide the following polygons (Fig 11.17) into parts (triangles and trapezium) to find out its area



(ii) Polygon ABCDE is divided into parts as shown below (Fig 11.18). Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.

Sol: $FH = AH - AF = 6 - 3 = 3\ cm, HD = AD - AH = 8 - 6 = 2\ cm$

$Area\ of\ \Delta AFB = \frac{1}{2} \times AF \times BF = \frac{1}{2} \times 3 \times 2 = 3\ cm^2$



Area of trapezium FBCH $= \text{FH} \times \frac{(\text{BF} + \text{CH})}{2}$

$= 3 \times \frac{(2+3)}{2}$

$= \frac{15}{2} = 7.5\ cm^2$

$FH = AH - AF = 6 - 3 = 3\ cm$

$Area\ of\ \Delta CHD = \frac{1}{2} \times HD \times CH = \frac{1}{2} \times 2 \times 3 = 3\ cm^2$

$Area\ of\ \Delta ADE = \frac{1}{2} \times AD \times GE = \frac{1}{2} \times 8 \times 2.5 = 10\ cm^2$

The area of polygon $ABCDE$

$= Area\ of\ \Delta AFB +$ Area of trapezium FBCH + Area of ΔCHD + Area of ΔADE

$= 3 + 7.5 + 3 + 10 = 23.5\ cm^2$

(iii) Find the area of polygon MNOPQR (Fig 11.19) if MP = 9 cm, MD = 7 cm, MC = 6 cm, MB = 4 cm, MA = 2 cm NA, OC, QD and RB are perpendiculars to diagonal MP.

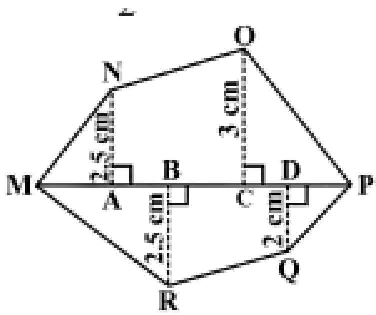
Sol: $AC = MC - MA = 6 - 2 = 4\ cm$

$CP = MP - MC = 9 - 6 = 3\ CM$

$DP = MP - MD = 9 - 7 = 2cm$

$BD = MD - MB = 7 - 4 = 3\ cm$

$Area\ of\ \Delta MAN = \frac{1}{2} \times MA \times NA = \frac{1}{2} \times 2 \times 2.5 = 2.5\ cm^2$



$$Area\ of\ trapezium\ \text{ACON} = \text{AC} \times \frac{(\text{AN}+\text{CO})}{2}$$

$$= 4 \times \frac{(2.5+3)}{2} = 2 \times 5.5 = 11cm^2$$

Area of ΔOCP $= \frac{1}{2} \times CP \times CO = \frac{1}{2} \times 3 \times 3 = 4.5\ cm^2$

Area of ΔPDQ $= \frac{1}{2} \times PD \times DQ = \frac{1}{2} \times 2 \times 2 = 2\ cm^2$

Area of trapezium BDQR $= BD \times \frac{(BR+DQ)}{2}$

$$= 3 \times \frac{(2.5+2)}{2} = 3 \times 2.25 = 6.75cm^2$$

Area of ΔMBR $= \frac{1}{2} \times MB \times BR = \frac{1}{2} \times 4 \times 2.5 = 5\ cm^2$

The area of polygon MNOPQR$=2.5 + 11 + 4.5 + 2 + 6.75 + 5 = 31.75\ cm^2$

Example 1: The area of a trapezium shaped field is 480 m² , the distance between two parallel sides is 15 m and one of the parallel side is 20 m. Find the other parallel side.

Sol: $h = 15m\ ,\ a = 20\ m,\ b = ?$

The given area of trapezium = 480 m² .

$$\frac{1}{2} \times h \times (a+b) = 480$$

$$\frac{1}{2} \times 15 \times (20+b) = 480$$

$$20 + b = \frac{480 \times 2}{15} = 32 \times 2 = 64$$

$$20 + b = 64$$

$$b = 64 - 20 = 44\ m$$

Hence the other parallel side of the trapezium is 44 m

Example 2: The area of a rhombus is 240 cm² and one of the diagonals is 16 cm. Find the other diagonal.

Sol: The area of a rhombus = 240 cm², $d_1 = 16cm, d_2 = ?$

$$\frac{1}{2} \times d_1 \times d_2 = 240$$

$$\frac{1}{2} \times 16 \times d_2 = 240$$

$$d_2 = \frac{240 \times 2}{16} = 30\ cm$$

Hence the length of the second diagonal is 30 cm.

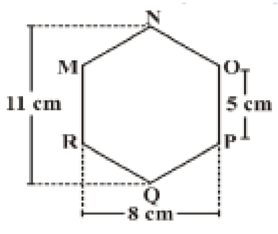
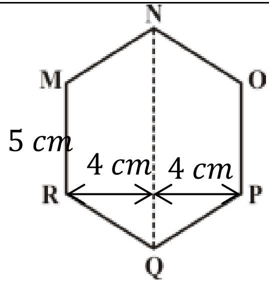


Fig 11.20

Example 3: There is a hexagon MNOPQR of side 5 cm (Fig 11.20). Aman and Ridhima divided it in two different ways (Fig 11.21). Find the area of this hexagon using both ways

Solution: Aman’s method:

Area of trapezium MNQR $= h \times \frac{(a+b)}{2}$

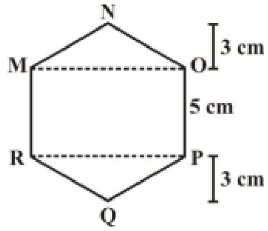


$$= 4 \times \frac{(11+5)}{2} = 2 \times 16 = 32\ cm^2$$

Similarly Area of trapezium NQRO= 32 cm^2

So, the area of hexagon MNOPQR = 2 × 32 = 64 cm^2 .

Ridhima's method:



Δ MNO and Δ RPQ are congruent triangles with altitude 3 cm and base 8cm

$$Area\ of\ \Delta MNO = \frac{1}{2} \times b \times h = \frac{1}{2} \times 8 \times 3 = 12\ cm^2$$

Also area of ΔMNO =12 cm^2

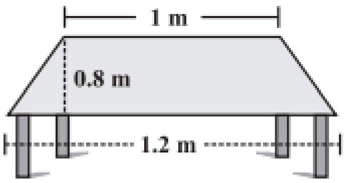
Area of rectangle MOPR = 8 × 5 = 40 cm^2 .

Now, area of hexagon MNOPQR = 40 + 12 + 12 = 64 cm^2 .

EXERCISE 9.1

1. The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

Sol: Given $a = 1m$, $b = 1.2\ m$ and $h = 0.8\ m$



$$\text{The area of table} = h \times \frac{(a+b)}{2} = 0.8 \times \frac{(1+1.2)}{2}$$

$$= 0.4 \times 2.2 = 0.88\ m^2$$

2. The area of a trapezium is 34 cm² and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the other parallel side.

Sol: Given $a = 10cm, h = 4cm, b = ?$

The area of a trapezium = 34 cm^2

$$h \times \frac{(a+b)}{2} = 34$$

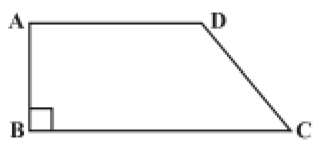
$$4 \times \frac{(10+b)}{2} = 34$$

$$10 + b = \frac{34}{2} = 17$$

$$b = 17 - 10 = 7$$

Length of the other parallel side=7 cm

3. Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.



Sol: Length of the fence of a trapezium shaped field ABCD = 120 m

AB+BC+CD+DA=120 m

AB+48+17+40=120

AB+105=120

AB=120-105

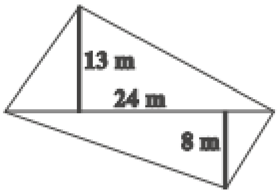
AB=15 m

Here $a = BC = 48\ m, b = AD = 40m\ \ and\ \ h = AB = 15m$

$$The\ area\ of\ the\ field = h \times \frac{(a+b)}{2}$$

$$= 15 \times \frac{(48+40)}{2} = 15 \times 44 = 660\ m^2$$

4. **The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field**



Sol: Here $d = 24m,\ h_1 = 13\ m,\ \ h_2 = 8\ m$

$$The\ area\ of\ the\ field = \frac{1}{2}d(h_1 + h_2)$$

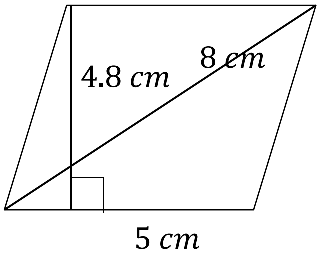
$$= \frac{1}{2} \times 24 \times (13+8) = 12 \times 21 = 252\ m^2$$

5. **The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.**

Sol: Here $d_1 = 7.5\ cm,\ \ d_2 = 12\ cm$

The area of rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 7.5 \times 12 = 7.5 \times 6 = 45\ cm^2$

6. **Find the area of a rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.**



Sol: Side of rhombus(S)=5 cm

Altitude (h)=4.8 cm

We know that rhombus is also a parallelogram.

Area of rhombus(parallelogram) $=$ Base $\times$ Height

$= 5 \times 4.8 = 24\ cm^2$

One of its diagonal$(d_1) = 8\ cm$

Area of rhombus $= 24\ cm^2$

$$\frac{1}{2} \times d_1 \times d_2 = 24 \Rightarrow \frac{1}{\cancel{2}_1} \times \cancel{8}^4 \times d_2 = 24$$

$$d_2 = \frac{24}{4} = 6\ cm$$

The length of the other diagonal=6 cm

7. **The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m² is ₹ 4.**

Sol: Diagonals of each tile $d_1 = 45\ cm,\ d_2 = \ 30\ cm$

Area of each tile $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 45 \times 30 = 45 \times 15 = 675\ cm^2$

$Area\ of\ 3000\ tiles = 3000 \times 675\ cm^2$

$= 2025000\ cm^2$

$= \frac{2025000}{10000}\ m^2 = 202.5\ m^2$

Cost of polishing the floor per 1 m^2=4

Cost of polishing 202.5 $m^2 = 4 \times 202.5 = 810$

total cost of polishing the floor=810

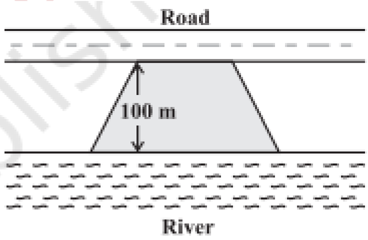
8. **Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m² and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river**

Sol: Let side along the road(a)=$x\ m$

Side along the river(b)=$2x\ m$

Distance between two sides(h)=100 m

Area of trapezium=10500 m²



$h \times \frac{(a+b)}{2} = 10500$

$100 \times \frac{(x+2x)}{2} = 10500$

$\frac{3x}{2} = \frac{10500}{100}$

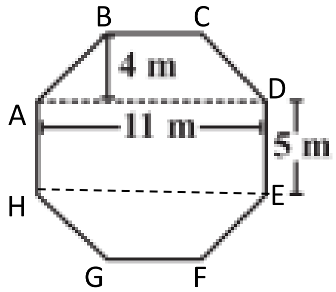
$x = \frac{105 \times 2}{3} = 35 \times 2 = 70$

Side along the road=x=70 m

The length of the side along the river=$2 \times 70 = 140\ m$

9. **Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.**

Sol: Given is a regular octagon . So, all sides are equal.



Area of trapezium ABCD $= h \times \frac{(a+b)}{2}$

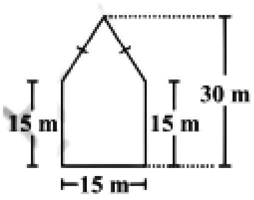
$= 4 \times \frac{(11+5)}{2} = 2 \times 16 = 32\ m^2$

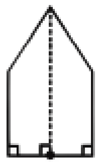
Area of rectangle ADEH=$AD \times DE = 11 \times 5 = 55\ m^2$

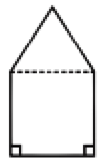
Area of octagonal surface $= 2 \times$ Area of trapezium + Area of rectangle

$= 2 \times 32 + 55 = 64 + 55 = 119 m^2$

10. There is a pentagonal shaped park as shown in the figure. For finding its area Jyoti and Kavita divided it in two different ways. Find the area of this park using both ways. Can you suggest some other way of finding its area?




Joyti's diagram


Kavita's diagram

Sol: Finding area by Jyothi's diagram:

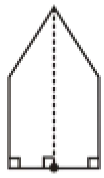
Area of trapezium ABFE

$$= EF \times \frac{(AE + BF)}{2}$$

$$= \frac{15}{2} \times \frac{(15 + 30)}{2} = \frac{15 \times 45}{4} m^2$$

Area of pentagon ABCDE = 2 × Area of trapezium ABFE

$$= 2 \times \frac{15 \times 45}{4} = \frac{675}{2} = 337.5\ m^2$$

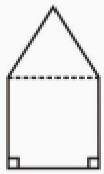

Joyti's diagram

Finding area by Kavita's diagram:

Area of $\Delta ABC = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 15 \times 15$

$= \frac{225}{2} = 112.5\ m^2$


Kavita's diagram

Area of square ACDE $= s \times s$

$= 15 \times 15 = 225\ m^2$

Area of pentagon ABCDE = Area of ΔABC + Area of square ACDE

$= 112.5 + 225 = 337.5\ m^2$

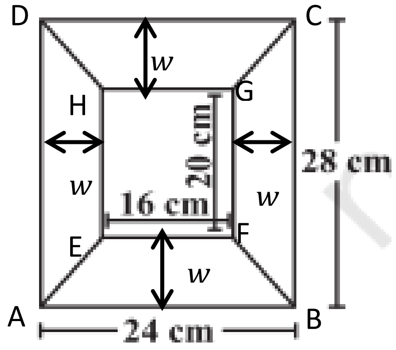
11. Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimensions 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same

Sol: Outer dimentions $L = 28cm \quad and \quad B = 24\ cm$

Inner dimentions $l = 20cm \quad and \quad b = 16\ cm$

The width of each section is same

Width $= w = \frac{L - l}{2} = \frac{28 - 20}{2} = \frac{8}{2} = 4\ cm \Rightarrow h = 4cm$



Area of trapezium ABFE $= h \times \frac{(a + b)}{2}$

$= 4 \times \frac{(24 + 16)}{2} = 4 \times \frac{40}{2} = 80\ cm^2$

Similarly area of trapezium DCGH= 80 cm^2

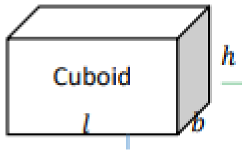
Area of trapezium BCGF $= h \times \frac{(a+b)}{2}$

$= 4 \times \frac{(28+20)}{2} = 2 \times 48 = 96\ cm^2$

Similarly area of trapezium ADHE= 96 cm^2

Solid Shapes

Cuboid :



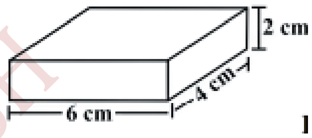
$Length = l\ , Breadth = b, \quad Height = h$

(i) Lateral surface area(LSA)=$2lh + 2bh = 2h(l+b)$

(ii) Toatl surface area(TSA)=$2(lb + bh + hl)$

TRY THESE

Find the total surface area of the following cuboids.



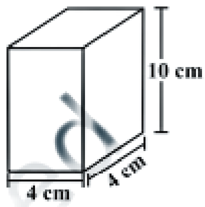
(i) $\boldsymbol{l = 6cm, \quad b = 4cm, \quad h = 2cm}$

Toatl surface area (TSA) of cuboid=$2(lb + bh + hl)$

$= 2(6 \times 4 + 4 \times 2 + 2 \times 6)$

$= 2(24 + 8 + 12)$

$= 2 \times 44 = 88\ cm^2$



(ii) $\boldsymbol{l = 4cm, \quad b = 4cm, \quad h = 10cm}$

Toatl surface area (TSA) of cuboid=$2(lb + bh + hl)$

$= 2(4 \times 4 + 4 \times 10 + 10 \times 4)$

$= 2(16 + 40 + 40)$

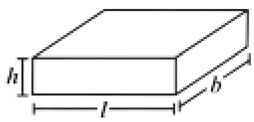
$= 2 \times 96 = 88\ cm^2$

THINK, DISCUSS AND WRITE

1. **Can we say that the total surface area of cuboid = lateral surface area + 2 × area of base?**

Sol: Lateral surface area=$2lh + 2bh = 2h(l+b)$

Area of base=lb



lateral surface area + 2 × area of base

$= 2lh + 2bh + 2 \times lb$

$= 2(lh + bh + lb)$

$= Total\ surface\ area\ of\ cuboid$

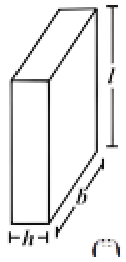
Yes, we can say that the total surface area of cuboid = lateral surface area + 2 × area of base.

2. **If we interchange the lengths of the base and the height of a cuboid (Fig 11.33(i)) to get another cuboid (Fig 11.33(ii)), will its lateral surface area change?**

Sol: LSA of cuboid (ii) $=2lb+2lh)=2l(b+h)$

But LSA of cuboid (i)$=2lh+2bh=2h(l+b)$

Hence the lateral surface area will change.

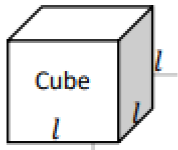


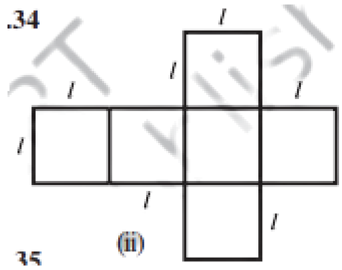
Cube,

Side of cube$=l$

(i) Larerl surface area of cube=$4l^2$

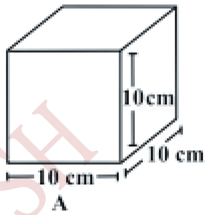
(ii) Total surface area of cube=$6l^2$





TRY THESE

Find the surface area of cube A and lateral surface area of cube B .

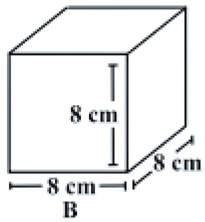


Sol: For cube A : $l=10cm$

The surface area of cube A $=6l^2=6\times10^2=6\times100=600\ cm^2$

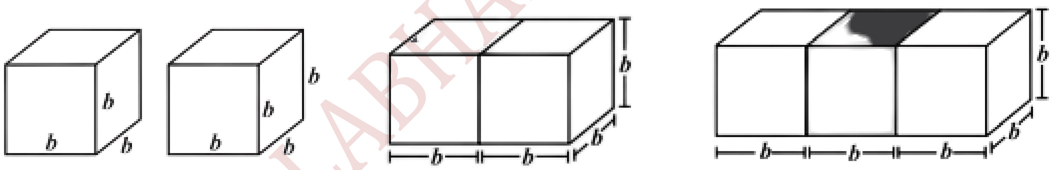
For cube B : $l=8\ cm$

The lateral surface area of cube B $=4l^2=4\times8^2=4\times64=256cm^2$



THINK, DISCUSS AND WRITE

(i) Two cubes each with side b are joined to form a cuboid (Fig 11.37). What is the surface area of this cuboid? Is it $12b^2$? Is the surface area of cuboid formed by joining three such cubes, $18b^2$? Why?



Sol: Cube has six faces normally when two equal cubes are placed together, two side faces are not visible.

We left with $12-2=10$ squared faces

$\therefore\ Surface\ area=10b^2$

When three equal cubes are placed together, four side faces are not visible.

We left with $18-4=14$ squared faces

$\therefore\ Surface\ area=14b^2$

(ii) How will you arrange 12 cubes of equal length to form a cuboid of smallest surface area?

Sol: Case 1: $12=12\times1\times1$

$Surface\ area=2(lb+bh+hl)$

$=2(12\times1+1\times1+1\times12)=2\times25=50\ square\ units$

Case 2: $12=6\times2\times1$

Surface area $= 2(6 \times 2 + 2 \times 1 + 1 \times 6) = 2(12 + 2 + 6) = 2 \times 20 = 40$ square units

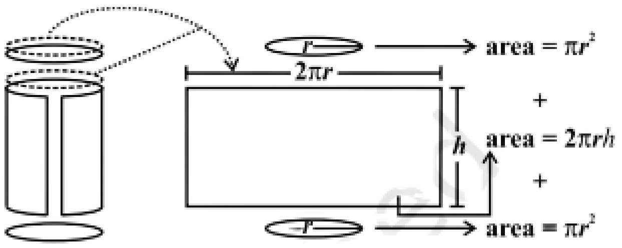
Case 3: $12 = 4 \times 3 \times 1$

$Surface\ area = 2(4 \times 3 + 3 \times 1 + 1 \times 4) = 2(12 + 3 + 4) = 2 \times 19 = 38\ square\ units$

Case 4: $12 = 3 \times 2 \times 2$

Surface area $= 2(3 \times 2 + 2 \times 2 + 2 \times 3) = 2(6 + 4 + 6) = 2 \times 16 = 32$ square units

Cylinders



The lateral (or curved) surface area of a cylinder $= 2\pi rh$

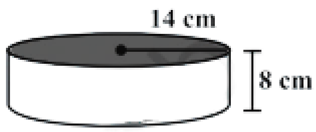
The total surface area of a cylinder $= \pi r^2 + 2\pi rh + \pi r^2$

$$= 2\pi r^2 + 2\pi rh = 2\pi r\,(r + h)$$

TRY THESE

Find total surface area of the following cylinders

1) $\boldsymbol{r = 14\ cm, h = 8cm}$



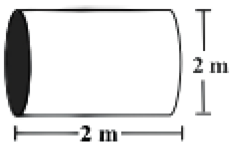
Total surface area of the cylinder$= 2\pi r\,(r + h)$

$= 2 \times \frac{22}{7} \times 14 \times (14 + 8)\ cm^2$

$= 2 \times \frac{22}{7} \times 14 \times 22\ cm^2$

$= 2 \times 22 \times 2 \times 22 = 1936\ cm^2$

2) $\boldsymbol{d = 2m \Rightarrow r = \frac{2}{2} = 1\ cm, h = 2cm}$



Total surface area of the cylinder$= 2\pi r\,(r + h)$

$= 2 \times \frac{22}{7} \times 1 \times (1 + 2)\ cm^2$

$= 2 \times \frac{22}{7} \times 3\ cm^2$

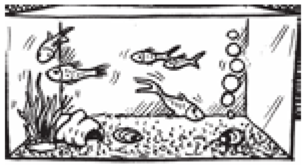
$= \frac{132}{7} = 18.9\ cm^2$

Example 4: An aquarium is in the form of a cuboid whose external measures are 80 cm × 30 cm × 40 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed?

Sol: The length of the aquarium = l = 80 cm

Width of the aquarium = b = 30 cm

Height of the aquarium = h = 40 cm



Area of the base = l × b = 80 × 30 = 2400 cm^2

Area of the side face = b × h = 30 × 40 = 1200 cm^2

Area of the back face = l × h = 80 × 40 = 3200 cm^2

Required area = Area of the base + area of the back face + (2 × area of a side face)

= 2400 + 3200 + (2 × 1200) = 8000 cm^2

Hence the area of the coloured paper required is 8000 cm^2.

Example 5: The internal measures of a cuboidal room are 12 m × 8 m × 4 m. Find the total cost of whitewashing all four walls of a room, if the cost of white washing is ₹ 5 per m2 . What will be the cost of white washing if the ceiling of the room is also whitewashed.

Sol: $l = 12\ m, b = 8\ m, h = 4\ m$

Area of the four walls of the room (LSA)=$2h(l + b)$

$$= 2 \times 4 \times (12 + 8) = 8 \times 20 = 160\ m^2$$

Cost of white washing per m^2 = ₹ 5

The total cost of white washing four walls of the room = ₹ (160 × 5) = ₹ 800

Area of ceiling= $l \times b = 12 \times 8 = 96\ m^2$

Cost of white washing the ceiling = ₹ (96 × 5) = ₹ 480

So the total cost of white washing = ₹ (800 + 480) = ₹ 1280

Example 6: In a building there are 24 cylindrical pillars. The radius of each pillar is 28 cm and height is 4 m. Find the total cost of painting the curved surface area of all pillars at the rate of ₹ 8 per m^2 .

Sol : Radius of cylindrical pillar, r = 28 cm = 0.28 m

Height, h = 4 m

curved surface area of a cylinder $= 2\pi rh$

Curved surface area of a pillar $= 2 \times \frac{22}{\cancel{7}} \times \cancel{0.28}^{0.04} \times 4$

$$= 2 \times 22 \times 0.04 \times 4 = 7.04\ m^2$$

Curved surface area of 24 such pillar = 7.04 × 24 = 168.96 m^2

Cost of painting an area of 1 m^2 = ₹ 8

Therefore, cost of painting 1689.6 m^2 = 168.96 × 8 = ₹ 1351.68

Ex 7: Find the height of a cylinder whose radius is 7 cm and the total surface area is 968 cm^2.

Sol: Let height of the cylinder = h, radius = r = 7cm

Total surface area $= 968\ cm^2$

$2\pi r\ (h + r) = 968$

$2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times (h + 7) = 968$

$h + 7 = \frac{968}{2 \times 22} = \frac{484}{22} = 22$

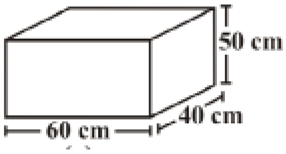
$h = 22 - 7 = 15\ cm$

Hence, the height of the cylinder is 15 cm.

EXERCISE 11.3

1. There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?

Sol: **(a)** $\boldsymbol{l = 60\ cm, b = 40\ cm, h = 50\ cm.}$



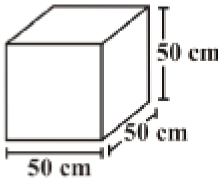
TSA of box$(a) = 2(lb + bh + lh)$

$= 2(60 \times 40 + 40 \times 50 + 60 \times 50)$

$= 2(2400 + 2000 + 3000)$

$= 2 \times 7400 = 14800\ cm^2$

(b) $\boldsymbol{l = 50\ cm, b = 50\ cm, h = 50\ cm.}$



TSA of box$(b) = 6a^2$

$= 6 \times 50^2 = 6 \times 2500 = 15000\ cm^2$

TSA of box$(a)$is lesser than The TSA of box(b)

So, box (a)requires the lesser amount of material to make .

2. A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suitcases?

Sol: $l = 80\ cm, b = 48\ cm, h = 24\ cm.$

$Total\ surface\ area\ of\ suitcase = 2(lb + bh + lh)$

$= 2(80 \times 48 + 48 \times 24 + 80 \times 24)$

$= 2(3840 + 1152 + 1920)$

$= 2 \times 6912 = 13824\ cm^2$

Tarpaulin required for 1 suitcase=13824 cm^2

Area of Tarpaulin required for 100 suitcase=100 × 13824 $cm^2 = 1382400\ \ cm^2$

Width of given tarpaulin=96 cm.

Length of required tarpaulin $= \frac{\text{Area of tarpaulin required}}{\text{width of tarpaulin}}$

$$= \frac{1382400\ \ cm^2}{96\ cm} = 14400\ cm = 144 \times 100\ cm = 144\ m$$

3. Find the side of a cube whose surface area is 600 cm.

Sol: surface area of cube= 600 cm

$6l^2 = 600$

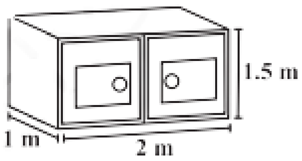
$$l^2 = \frac{600}{6} = 100$$

$$l = \sqrt{100} = 10$$

$\therefore$ *side of cube* $= 10\ cm.$

4. Rukhsar painted the outside of the cabinet of measure 1 m × 2 m × 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet.

Sol: $l = 1\ m, b = 2\ m, h = 1.5\ m$



Total surface area of cabinet $= 2(lb + bh + lh)$

$= 2(1 \times 2 + 2 \times 1.5 + 1 \times 1.5)$

$= 2(2 + 3 + 1.5) = 2 \times 6.5 = 13\ m^2$

The bottom area of the cabinet $= l \times b = 1 \times 2 = 2\ m^2$

Painted area $=$ Total surface area of cabinet $-$ The bottom area of the cabinet

$= 13 - 2 = 11 m^2$

5. Daniel is painting the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m² of area is painted. How many cans of paint will she need to paint the room?

Sol: $l = 15\ m, b = 10\ m, h = 7\ m$

Total surface area of hall $= 2(lb + bh + lh)$

$= 2(15 \times 10 + 10 \times 7 + 15 \times 7)$

$= 2(150 + 70 + 105) = 2 \times 325 = 650\ m^2.$

The bottom area of hall $= l \times b = 15 \times 10 = 150\ m^2$

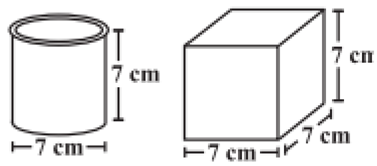
Painted area $=$ Total surface area of hall $-$ The bottom area of hall

$= 650 - 150 = 500\ m^2.$

Area painted by 1 can $= 100\ m^2.$

Number of cans required $= \dfrac{\text{Painted area}}{\text{Area painted by 1 can}} = \dfrac{500}{100} = 5$

6. Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area?



Sol: The two figures are like prisms with same height 7 cm.

First figure is cylinder it has curved surface area.

Second figure is cube it has only plan surfaces.

Cylinder:

Diameter=d=7 cm

$$Radius = r = \frac{7}{2}\ cm$$

Height=h=7 cm.

Lateral surface area of cylinder $= 2\pi rh$

$= \cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times 7 = 22 \times 7 = 154\ cm^2$

Cube:

Side of cube $= l = 7\ cm$

Lateral surface area of cube $= 4l^2 = 4 \times 7 \times 7 = 196\ cm^2$

So, cube has larger surface area.

7. A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

Sol: Radius of cylinder=r=7 m

Height of cylinder=h=3 m.

Total surface area of cylinder $= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (3 + 7) = 44 \times 10 = 440\ m^2$

Required sheet of metal=440 m^2

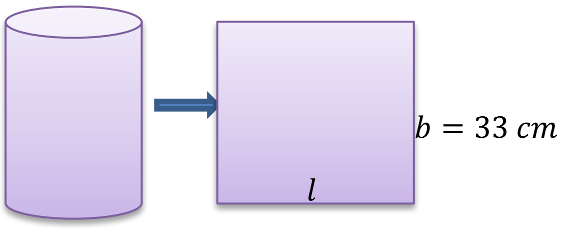
8. The lateral surface area of a hollow cylinder is 4224 cm² . It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

Sol: Area of rectangular sheet=curved surface area of cylinder

$l \times b = 4224$

$l \times 33 = 4224$

$l = \frac{4224}{33} = 128\ cm$



$Perimeter\ of\ rectangular\ sheet = 2(l + b)$

$= 2 \times (128 + 33) = 2 \times 161 = 322\ cm$

9. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.

Sol: Diameter of a road roller = 84 cm

Radius of road roller $= r = \frac{84}{2} = 42\ cm$

Length $= h = 1\ m = 100\ cm$

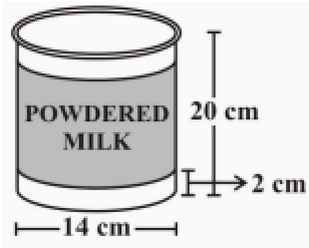
Curved surface area of roller $= 2\pi rh$.

$= 2 \times \frac{22}{\cancel{7}} \times \cancel{42}^{6} \times 100 = 44 \times 600 = 26400\ cm^2$

The area of road covered in 750 complete revolutions $= 750 \times$ Curved surface area of roller

$= 750 \times 26400 = 19800000\ cm^2 = 1980 \times 10000\ cm^2 = 1980\ m^2$

10. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2 cm from top and bottom, what is the area of the label ?



Sol: Label is in cylinder shape

Diameter=d=14 cm

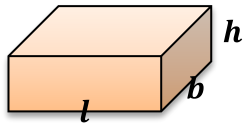
$Radius = r = \frac{14}{2} = 7\ cm$

$Height = h = 20 - 2 \times 2 = 20 - 4 = 16\ cm.$

Area of label=CSA of cylinder

$= 2\pi rh$

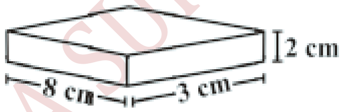
$= 2 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 16 = 44 \times 16 = 704\ cm^2$



Volume of cuboid:

$$\boldsymbol{Volume\ of\ cuboid = \text{area of the base} \times \text{height} = (l \times b) \times h = lbh}$$

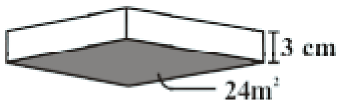
TRY THESE



(i) Volume of cuboid $= lbh$

$= 8\ cm \times 3cm \times 2\ cm$

$= 48\ cm^3$



Volume of cuboid = area of the base × height

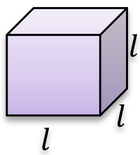
$= 24\ m^2 \times 3\ cm = 24\ m^2 \times \frac{3}{100}\ m$

$= \frac{24 \times 3}{100}\ m^3 = \frac{92}{100}\ m^3 = 0.92\ m^3$

Volume of cube:

$$\boldsymbol{Volume\ of\ cube = \text{area of the base} \times \text{height}}$$

$$\boldsymbol{= (l \times l) \times l = l^2 \times l = l^3}$$



TRY THESE

Find the volume of the following cubes.

(a) With a side 4 cm.

Sol: side (l)=4 cm

Volume of cube $= l^3 = l \times l \times l = 4 \times 4 \times 4 = 64\ cm^3$

(b) With a side 1.5 m

Sol: side (l)=1.5 m

Volume of cube $= l^3 = l \times l \times l = 1.5 \times 1.5 \times 1.5 = 3.375\ m^3$

DO THIS

Arrange 64 cubes of equal size in as many ways as you can to form a cuboid. Find the surface area of each arrangement. Can solid shapes of same volume have same surface area?

Arrangement	$Volume = l \times b \times h$	$Surface\ area = 2(lb + bh + lh)$
$1 \times 1 \times 64$	64	$2(1 \times 1 + 1 \times 64 + 1 \times 64) = 2(1 + 64 + 64) = 258$
$1 \times 2 \times 32$	64	$2(1 \times 2 + 2 \times 32 + 1 \times 32) = 2(2 + 64 + 32) = 196$
$1 \times 4 \times 16$	64	$2(1 \times 4 + 4 \times 16 + 1 \times 16) = 2(4 + 64 + 16) = 168$
$1 \times 8 \times 8$	64	$2(1 \times 1 + 1 \times 64 + 1 \times 64) = 2(1 + 64 + 64) = 258$
$2 \times 2 \times 16$	64	$2(2 \times 2 + 2 \times 16 + 2 \times 16) = 2(4 + 32 + 32) = 136$
$2 \times 4 \times 8$	64	$2(2 \times 4 + 4 \times 8 + 2 \times 8) = 2(8 + 32 + 16) = 112$
$4 \times 4 \times 4$	64	$2(4 \times 4 + 4 \times 4 + 4 \times 4) = 2(16 + 16 + 16) = 96$

Here volume is same but surface area is different.

From above , we conclude that solid shapes of same volume does not have same surface area.

THINK, DISCUSS AND WRITE

A company sells biscuits. For packing purpose they are using cuboidal boxes: box A →3 cm × 8 cm × 20 cm, box B → 4 cm × 12 cm × 10 cm. What size of the box will be economical for the company? Why? Can you suggest any other size (dimensions) which has the same volume but is more economical than these?

Sol: Box A:

$l = 3\ cm\ , b = 8\ cm, h = 20\ cm$

Total surface area of box A $= 2(lb + bh + lh) = 2(3 \times 8 + 8 \times 20 + 3 \times 20)$

$= 2(24 + 160 + 60) = 2 \times 244 = 488\ cm^2$

Box B:

$l = 4\ cm\ , b = 12cm, h = 10\ cm$

Total surface area of box A $= 2(lb + bh + lh) = 2(4 \times 12 + 12 \times 10 + 4 \times 10)$

$= 2(48 + 120 + 40) = 2 \times 208 = 416\ cm^2$

Box B will be economical for the company.

Suggested Box:

$l = 6\ cm\ , b = 8\ cm, h = 10\ cm$

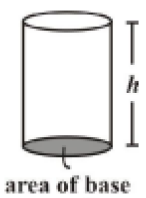
Total surface area of box A $= 2(lb + bh + lh) = 2(6 \times 8 + 8 \times 10 + 6 \times 10)$

$= 2(48 + 80 + 60) = 2 \times 188 = 376\ cm^2$

Cylinder

Volume of cylinder = area of base × height

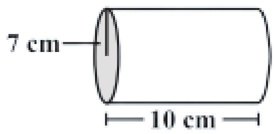
$$= \boldsymbol{\pi r^2 \times h = \pi r^2 h}$$



TRY THESE

Find the volume of the following cylinders.

(i) Radius$(r) = 7\ cm.$

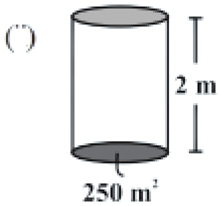


Height $(h) = 10\ cm.$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10 = 440\ cm^3$

(ii) Height $(h) = 2\ m.$



Area of base $(A) = 250\ m^2$

Volume of cylinder = Area of base × height

$= 250\ m^2 \times 2\ m = 500\ m^3$

Volume and Capacity

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

$$1\text{mL} = 1\ cm^3$$
$$1\text{ L} = 1000\ cm^3$$
$$1m^3 = 1000000\ cm^3 = 1000\text{L}$$

Ex 8: Find the height of a cuboid whose volume is 275 cm³ and base area is 25 cm².

Sol: Volume of a cuboid = 275 cm³

Base area × Height=275 cm³

25 × Height = 275

Height $= \frac{275}{25} = 11\ cm$

Height of the cuboid is 11 cm.

Ex 9: A godown is in the form of a cuboid of measures 60 m × 40 m × 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 0.8 m³ ?

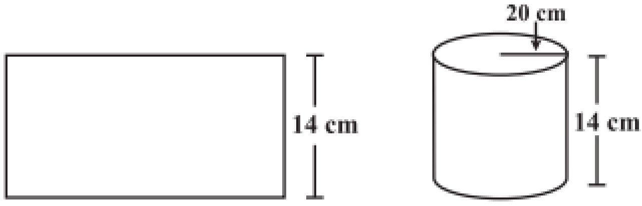
$Sol: Volume\ of\ one\ box\ = 0.8 = \frac{8}{10}\ m^3$

$Volume\ of\ godown = 60 \times 40 \times 30\ m^3$

Required number of boxes $= \frac{\text{Volume of godown}}{\text{Volume of one box}} = \frac{60 \times 40 \times 30}{\frac{8}{10}} = \frac{60 \times \cancel{40}^{5} \times 30 \times 10}{\cancel{8}}$

$= 60 \times 5 \times 30 \times 10 = 90{,}000$

Ex 10: A rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder (Take π=$\frac{22}{7}$)



Sol: Height$(h) = 14\ cm$

Radius$(r) = 20\ cm$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{\cancel{7}} \times 20 \times 20 \times \cancel{14}^2 = 22 \times 20 \times 20 \times 2$

$= 17600\ cm^3$

Ex11: A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

Sol: Let radius of the cylinder $= r$ and height $= h = 4\ cm$

Perimeter of the base of the cylinder=Length of paper

$2\pi r = 11$

$2 \times \frac{22}{7} \times r = 11$

$r = \frac{11 \times 7}{2 \times 22} = \frac{7}{4}\ cm$

Volume of cylinder $= \pi r^2 h = \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{4} \times \frac{7}{4} \times 4 = \frac{\cancel{22}^{11} \times 7}{\cancel{4}_2} = \frac{77}{2} = 38.5\ cm^3$

EXERCISE 9.3

1. **Given a cylindrical tank, in which situation will you find surface area and in which situation volume.**

(a) To find how much it can hold.

Sol: Volume

(b) Number of cement bags required to plaster it.

Sol: Surface area.

(c) To find the number of smaller tanks that can be filled with water from it.

Sol: Volume.

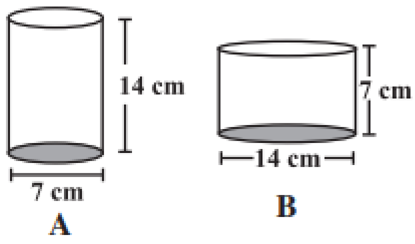
2. **Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area?**

Sol: The diameter of cylinder B is double to cylinder A. So, The cylinder B volume is greater.

Cylinder A:

Diamter(d) $= 7$cm; Radius$(r) = \frac{7}{2}\ cm.$

Height(h) $= 14\ cm.$



Volume of cylinder $A = \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 14 = 539\ cm^3$

Surface area(TSA)$of\ A = 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \left(14 + \frac{7}{2}\right) = 22 \times \frac{35}{2} = 11 \times 35 = 385\ cm^2$

Cylinder B:

Diamter(d) $=$ 14cm; Radius(r) $= \frac{14}{2} = 7\ cm.$

Height$(h) = 7\ cm.$

Volume of cylinder $A = \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 7 = 1078\ cm^3$

Surface area$(TSA) of\ B = 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 7 \times (7 + 7) = 44 \times 14 = 616\ cm^2$

From above cylinder B has greater volume and also greater surface area.

Conclusion: The cylinder with greater volume has also greater surface area.

3. Find the height of a cuboid whose base area is 180 cm² and volume is 900 cm³ ?

Sol: Base area of cuboid (A)=180 cm^2

Volume of cuboid=900 cm^3

Base area ×Height=900 cm^3

180×h=900

$h = \frac{900}{180} = 5\ cm$

∴ Height of cuboid=5 cm.

4. A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?

Sol: Volume of cuboid= 60 cm × 54 cm × 30 cm=60×54×30 cm^3

Volume of cube=6 cm×6 cm×6 cm=6×6×6 cm^3

No. of cubes that can be placed in cuboid $= \frac{\text{Volume of cuboid}}{\text{Volume of cube}} = \frac{60 \times 54 \times 30}{6 \times 6 \times 6}$

$= 10 \times 9 \times 5 = 450$

5. Find the height of the cylinder whose volume is 1.54 m³ and diameter of the base is 140 cm?

Sol: Diameter of the base of cylinder (d)= 140 cm

Radius$(r) = \frac{d}{2} = \frac{140}{2} = 70\ cm = \frac{70}{100} m = \frac{7}{10}\ m.$

Volume of cylinder=1.54 m^3

$\pi r^2 h = 1.54$

$\frac{22}{\cancel{7}} \times \frac{\cancel{7}}{10} \times \frac{7}{10} \times h = \frac{154}{100}$

$h = \frac{154 \times 10 \times 10}{100 \times 22 \times 7} = \frac{154}{154} = 1m$

$\therefore$ The height of the cylinder=1 m.

6. **A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?**

$\boxed{1\ m^3 = 1000\ L}$

Sol: Milk tank (Cylinder)radius $= 1.5\ m = \frac{15}{10} = \frac{3}{2}\ m$

Length (h)=7 m.

Volume of tank $= \pi r^2 h = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 7 = \frac{11 \times 9}{2} = \frac{99}{2} = 49.5\ m^3$

The quantity of milk in litres that can be stored in the tank=$49.5 \times 1000\ L = 48500\ L$

7. **If each edge of a cube is doubled, (i) how many times will its surface area increase? (ii) how many times will its volume increase?**

Sol: Let side of cube=l

Surface area=$6\ l^2,\ Volume = l^3$

If each edge of cube is doubled then side=$2l$

Surface area of new cube =$6\ (2l)^2 = 6 \times 4l^2 = 24l^2 = 4 \times (6\ l^2)$

$\therefore$ Surface area of cube increased 4 times

Volume of new cube $= (2l)^3 = 8 \times l^3$

$\therefore$ Volume of cube increased 8 times.

8. **Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3 , find the number of hours it will take to fill the reservoir.**

Sol: Volume of cuboidal reservoir=108 m^3 =108×1000 L=108000 L

Rate of pouring the water per 1minute= 60 litres.

Number of minutes take to fill the reservoir $= \frac{\text{Volume of reservoir}}{60\ \text{L}}$

$$= \frac{10800\cancel{0}}{6\cancel{0}} = 1800\ minutes$$

It will take 1800 minutes = 30 hours to fill the reservoir.

AP VIII CLASS-CBSE (2023-24)
EXPONENTS AND POWERS (Notes)

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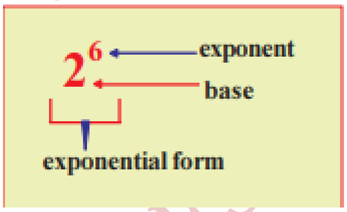
1. $a \times a = a^2$ ('a' squared)
2. $a \times a \times a = a^3$ ('a' cubed)
3. $a \times a \times a \times a = a^4$ ('a' raised to the power of 4)
4. $a \times a \times a \times a \times a = a^5$ ('a' raised to the power of 5)
5. $a \times a \times a \times a \times a \times \ldots' n' = a^n$, Where 'a' is the base and 'n' is the exponent. This is said to be exponential form.

$\underbrace{a^n}_{Exponential\ form}$ ← $Exponent$, ← $Base$



6. For any nonzero integer 'a' and integers 'm' and 'n'

(i) $a^m \times a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $a^m \times b^m = (a \times b)^m$

(iv) $\frac{a^m}{a^n} = a^{m-n}\ if\ m > n$

(v) $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}\ if\ n > m$

(vi) $a^0 = 1\ (a \neq 0)$

(vii) $\frac{a^m}{a^n} = \left(\frac{a}{b}\right)^m$

(viii) $1^{any\ number} = 1$

(ix) $(-1)^{\text{even number}} = 1$

(x) $(-1)^{\text{odd number}} = -1$

7. For any non-zero integer, $a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m .Also $\frac{1}{a^{-m}} = a^m$. $(a^m \times a^{-m} = 1)$

$(i)\ \frac{1}{a^{-1}} = a\ ;\ a^{-1} = \frac{1}{a}$ $\quad (ii)\ \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ $\quad (iii)\ \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

8.

$2^2 = 4$	$2^{10} = 1024$	$4^4 = 256$	$7^2 = 49$
$2^3 = 8$	$3^2 = 9$	$4^5 = 1024$	$7^3 = 343$
$2^4 = 16$	$3^3 = 27$	$5^2 = 25$	$8^2 = 64$
$2^5 = 32$	$3^4 = 81$	$5^3 = 125$	$8^3 = 512$
$2^6 = 64$	$3^5 = 243$	$5^4 = 625$	$9^2 = 81$
$2^7 = 128$	$3^6 = 729$	$6^2 = 36$	$9^3 = 729$
$2^8 = 256$	$4^2 = 16$	$6^3 = 216$	$10^2 = 100$
$2^9 = 512$	$4^3 = 64$	$6^4 = 1296$	$10^3 = 1000$

TRY THESE

Find the multiplicative inverse of the following

The multiplicative inverse of $a^{-m} = a^m$

(i) The multiplicative inverse of $2^{-4} = 2^4$

(ii) The multiplicative inverse of $10^{-5} = 10^5$

(iii) The multiplicative inverse of $7^{-2} = 7^2$

(iv) The multiplicative inverse of $5^{-3} = 5^3$

(v) The multiplicative inverse of $10^{-100} = 10^{100}$

Expand the following numbers using exponents.

(i) 1025.63

Sol: $1025.63 = 1000 + 20 + 3 + \frac{6}{10} + \frac{3}{100}$

$= 1 \times 1000 + 2 \times 10 + 3 \times 1 + 6 \times \frac{1}{10} + 3 \times \frac{1}{100}$

$= 1 \times 10^3 + 2 \times 10^1 + 3 \times 10^0 + 6 \times \frac{1}{10^1} + 3 \times \frac{1}{10^2}$

$= 1 \times 10^3 + 2 \times 10^1 + 3 \times 10^0 + 6 \times 10^{-1} + 3 \times 10^{-2}$

$\boxed{\frac{1}{a^m} = a^{-m}}$

(ii) 1256.249

Sol: $1256.249 = 1000 + 200 + 50 + 6 + \frac{2}{10} + \frac{4}{100} + \frac{9}{1000}$

$= 1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1 + 2 \times \frac{1}{10} + 4 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 2 \times \frac{1}{10^1} + 4 \times \frac{1}{10^2} + 9 \times \frac{1}{10^3}$

$= 1 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 6 \times 10^0 + 2 \times 10^{-1} + 4 \times 10^{-2} + 9 \times 10^{-3}$

Simplify and write in exponential form.

$(i)\ (-2)^{-3} \times (-2)^{-4} = (-2)^{-3+(-4)} = (-2)^{-7}$

$(ii)\ p^3 \times p^{-10} = p^{3+(-10)} = p^{-7}$

$(iii)\ 3^2 \times 3^{-5} \times 3^6 = 3^{2+(-5)+6} = 3^3$

Example 1: Find the value of

$(i) 2^{-3} = \frac{1}{2^3} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$

$(ii) \frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

$$a^{-m} = \frac{1}{a^m}$$

$$\frac{1}{a^{-m}} = a^m$$

Example 2: Simplify

$(i)\ (-4)^5 \times (-4)^{-10}$

Sol: $(-4)^5 \times (-4)^{-10} = (-4)^{5+(-10)} = (-4)^{-5} = \frac{1}{(-4)^5}$

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(ii) $2^5 \div 2^{-6} = \frac{2^5}{2^{-6}} = 2^{5-(-6)} = 2^{5+6} = 2^{11}$

Example 3: Express 4^{-3} as a power with the base 2.

Sol: $4 = 2 \times 2 = 2^2$

$4^{-3} = (2^2)^{-3} = 2^{2\times(-3)} = 2^{-6}$

$(a^m)^n = a^{mn}$

Example 4: Simplify and write the answer in the exponential form.

(i) $\left(\mathbf{2^5 \div 2^8}\right)^{\mathbf{5}} \times \mathbf{2^{-5}}$

Sol: $(2^5 \div 2^8)^5 \times 2^{-5}$

$= (2^{5-8})^5 \times 2^{-5}$

$= (2^{-3})^5 \times 2^{-5}$

$= 2^{-15} \times 2^{-5} = 2^{-15-5}$

$= 2^{-20} = \frac{1}{2^{20}}$

(ii) $\mathbf{(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}}$

$a^m \times b^m = (a \times b)^m$

Sol: $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = 100^{-3} = \frac{1}{100^3}$

(iii) $\mathbf{\frac{1}{8} \times (3)^{-3}}$

Sol: $\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \times (3)^{-3} = (2)^{-3} \times (3)^{-3} = [2 \times 3]^{-3} = 6^{-3} = \frac{1}{6^3}$

(iv) $\mathbf{(-3)^4 \times \left(\frac{5}{3}\right)^4}$

$(-a)^n = a^n$ *if n is even*

$(-a)^n = -a^n$ *if n is odd*

Sol: $(-3)^4 \times \left(\frac{5}{3}\right)^4 = 3^4 \times \frac{5^4}{3^4} = 5^4$

Example 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Sol: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

If bases($\neq 0, \pm 1$) are equal, then their exponents must be equal.

$m + 6 = 7$

$m = 7 - 6 = 1$

Example 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.

Sol: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$

Example 7: Simplify (i) $\left\{\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-2}$

Sol: $\left\{\left(\frac{1}{3}\right)^{-2}-\left(\frac{1}{2}\right)^{-3}\right\}\div\left(\frac{1}{4}\right)^{-2}=\left\{\left(\frac{3}{1}\right)^{2}-\left(\frac{2}{1}\right)^{3}\right\}\div\left(\frac{4}{1}\right)^{2}$

$=(3^2-2^3)\div 4^3=(9-8)\div 16=\frac{1}{16}$

(*ii*) $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5}$

Sol: $\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-5}=\left(\frac{8}{5}\right)^{7}\times\left(\frac{5}{8}\right)^{5}=\frac{8^7}{5^7}\times\frac{5^5}{8^5}=\frac{8^{7-5}}{5^{7-5}}=\frac{8^2}{5^2}=\frac{16}{25}$

EXERCISE 10.1

1. Evaluate

(*i*) $3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

(*ii*) $(-4)^{-2}=\frac{1}{(-4)^2}=\frac{1}{(-4)(-4)}=\frac{1}{16}$

(*iii*) $\left(\frac{1}{2}\right)^{-5}=\left(\frac{2}{1}\right)^{5}=2^5=32$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5\div(-4)^8=(-4)^{5-8}=(-4)^{-3}=\frac{1}{(-4)^3}$

(ii) $\left(\frac{1}{2^3}\right)^{2}=\frac{1^2}{(2^3)^2}=\frac{1}{2^6}=\left(\frac{1}{2}\right)^6$

(iii) $(-3)^4\times\left(\frac{5}{3}\right)^4=3^4\times\frac{5^4}{3^4}=5^4$

(iv) $(3^{-7}\div 3^{-10})\times 3^{-5}=\frac{3^{-7}}{3^{-10}}\times 3^{-5}=\frac{3^{10}}{3^7}\times\frac{1}{3^5}=\frac{3^{10}}{3^{12}}=\frac{1}{3^{12-10}}=\frac{1}{3^2}=\left(\frac{1}{3}\right)^2$

(v) $2^{-3}\times(-7)^{-3}=(2\times -7)^{-3}=(-14)^{-3}=\frac{1}{(-14)^3}=\left(\frac{1}{-14}\right)^3$

3. Find the value of

(i) $(3^0+4^{-1})\times 2^2=\left(1+\frac{1}{4}\right)\times 4=\frac{5}{4}\times 4=5$

(ii) $(2^{-1}\times 4^{-1})\div 2^{-2}=\left(\frac{1}{2}\times\frac{1}{4}\right)\div\frac{1}{2^2}=\frac{1}{8}\div\frac{1}{4}=\frac{1}{8}\times\frac{4}{1}=\frac{1}{2}$

(iii) $\left(\frac{1}{2}\right)^{-2}+\left(\frac{1}{3}\right)^{-2}+\left(\frac{1}{4}\right)^{-2}=2^2+3^2+4^2=4+9+16=29$

(*iv*) $(3^{-1}+4^{-1}+5^{-1})^0=1$

$$(v)\ \left\{\left(\frac{-2}{3}\right)^{-2}\right\}^{2}=\left(\frac{-2}{3}\right)^{-2\times2}=\left(\frac{-2}{3}\right)^{-4}=\left(\frac{-3}{2}\right)^{4}=\frac{(-3)^4}{2^4}=\frac{81}{16}$$

4. Evaluate

$$(i)\ \frac{8^{-1}\times5^3}{2^{-4}}=\frac{2^4\times5^3}{8^1}=\frac{16\times125}{8}=2\times125=250$$

$$(ii)\ (5^{-1}\times2^{-1})\times6^{-1}=\left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6}=\frac{1}{10}\times\frac{1}{6}=\frac{1}{60}$$

5. Find the value of m for which $5^m\div5^{-3}=5^5$

Sol: $5^m\div5^{-3}=5^5$

$5^{m-(-3)}=5^5$

$5^{m+3}=5^5$

Bases (≠0,+1,-1) same, so their exponents must be equal.

$m+3=5$

$m=5-3$

$m=2$

6. Evaluate

$$\text{(i)}\ \left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=(3-4)^{-1}=(-1)^{-1}=\frac{1}{(-1)}=-1$$

$$(ii)\ \left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\left(\frac{8}{5}\right)^{7}\times\left(\frac{5}{8}\right)^{4}=\frac{8^7}{5^7}\times\frac{5^4}{8^4}=\frac{8^{7-4}}{5^{7-4}}=\frac{8^3}{5^3}=\frac{512}{125}$$

7. Simplify

$$\boldsymbol{(i)\ \frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}(t\neq0)}$$

Sol: $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}$

$=\frac{5^1\times t^{-4}}{5^{-3}\times2\times t^{-8}}$

$=\frac{5^{1+3}\times t^{-4+8}}{2}$

$=\frac{5^4\times t^4}{2}=\frac{625}{2}t^4$

$$\boldsymbol{(ii)\ \frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}}$$

Sol: $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}=\frac{3^{-5}\times(2\times5)^{-5}\times5^3}{5^{-7}\times(2\times3)^{-5}}$

$=\frac{3^{-5}\times(2\times5)^{-5}\times5^3}{5^{-7}\times(2\times3)^{-5}}$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^{3}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$
$$= 5^{-5+3+7} = 5^{5}$$

Use of Exponents to Express Small Numbers and large Numbers in Standard Form

1. The distance from the Earth to the Sun is **149,600,000,000 m.=1.496×10^{11}m**
2. The speed of light is **300,000,000 m/sec=3×10^{8} m/sec.**
3. Thickness of Class VII Mathematics book is 20 mm
4. The average diameter of a Red Blood Cell is **0.000007 mm=7×10^{-6}mm**
5. The thickness of human hair is in the range of **0.005 cm to 0.01 cm=**$\boldsymbol{5 \times 10^{-3} cm\ to\ 10^{-2} cm}$
6. The distance of moon from the Earth is $\mathbf{384,467,000\ m = 3.84467 \times 10^{8}}\ \boldsymbol{m}$.
7. The size of a plant cell is **0.00001275 m=** $\mathbf{1.275 \times 10^{-5}}\boldsymbol{m}$
8. Average radius of the Sun is **695000 km=** $\mathbf{6.95 \times 10^{5}}\boldsymbol{km}$
9. Mass of propellant in a space shuttle solid rocket booster is **503600 kg=** $\mathbf{5.036 \times 10^{5}}\boldsymbol{kg}$
10. Thickness of a piece of paper is **0.0016 cm=** $\mathbf{1.6 \times 10^{-3}}\boldsymbol{cm}$
11. Diameter of a wire on a computer chip is **0.000003 m=** $\mathbf{3 \times 10^{-6}}\boldsymbol{m}$
12. The height of Mount Everest is 8848 m=8.848×10^{3}m.

TRY THESE

1. Write the following numbers in standard form

$(i)\mathbf{0.000000564} = 5.64 \times 10^{-7}$

$(\mathbf{ii})\mathbf{0.0000021} = 2.1 \times 10^{-6}$

$(\mathbf{iii})\mathbf{21600000} = 2.16 \times 10^{8}$

$(\mathbf{iv})\mathbf{15240000} = 1.524 \times 10^{7}$

Comparing very large and very small numbers

(i) Diameter of the Sun $= 1.4 \times 10^{9}$ m

(ii) Diameter of the earth $= 1.2756 \times 10^{7}$ m

(iii) Size of Red Blood cell $= 0.000007$ m $= 7 \times 10^{-6}$ m

(iv) Size of plant cell $= 0.00001275 = 1.275 \times 10^{-5}$ m

(v) Mass of earth is 5.97×10^{24} kg

(vi) Mass of moon is 7.35×10^{22} kg

(vii) Total mass of earth and moon $= 5.97 \times 10^{24}$ kg $+ 7.35 \times 10^{22}$ kg.

$= 5.97 \times 10^{2} \times 10^{22}\ kg + 7.35 \times 10^{22}\ kg.$

$= 5.97 \times 100 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg.

$= 597 \times 10^{22}$ kg $+ 7.35 \times 10^{22}$ kg.

$= (597 \ + 7.35) \times 10^{22}$ kg.

$= 604.35 \times 10^{22} kg.$

$(viii)$ Distance between Sun and Earth $= 1.496 \times 10^{11}$m

(ix) Distance between Earth and Moon $= 3.84 \times 10^{8}$m

(x) Distance between Sun and Moon $= 1.496 \times 10^{11}\text{m} - 3.84 \times 10^{8}\text{m}$

$= 1.496 \times 10^{3} \times 10^{8}m - 3.84 \times 10^{8}m$

$= 1.496 \times 1000 \times 10^{8}m - 3.84 \times 10^{8}\text{m}$

$= 1496 \times 10^{8}m - 3.84 \times 10^{8}m$

$= (1496 - 3.84) \times 10^{8}m$

$= 1492.16 \times 10^{8}m$

Example 8: Express the following numbers in standard form.

$(i)\mathbf{0.000035} = 3.5 \times 10^{-5}$

$(ii)\mathbf{4050000} = 4.05 \times 10^{6}$

Example 9: Express the following numbers in usual form

$(i)\mathbf{3.52 \times 10^{5}} = 352000$

$(ii)\mathbf{7.54 \times 10^{-4}} = 0.000754$

$(iii)\ \mathbf{3 \times 10^{-5}} = 0.00003$

EXERCISE 10.2

1. Express the following numbers in standard form

(i) $\mathbf{0.0000000000085} = 8.5 \times 10^{-12}$

(ii) $\mathbf{0.00000000000942} = 9.42 \times 10^{-12}$

(iii) $\mathbf{6020000000000000} = 6.02 \times 10^{15}$

(iv) $\mathbf{0.00000000837} = 8.37 \times 10^{-9}$

(v) $\mathbf{31860000000} = 3.186 \times 10^{10}$

2. Express the following numbers in usual form.

(i) $\mathbf{3.02 \times 10^{-6}} = 0.00000302$

(ii) $\mathbf{4.5 \times 10^{4}} = 45000$

(iii) $\mathbf{3 \times 10^{-8}} = 0.00000003$

(iv) $\mathbf{1.0001 \times 10^{9}} = 1000100000$

(v) $\mathbf{5.8 \times 10^{12}} = 5800000000000$

$(vi)\mathbf{3.61492 \times 10^{6}} = 3614920$

3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m $= \frac{1}{10^{6}} = 1 \times 10^{-6}m$

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb $= 1.6 \times 10^{-19}$ coulomb

(iii) Size of a bacteria is 0.0000005 m $= 5 \times 10^{-7} m$

(iv) Size of a plant cell is 0.00001275 m $= 1.275 \times 10^{-5} m$

(v) Thickness of a thick paper is 0.07 mm $= 7 \times 10^{-2} mm$

4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Sol: Thickness of book=20mm

Thickness of paper sheet=0.016 mm

Number of books=5 and Number of paper sheets=5

The total thickness of the stack=5×20+5×0.016

=100+0.08=100.08

$=1.0008\times10^{2}$ mm

If x and y are any two quantities such that both of them increase or decrease together and $\frac{x}{y}$ is constant (say k), then we say that x and y are in direct proportion. This is written as $x \propto y$ and read as x is directly proportional to y.

x *and* y are in direct proportion, if $\frac{x}{y} = k \Rightarrow x = ky$ *where k is constant of proportion*

If y_1 and y_2 are the values of y corresponding to the values of x_1 and x_2 of x respectively,

$$\text{then } \frac{x_1}{y_1} = \frac{x_2}{y_2} \ (or) \frac{x_1}{x_2} = \frac{y_1}{y_2}$$

TRY THESE

1. **Observe the following tables and find if x and y are directly proportional.**

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

$$\frac{x_1}{y_1} = \frac{20}{40} = \frac{1}{2}; \quad \frac{x_2}{y_2} = \frac{17}{34} = \frac{1}{2}; \quad \frac{x_3}{y_3} = \frac{14}{28} = \frac{1}{2}; \quad \frac{x_4}{y_4} = \frac{11}{22} = \frac{1}{2}; \quad \frac{x_5}{y_5} = \frac{8}{16} = \frac{1}{2};$$

$$\frac{x_6}{y_6} = \frac{5}{10} = \frac{1}{2}; \quad \frac{x_7}{y_7} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \frac{x}{y} = \frac{1}{2} = constant \ \Rightarrow x \text{ and } y \text{ are directly proportional}$$

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

$$\frac{x_1}{y_1} = \frac{6}{4} = \frac{3}{2}; \quad \frac{x_2}{y_2} = \frac{10}{8} = \frac{5}{4}; \quad \frac{x_3}{y_3} = \frac{14}{12} = \frac{7}{6}; \quad \frac{x_4}{y_4} = \frac{18}{16} = \frac{9}{8};$$

$$\frac{x_5}{y_5} = \frac{22}{20} = \frac{11}{10}; \quad \frac{x_6}{y_6} = \frac{26}{24} = \frac{13}{12}; \quad \frac{x_7}{y_7} = \frac{30}{28} = \frac{15}{14}$$

$$\therefore \frac{x}{y} \textit{ is not constant } \Rightarrow x \textit{ and } y \textit{ are not in directly proportional}$$

(iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

$\frac{x_1}{y_1} = \frac{5}{15} = \frac{1}{3};$ $\frac{x_2}{y_2} = \frac{8}{24} = \frac{1}{3};$ $\frac{x_3}{y_3} = \frac{12}{36} = \frac{1}{3};$

$\frac{x_4}{y_4} = \frac{15}{60} = \frac{1}{4};$ $\frac{x_5}{y_5} = \frac{18}{72} = \frac{1}{4};$ $\frac{x_6}{y_6} = \frac{20}{10} = \frac{1}{5}$

$\therefore \frac{x}{y}\, is\ not\ constant \ \Rightarrow x\ and\ y\ are\ not\ in\ directly\ proportional$

2. **Principal = ₹ 1000, Rate = 8% per annum. Fill in the following table and find which type of interest (simple or compound) changes in direct proportion with time period.**

Sol: P=1000; R=8%

(i) T=1 y

$$S.I = \frac{P \times R \times T}{100} = \frac{1000 \times 8 \times 1}{100} = ₹80$$

$$C.I = P\left(1 + \frac{R}{100}\right)^T - P = 1000\left(1 + \frac{8}{100}\right)^1 - 1000$$

$$= 1000 \times \frac{108}{100} - 1000$$

$$= 1080 - 1000 = ₹80$$

(ii) T=2 y

$$S.I = \frac{P \times R \times T}{100} = \frac{1000 \times 8 \times 2}{100} = ₹160$$

$$C.I = P\left(1 + \frac{R}{100}\right)^T - P = 1000\left(1 + \frac{8}{100}\right)^2 - 1000$$

$$= 1000 \times \frac{108}{100} \times \frac{108}{100} - 1000$$

$$= 1166.40 - 1000 = ₹166.40$$

(iii) T=3 y

$$S.I = \frac{P \times R \times T}{100} = \frac{1000 \times 8 \times 3}{100} = ₹240$$

$$C.I = P\left(1 + \frac{R}{100}\right)^T - P = 1000\left(1 + \frac{8}{100}\right)^3 - 1000$$

$$= 1000 \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100} - 1000$$

$$= 1259.70 - 1000 = ₹259.70$$

$$For\ 1year\ \frac{S.I}{T} = \frac{80}{1} = 80$$

$$For\ 2years\ \frac{S.I}{T} = \frac{160}{2} = 80$$

$$For\ 3year\ \frac{S.I}{T} = \frac{240}{3} = 80$$

$$\frac{S.I}{T}\ is\ constant$$

Simple interest (S.I) and time period (T) are in direct proportion.

$$For\ 1year\ \frac{C.I}{T} = \frac{80}{1} = 80$$

$$For\ 2years\ \frac{C.I}{T} = \frac{166.40}{2} = 83.20$$

$$For\ 3year\ \frac{C.I}{T} = \frac{259.70}{3} = 80$$

C.I and time period (T) are not in direct proportion.

Example 1: The cost of 5 metres of a particular quality of cloth is ₹ 210. Tabulate the cost of 2, 4, 10 and 13 metres of cloth of the same type.

Solution: .

Length of cloth: x (m)	$5(x_1)$	$2(x_2)$	$4(x_3)$	$10(x_4)$	$13(x_5)$
Cost: y(₹)	$210(y_1)$	y_2	y_3	y_4	y_5

Length of cloth is directly proportional to cost of cloth.

$$(i)\frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{5}{210} = \frac{2}{y_2} \Rightarrow y_2 = \frac{2 \times 210}{5} = 2 \times 42 = 84$$

$$(i)i\frac{x_1}{y_1} = \frac{x_3}{y_3} \Rightarrow \frac{5}{210} = \frac{4}{y_3} \Rightarrow y_3 = \frac{4 \times 210}{5} = 4 \times 42 = 168$$

$$(iii)\frac{x_1}{y_1} = \frac{x_4}{y_4} \Rightarrow \frac{5}{210} = \frac{10}{y_4} \Rightarrow y_4 = \frac{10 \times 210}{5} = 10 \times 42 = 420$$

$$(vi)\frac{x_1}{y_1} = \frac{x_5}{y_5} \Rightarrow \frac{5}{210} = \frac{13}{y_5} \Rightarrow y_5 = \frac{13 \times 210}{5} = 13 \times 42 = 546$$

Example 2: An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Sol:

height of the object (in metres)	$14(x_1)$	$x(x_2)$
length of the shadow (in metres)	$10(y_1)$	$15(y_1)$

Length of the shadow is directly proportional to height of the object

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{14}{10} = \frac{x}{15}$$

$$\Rightarrow x \times 10 = 14 \times 15$$

$$\Rightarrow x = \frac{14 \times 15}{10} = 21$$

Height of the tree is 21 metres.

Example 3: If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

Sol:

Number of sheets	$12(x_1)$	$x(x_2)$
Weight of sheets (in grams)	$40(y_1)$	$2500(y_2)$

Number of sheets is directly proportional to Weight of sheets.

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} \Rightarrow \frac{12}{40} = \frac{x}{2500}$$

$$\Rightarrow x \times 40 = 12 \times 2500$$

$$\Rightarrow x = \frac{12 \times 2500}{40} = 750$$

Thus, the required number of sheets of paper $= 750$

Example 4: A train is moving at a uniform speed of 75 km/hour. (i) How far will it travel in 20 minutes? (ii) Find the time required to cover a distance of 250 km.

Sol:

Distance travelled (in km)	$75(x_1)$	$x(x_2)$	$250(x_3)$
Time taken (in minutes)	$60(y_1)$	$20(y_2)$	$y(y_5)$

The distance covered would be directly proportional to time. (Speed is uniform)

$$(i)\ \frac{75}{60} = \frac{x}{20} \Rightarrow x \times 60 = 75 \times 20$$

$$\Rightarrow x = \frac{75 \times 20}{60} = 25$$

So, the train will cover a distance of 25 km in 20 minutes.

$$(ii)\ \frac{75}{60} = \frac{250}{y} \Rightarrow y \times 75 = 250 \times 60$$

$$\Rightarrow y = \frac{250 \times 60}{75} = 200$$

The time required to cover a distance of 250 km is 200 minutes=3 hours 20 minutes.

Example 5: The scale of a map is given as 1:30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.

Sol:

Distance on map (in cm)	1	4
Actual distance(in cm)	30000000	y

Distance on map is directly proportional to Actual distance.

$$\frac{1}{30000000} = \frac{4}{y}$$

$$\boxed{\begin{array}{c} 1\ km = 100\ m \\ 1\ km = 100 \times 100 cm \end{array}}$$

$$\Rightarrow y = 12000000cm = \frac{12000000}{100 \times 100} km = 1200\ km$$

The actual distance between two cities=1200km

EXERCISE 11.1

1. Following are the car parking charges near a railway station up to

parking time(x)(hours)	$4(x_1)$	$8(x_2)$	$12(x_3)$	$24(x_4)$
parking charges(y)	$60(y_1)$	$100(y_1)$	$140(y_1)$	$180(y_1)$

$$\frac{x_1}{y_1} = \frac{4}{60} = \frac{1}{15};\ \frac{x_2}{y_2} = \frac{8}{100} = \frac{2}{25};\ \frac{x_3}{y_3} = \frac{12}{140} = \frac{3}{35};\ \frac{x_4}{y_4} = \frac{24}{180} = \frac{2}{15}$$

$$\therefore \frac{x}{y} \textit{is not constant } \Rightarrow x \textit{ and } y \textit{ are not in directly proportional}$$

The parking charges are not in direct proportion to the parking time.

2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment(x)	$1(x_1)$	$4(x_2)$	$7(x_3)$	$12(x_4)$	$20(x_5)$
Parts of base(y)	$8\ (y_1)$	y_2	y_3	y_4	y_5

$$x, y \textit{ are in direct proportion}$$

$$\frac{y_2}{x_2} = \frac{y_1}{x_1} \Rightarrow \frac{y_2}{4} = \frac{8}{1} \Rightarrow y_2 = 4 \times 8 = 32$$

$$\frac{y_3}{x_3} = \frac{y_1}{x_1} \Rightarrow \frac{y_3}{7} = \frac{8}{1} \Rightarrow y_3 = 7 \times 8 = 56$$

$$\frac{y_4}{x_4} = \frac{y_1}{x_1} \Rightarrow \frac{y_4}{12} = \frac{8}{1} \Rightarrow y_4 = 12 \times 8 = 96$$

$$\frac{y_5}{x_5} = \frac{y_1}{x_1} \Rightarrow \frac{y_5}{20} = \frac{8}{1} \Rightarrow y_5 = 20 \times 8 = 160$$

3. **In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?**

$Sol: x_1 = 1\ , y_1 = 75\ ml$

$If\ x_2 =?\ , y_2 = 1800\ ml$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x_2}{1800} = \frac{1}{75} \Rightarrow x_2 = \frac{1800}{75} = 24$$

4. **A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?**

Sol:

Number of bottles(x)	$840(x_1)$	$x(x_2)$
Time(hours)(y)	$6(y_1)$	$5(y_2)$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x}{5} = \frac{840}{6} \Rightarrow x = \frac{840 \times 5}{6} = 140 \times 5 = 700$$

700 bottles will it fill in five hours.

5. **A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the actual length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?**

Sol:

Enlarged length(cm) (x)	$5(x_1)$	$x(x_2)$
Enlarged times(y)	$50{,}000(y_1)$	$20{,}000(y_2)$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x}{20000} = \frac{5}{50000} \Rightarrow x = \frac{20000 \times 5}{50000} = 2$$

Required length=2cm

6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

Sol:

Length of modal ship(x) (cm)	$9(x_1)$	$x(x_2)$
Length of original ship(y)(m)	$12(y_1)$	$28(y_2)$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x}{28} = \frac{9}{12} \Rightarrow x = \frac{9 \times 28}{12} = 21$$

Required length of model ship =21 cm

7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

Sol:

Number of Crystals(x)	$9 \times 10^6(x_1)$	x_2	x_3
Weight of sugar(kg) (y)	$2(y_1)$	$5(y_2)$	$1.2(y_3)$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x_2}{5} = \frac{9 \times 10^6}{2} \Rightarrow x_2 = \frac{9 \times 10^6 \times 5}{2} = 22.5 \times 10^6$$

$$\frac{x_3}{y_3} = \frac{x_1}{y_1} \Rightarrow \frac{x_3}{1.2} = \frac{9 \times 10^6}{2} \Rightarrow x_3 = \frac{9 \times 10^6 \times 1.2}{2} = 5.4 \times 10^6$$

(i) 5 kg of sugar contains $22.5 \times 10^6 = 225 \times 10^5$ crystals

(ii) 1.2 kg of sugar contains $5.4 \times 10^6 = 54 \times 10^5$ crystals

8. **Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?**

Sol:

Distance in the map(cm) (x)	1	x
Distance on road(km) (y)	18	72

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x}{72} = \frac{1}{18} \Rightarrow x = \frac{72}{18} = 4$$

Required distance=4cm

9. **A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5m long.**

Sol:

Length of pole(m)	5.60	10.50	x
Length of shadow(m)	3.20	y	5

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{10.50}{y} = \frac{5.60}{3.20} \Rightarrow y = \frac{10.50 \times 3.20}{5.60} = 6$$

If the height of the pole is 10.5 m, then length of the shadow is 6 m

$$\frac{x_3}{y_3} = \frac{x_1}{y_1} \Rightarrow \frac{x}{5} = \frac{5.60}{3.20} \Rightarrow y = \frac{5 \times 5.60}{3.20} = 8.75$$

If the height of the pole is 5 m, then length of the shadow is 8.75 m.

10. **A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?**

Sol:

Distance(km)	$14(x_1)$	$x(x_2)$
Time(Minutes)	$25(y_1)$	$300(y_2)$

x, y *are in direct proportion*

$$\frac{x_2}{y_2} = \frac{x_1}{y_1} \Rightarrow \frac{x}{300} = \frac{14}{25} \Rightarrow x = \frac{300 \times 14}{25} = 12 \times 14 = 166\ km$$

$5\ hours = 300\ minutes$

The truck travels 166 km in 5 hours.

INVERSE PROPORTION:

Two quantities x and y are said to be in inverse proportion if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant.

That is, if xy = k, then x and y are said to vary inversely

(x varies inversely with y and y varies inversely with x. Thus two quantities x and y are said to vary in inverse proportion.)

$If\ x\ and\ y\ are\ in\ inverse\ proportion\ then\ x \propto \frac{1}{y}$

$\Rightarrow xy = k\ (k\ is\ constant\ of\ proportianality)$

If y_1 , y_2 are the values of y corresponding to the values x_1 , x_2 of x respectively then

$x_1\, y_1 = x_2\, y_2\ (= k), or\ \frac{x_1}{x_2} = \frac{y_2}{y_1}$

DO THIS

Take a squared paper and arrange 48 counters on it in different number of rows.

Number of rows(R)	(R_1) 2	(R_2) 3	(R_3) 4	(R_4) 6	(R_5) 8
Number of Columns(C)	(C_1) 24	(C_2) 16	(C_3) 12	(C_4) 8	(C_5) 6

(i) $\boldsymbol{Is\ R_1 : R_2 = C_2 : C_1}$?

$Sol: R_1 : R_2 = 2:3 \quad ; \quad C_2 : C_1 = 16:24 = 2:3$

$Yes\ R_1 : R_2 = C_2 : C_1$

(ii) $\boldsymbol{Is\ R_3 : R_4 = C_4 : C_3}$?

$Sol: R_3 : R_4 = 4:6 = 2:3 \quad and \quad C_4 : C_3 = 8:12 = 2:3$

$Yes\ R_3 : R_4 = C_4 : C_3$

(iii) Are R and C inversely proportional to each other?

Sol: $R \times C = 48(constant). So, R\ and\ C\ inversely\ proportional\ to\ each\ other.$

TRY THESE

Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	$50(x_1)$	$40(x_2)$	$30(x_3)$	$20(x_4)$
y	$5(y_1)$	$6(y_2)$	$7(y_3)$	$8(y_4)$

Sol:

$x_1y_1 = 50 \times 5 = 250$

$x_2y_2 = 40 \times 6 = 240$

$x_1y_1 \neq x_2y_2$

$x, y\ are\ not\ in\ inverse\ proportion.$

(ii)

x	$100(x_1)$	$200(x_2)$	$300(x_3)$	$400(x_4)$
y	$60(y_1)$	$30(y_2)$	$20(y_3)$	$15(y_4)$

Sol:

$x_1y_1 = 100 \times 60 = 6000$

$x_2y_2 = 200 \times 30 = 6000$

$x_3y_3 = 300 \times 20 = 6000$

$x_4y_4 = 400 \times 15 = 6000$

$xy = constant$

So, x, y are in inverse proportion.

(iii)

x	$90(x_1)$	$60(x_2)$	$45(x_3)$	$30(x_4)$	$20(x_5)$	$5(x_6)$
y	$10(y_1)$	$15(y_2)$	$20(y_3)$	$25(y_4)$	$30(y_5)$	$35(y_6)$

Sol:

$x_1y_1 = 90 \times 10 = 900$

$x_2y_2 = 60 \times 15 = 900$

$x_3y_3 = 45 \times 20 = 900$

$x_4y_4 = 20 \times 30 = 600$

$x_3y_3 \neq x_4y_4$

So, x, y are not in inverse proportion.

Example 7: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Sol:

Number of pipes	$6(x_1)$	$5(x_1)$
Time (in minutes)	$80(y_1)$	$y(y_2)$

Numbers of pipes inversely proportional to time takes fill the tank.

$$x_2\, y_2 = x_1\, y_1 \quad \Rightarrow 5 \times y = 6 \times 80$$

$$\Rightarrow y = \frac{6 \times 80}{5} = 6 \times 16 = 96$$

Thus, time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

Example 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Sol:

Number of students	$100(x_1)$	$125(x_1)$
Number of days	$20(y_1)$	$y(y_2)$

Food provision for number of students is inversely proportional to number of days.

$$x_2\, y_2 = x_1\, y_1 \quad \Rightarrow 125 \times y = 100 \times 20$$

$$\Rightarrow y = \frac{100 \times 20}{125} = 16$$

Thus, the provisions will last for 16 days, if 25 more students join the hostel.

Example 9: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Sol:

Number of hours	$48(x_1)$	$30(x_1)$
Number of workers	$15(y_1)$	$y(y_2)$

The number of hours and number of workers vary in inverse proportion.

$$30 \times y = 48 \times 15$$

$$\Rightarrow y = \frac{48 \times 15}{30} = 24$$

To finish the work in 30 hours, 24 workers are required.

EXERCISE 11.2

1. **Which of the following are in inverse proportion?**

(i) The number of workers on a job and the time to complete the job.

Sol: If the number of workers decreases, the time to complete the job increases in the same proportion.

So, number of workers varies inversely to the number of days.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

Sol: If distance increases then the time taken for a journey is also increase.

So, time and distance are in direct proportion.

(iii) Area of cultivated land and the crop harvested.

Sol: If the area increases then the crop harvested also increase.

So, cultivated land and crop harvested are in direct proportion.

(iv) The time taken for a fixed journey and the speed of the vehicle.

Sol: As speed increases, time taken decreases in same proportion.

So the time taken varies inversely to the speed of the vehicle ,for the same distance

(v) The population of a country and the area of land per person.

Sol: If the population of a country increases then the area of land per person decrease.

So, the population of a country varies inversely the area of land per person.

2. In a Television game show, the prize money of ` 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an

individual winner is directly or inversely proportional to the number of winners?

Number of winners	**1**	**2**	**3**	**4**	**5**	**8**	**10**	**20**
Prize for each winner(in ₹)	**1,00,000**	**50,000**	**...**	**....**	**...**	**....**	**....**	**....**

Sol: Number of winners varies inversely to the prize for each winner.

$$x_1y_1 = 1 \times 1{,}00{,}000 = 1{,}00{,}000 = k$$

$$x_2y_2 = 2 \times 50{,}000 = 1{,}00{,}000 = k$$

$$x_3y_3 = 1{,}00{,}000$$

$$\therefore y_3 = \frac{1{,}00{,}000}{x_3} = \frac{1{,}00{,}000}{4} = 25{,}000$$

$Similarly$

$$y_4 = \frac{1{,}00{,}000}{x_4} = \frac{1{,}00{,}000}{5} = 20{,}000$$

$$y_5 = \frac{1{,}00{,}000}{x_5} = \frac{1{,}00{,}000}{8} = 12{,}500$$

$$y_6 = \frac{1{,}00{,}000}{x_6} = \frac{1{,}00{,}000}{10} = 10{,}000$$

$$y_7 = \frac{1{,}00{,}000}{x_7} = \frac{1{,}00{,}000}{20} = 5{,}000$$

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes(x)	$4(x_1)$	$6(x_2)$	$8(x_3)$	$10(x_4)$	$12(x_5)$
Angle between a pair of consecutive spokes(y)	$90°(y_1)$	$60°(y_2)$	y_3	y_4	y_5

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

Sol:

$$x_1y_1 = 4 \times 90° = 360°$$

$$x_2y_2 = 6 \times 60° = 360°$$

$$x_1y_1 = x_2y_2 \rightarrow x, y \ are\ in\ inverse\ proportion$$

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes

$$Required\ angle = \frac{360°}{15} = 24°$$

(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is

40°?

Sol: Required number of spokes $= \frac{360°}{40^0} = 9$

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Sol: $x_1 = 24 \; and \;\; y_1 = 5$

$x_2 = 20 \;\; and \; y_2 = ?$

$x \; and \; y \; are \; in \; inverse \; proportion$

$x_2 y_2 = x_1 y_1$

$20 \times y_2 = 24 \times 5$

$y_2 = \frac{24 \times 5}{20} = 6$

Hence, 20 children will get 6 sweets each

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long the food last if would there were 10 more animals in his cattle?

Sol:

Number of animals(x)	$20(x_1)$	$30(x_2)$
Enough food for days(y)	$6(y_1)$	y_2

Number of animals is inversely proportional to enough food for days

$x_2 y_2 = x_1 y_1$

$30 \times y_2 = 20 \times 6$

$y_2 = \frac{20 \times 6}{30} = 4$

If there were 10 more animals in cattle the food last for 4 days.

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Sol:

Number of persons(x)	3	4
Number of days(y)	4	y_2

Number of persons is inversely proportional to number of days.

$x_2 y_2 = x_1 y_1$

$4 \times y_2 = 3 \times 4$

$y_2 = \frac{3 \times 4}{4} = 3$

4 persons will take 3 days to complete the job

7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is

packed using 20 bottles in each box, how many boxes would be filled?

Sol:

Number of bottles in each box (x)	$12(x_1)$	$20(x_2)$
Number of boxes(y)	$25(y_1)$	y_2

$x_2 y_2 = x_1 y_1$

$20 \times y_2 = 12 \times 25$

$y_2 = \dfrac{12 \times 25}{20} = 15$

Hence 15 boxes will be filled with 20 bottles in each box.

8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Sol:

Number of days	63	54
Number of machines required	42	y

$$54 \times y = 63 \times 42$$

$$y = \frac{63 \times 42}{54} = 49$$

49 machines will be required to produce the same number of articles in 54 days.

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

Sol:

Speed of the car(km/h)	60	80
Time taken(h)	2	y

$80 \times y = 60 \times 2$

$y = \dfrac{60 \times 2}{80} = 1\dfrac{1}{2}$

$1\frac{1}{2}$ hours will it take when the car travels at the speed of 80 km/h.

10. Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

Number of persons	2	1
Number of days	3	y

Number of persons is inversely proportional to number of days

$$1 \times y = 2 \times 3$$

$$y = 6$$

$\therefore$ The job will be completed in 6 days.

(ii) How many persons would be needed to fit the windows in one day?

Number of persons	2	x
Number of days	3	1

Number of persons is inversely proportional to number of days

$$x \times 1 = 2 \times 3$$

$$x = 6$$

6 persons will need to fix the window in one day.

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Sol:

Number of periods	8	9
Duration for each period(minutes)	45	y

Number of periods is inversely proportional to duration for each period.

$$9 \times y = 8 \times 45$$

$$y = \frac{8 \times 45}{9} = 40$$

Hence, each period would be 40 minutes long.

CHAPTER 14

AP VIII CLASS-CBSE (2022-23)
Factorisation (Notes)
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1. $5xy = 5 \times x \times y$ *is the 'irreducible' form of* $5xy$

Since $5, x, y$ are cannot be expressed as a product of factors .

Distributive laws

$(i)\ a \times b + a \times c = a \times (b + c)$

$(ii)\ a \times b - a \times c = a \times (b - c)$

Factorisation of an algebraic expression

Factorisation of an algebraic expression means writing the given expression as a product **of** its factors. These factors can be numbers, variables**,** or an algebraic expression

Method of common factors:

Example 1: **Factorise $12a^2b + 15ab^2$**

Sol: $12a^2b = 2 \times 2 \times \underline{3} \times \underline{a} \times a \times \underline{b} = (3 \times a \times b) \times (2 \times 2 \times a) = 3ab \times 4a$

$15ab^2 = \underline{3} \times 5 \times \underline{a} \times \underline{b} \times b = (3 \times a \times b) \times (5 \times b) = 3ab \times 5b$

$12a^2b + 15ab^2 = 3ab \times 4a + 3ab \times 5b$

$= 3ab(4a + 5b)$

Example 2: **Factorise $10x^2 - 18x^3 + 14x^4$**

Sol: $10x^2 = \underline{2} \times 5 \times \underline{x} \times \underline{x} = (2 \times x \times x) \times (5) = 2x^2 \times 5$

$18x^3 = \underline{2} \times 3 \times 3 \times \underline{x} \times \underline{x} \times x = (2 \times x \times x) \times (3 \times 3 \times x) = 2x^2 \times 9x$

$14x^4 = \underline{2} \times 7 \times \underline{x} \times \underline{x} \times x \times x = (2 \times x \times x) \times (7 \times x \times x) = 2x^2 \times 7x^2$

$10x^2 - 18x^3 + 14x^4 = (2x^2 \times 5) - (2x^2 \times 9x) + (2x^2 \times 7x^2)$

$= 2x^2(5 - 9x + 7x^2)$

$= 2x^2(7x^2 - 9x + 5)$

TRY THESE

(i) **Factorise $12x + 36$**

Sol: $12x + 36 = (\underline{2} \times \underline{2} \times \underline{3} \times x) + (\underline{2} \times \underline{2} \times \underline{3} \times 3)$

$= 12 \times x + 12 \times 3$

$= 12(x + 3)$

(ii) Factorise $22y - 33z$

Sol : $22y - 33z = 2 \times \underline{11} \times y - 3 \times \underline{11} \times z$

$= 11 \times 2y - 11 \times 3z$

$= 11(2y - 3z)$

(ii) Factorise $14pq + 35pqr$

Sol: $14pq + 35pqr = (2 \times \underline{7} \times \underline{p} \times \underline{q}) + (5 \times \underline{7} \times \underline{p} \times \underline{q} \times r)$

$= 7pq \times 2 + 7pq \times 5r$

$= 7pq(2 + 5r)$

Example 3: Factorise $6xy - 4y + 6 - 9x$.

Sol: $6xy - 4y + 6 - 9x$

$= 6xy - 4y - 9x + 6$

$= \underline{2y} \times 3x - \underline{2y} \times 2 - \underline{3} \times 3x + \underline{3} \times 2$

$= 2y(3x - 2) - 3(3x - 2)$

$= (3x - 2)(2y - 3)$

EXERCISE 12.1

1. **Find the common factors of the given terms.**

(i) $12x, 36$

Sol: $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

Common factors of $12x$,36 are

$2, 2 \times 2, 2 \times 2 \times 3 \Rightarrow 2,4,6$

2. **Factorise the following expressions.**

(i) $7x - 42$

Sol: $7x - 42 = 7 \times x - 7 \times 6 = 7(x - 6)$

(ii) $6p - 12q$

Sol: $6p - 12q = 6 \times p - 6 \times 2q = 6(p - 2q)$

(iii) $7a^2 + 14a$

Sol: $7a^2 + 14a = 7a \times a + 7a \times 2 = 7a(a + 2)$

(iv) $-16z + 20z^3$

Sol: $-16z + 20z^3 = 4z \times (-4) + 4z \times 5z^2 = 4z(-4 + 5z^2)$

(v) $20l^2m + 30alm$

Sol: $20\,l^2\,m + 30\,a\,l\,m = 10lm \times 2l + 10lm \times 3a = 10lm(2l + 3a)$

(vi) $\boldsymbol{5\,x^2y - 15\,xy^2}$

Sol: $5\,x^2y - 15\,xy^2 = 5xy \times x - 5xy \times 3y = 5xy(x - 3y)$

(vii) $\boldsymbol{10\,a^2 - 15\,b^2 + 20\,c^2}$

Sol: $10\,a^2 - 15\,b^2 + 20\,c^2 = 5 \times 2a^2 - 5 \times 3b^2 + 5 \times 4c^2$

$= 5(2a^2 - 3b^2 + 4c^2)$

$(viii)$ $\boldsymbol{-4\,a^2 + 4\,ab - 4\,ca}$

Sol: $-4\,a^2 + 4\,ab - 4\,ca = 4a \times (-a) + 4a \times b + 4a \times (-c)$

$= 4a(-a + b - c)$

(ix) $\boldsymbol{x^2\,y\,z + x\,y^2z + x\,y\,z^2}$

Sol: $x^2\,y\,z + x\,y^2z + x\,y\,z^2 = xyz \times x + xyz \times y + xyz \times z$

$= xyz(x + y + z)$

(x) $\boldsymbol{ax^2y + bxy^2 + c\,x\,y\,z}$

Sol: $ax^2y + bxy^2 + c\,x\,y\,z = xy \times ax + xy \times by + xy \times cz$

$= xy(ax + by + cz)$

3. Factorise.

(i) $\boldsymbol{x^2 + x\,y + 8x + 8y}$

Sol: $x^2 + x\,y + 8x + 8y = (x \times x + x \times y) + (8 \times x + 8 \times y)$

$= x(x + y) + 8(x + y)$

$= (x + y)(x + 8)$

(ii) $\boldsymbol{15\,xy - 6x + 5y - 2}$

Sol: $15\,xy - 6x + 5y - 2 = 3x \times 5y - 3x \times 2 + 1 \times 5y - 1 \times 2$

$= 3x(5y - 2) + 1(5y - 2)$

$= (5y - 2)(3x + 1)$

(iii) $\boldsymbol{ax + bx - ay - by}$

Sol: $ax + bx - ay - by = x \times a + x \times b - y \times a - y \times b$

$= x(a + b) - y(a + b)$

$= (a + b)(x - y)$

(iv) $\boldsymbol{15\,pq + 15 + 9q + 25p}$

Sol: $15\,pq + 15 + 9q + 25p = 15\,pq + 25p + 9q + 15$

$= 5p \times 3q + 5p \times 5 + 3 \times 3q + 3 \times 5$

$= 5p(3q + 5) + 3(3q + 5)$

$= (3q + 5)(5p + 3)$

(v) $z - 7 + 7xy - xyz$

Sol: $z - 7 + 7xy - xyz$

$= z - 7 - xyz + 7xy$

$= 1 \times z - 1 \times 7 - xy \times z + xy \times 7$

$= 1(z - 7) - xy(z - 7)$

$= (z - 7)(1 - xy)$

Factorisation using identities

I. $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

II. $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$

III. $a^2 - b^2 = (a + b)(a - b)$

IV. $x^2 + (a + b)x + ab = (x + a)(x + b)$

Example 4: Factorise $x^2 + 8x + 16$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$x^2 + 8x + 16$

$= x^2 + 2 \times x \times 4 + 4^2$

$= (x + 4)^2$

Example 5: Factorise $4y^2 - 12y + 9$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$4y^2 - 12y + 9$

$= (2y)^2 - 2 \times 2y \times 3 + (3)^2$

$= (2y - 3)^2$

Example 6: Factorise $49p^2 - 36$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$49p^2 - 36 = (7p)^2 - (6)^2$

$= (7p + 6)(7p - 6)$

Example 7: Factorise $a^2 - 2ab + b^2 - c^2$

Sol: $a^2 - 2ab + b^2 - c^2$

$= (a^2 - 2ab + b^2) - c^2$

$= (a - b)^2 - c^2$ (*Identity II*)

$= [(a - b) + c][(a - b) - c]$ (*Identity III*)

$= (a - b + c)(a - b - c)$

Example 8: Factorise $m^4 - 256$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$m^4 - 256 = (m^2)^2 - (16)^2$

$= (m^2 + 16)(m^2 - 16)$

$= (m^2 + 16)[m^2 - 4^2]$

$= (m^2 + 16)(m + 4)(m - 4)$

Example 9: Factorise $x^2 + 5x + 6$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$x^2 + 5x + 6$

$= x^2 + (2 + 3)x + 2 \times 3$

$= (x + 2)(x + 3)$

$a + b = 2 + 3 = 5$ $a \times b = 2 \times 3 = 6$

Example 10: Find the factors of $y^2 - 7y + 12$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$y^2 - 7y + 12$

$= y^2 + (-3 - 4)y + (-3)(-4)$

$= (y - 3)(y - 4)$

(OR)

$y^2 - 7y + 12 = y^2 - 3y - 4y + 12$

$= y(y - 3) - 4(y - 3)$

$= (y - 3)(y - 4)$

$a + b = -3 - 4 = -7$ $a \times b = (-3)(-4) = -12$

Example 11: Obtain the factors of $z^2 - 4z - 12$

Sol: $x^2 + (a + b)x + ab = (x + a)(x + b)$

$z^2 - 4z - 12 = z^2 + (-6 + 2)z + (-6)(2)$

$= (z - 6)(z + 2)$

Example 12: Find the factors of $3m^2 + 9m + 6$.

Sol: $3m^2 + 9m + 6$

$= 3[m^2 + 3m + 2]$

$= 3[m^2 + (1 + 2)m + 1 \times 2]$

$= 3(m + 1)(m + 2)$

EXERCISE 12.2

1. Factorise the following expressions.

(i) $\boldsymbol{a^2 + 8a + 16}$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$a^2 + 8a + 16$

$= a^2 + 2 \times a \times 4 + 4^2$

$= (a + 4)^2$

$\boldsymbol{(ii) p^2 - 10p + 25}$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$p^2 - 10p + 25$

$= p^2 - 2 \times p \times 5 + 5^2$

$= (p - 5)^2$

$\boldsymbol{(iii)\ 25m^2 + 30m + 9}$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$25m^2 + 30m + 9$

$= (5m)^2 + 2 \times 5m \times 3 + (3)^2$

$= (5m + 3)^2$

$\boldsymbol{(iv)\ 49y^2 + 84yz + 36z^2}$

Sol: $a^2 + 2ab + b^2 = (a + b)^2$

$49y^2 + 84yz + 36z^2$

$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$

$= (7y + 6z)^2$

$\boldsymbol{(v)\ 4x^2 - 8x + 4}$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$4x^2 - 8x + 4$

$= 4[x^2 - 2x + 1]$

$= 4[x^2 - 2 \times x \times 1 + 1^2]$

$= 4(x - 1)^2$

$\boldsymbol{(vi)\ 121b^2 - 88bc + 16c^2}$

Sol: $a^2 - 2ab + b^2 = (a - b)^2$

$121b^2 - 88bc + 16c^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

$= (11b - 4c)^2$

$(\boldsymbol{vii})\ (\boldsymbol{l}+\boldsymbol{m})^{2}-\mathbf{4}\boldsymbol{lm}$

Sol: $(l+m)^2-4lm$

$=(l^2+2lm+m^2)-4lm$

$=l^2+2lm-4lm+m^2$

$=l^2-2lm+m^2$

$=(l-m)^2$

$(\boldsymbol{viii})\boldsymbol{a}^{4}+\mathbf{2}\boldsymbol{a}^{2}\boldsymbol{b}^{2}+\boldsymbol{b}^{4}$

Sol: $a^4+2a^2b^2+b^4$

$=(a^2)^2+2\times a^2\times b^2+(b^2)^2$

$=(a^2+b^2)^2$

2. Factorise.

$(i)4p^2-9q^2$

Sol: $a^2-b^2=(a+b)(a-b)$

$4p^2-9q^2=(2p)^2-(3q)^2$

$=(2p+3q)(2p-3q)$

$(\boldsymbol{ii})\ \mathbf{63}\boldsymbol{a}^{2}-\mathbf{112}\boldsymbol{b}^{2}$

Sol: $a^2-b^2=(a+b)(a-b)$

$63a^2-112b^2=7\times 9a^2-7\times 16b^2$

$=7[9a^2-16b^2]$

$=7[(3a)^2-(4b)^2]$

$=7(3a+4b)(3a-4b)$

$(\boldsymbol{iii})\ \mathbf{49}\boldsymbol{x}^{2}-\mathbf{36}$

Sol: $a^2-b^2=(a+b)(a-b)$

$49x^2-36=(7x)^2-(6)^2$

$=(7x+6)(7x-6)$

$(\boldsymbol{iv})\ \mathbf{16}\boldsymbol{x}^{5}-\mathbf{144}\boldsymbol{x}^{3}$

Sol: $a^2-b^2=(a+b)(a-b)$

$16x^5-144x^3=16x^3\times x^2-16x^3\times 9$

$=16x^3[x^2-9]$

$=16x^3[x^2-3^2]$

$=16x^3(x+3)(x-3)$

$(\boldsymbol{v})\ (\boldsymbol{l}+\boldsymbol{m})^{2}-(\boldsymbol{l}-\boldsymbol{m})^{2}$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$(l + m)^2 - (l - m)^2$

$= [(l + m) + (l - m)][(l + m) - (l - m)]$

$= [l + m + l - m][l + m - l + m]$

$= (2l)(2m)$

$= 4lm$

(vi) $9x^2 y^2 - 16$

Sol: $a^2 - b^2 = (a + b)(a - b)$

$9x^2 y^2 - 16 = (3xy)^2 - (4)^2$

$= (3xy + 4)(3xy - 4)$

$(vii)(x^2 - 2xy + y^2) - z^2$

Sol: $(x^2 - 2xy + y^2) - z^2$

$= (x - y)^2 - (z)^2$

$= [(x - y) + z][(x - y) - z]$

$= (x - y + z)(x - y - z)$

$(viii) 25a^2 - 4b^2 + 28bc - 49c^2$

Sol: $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 25a^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - (2b - 7c)^2$

$= [5a + (2b - 7c)][5a - (2b - 7c)]$

$= (5a + 2b - 7c)(5a - 2b + 7c)$

3. **Factorise the expressions.**

$(i) ax^2 + bx$

Sol: $ax^2 + bx = ax \times x + b \times x$

$= x(ax + b)$

(ii) $7p^2 + 21q^2$

Sol: $7p^2 + 21q^2 = 7 \times p^2 + 7 \times 3q^2$

$= 7(p^2 + 3q^2)$

$(iii) 2x^3 + 2xy^2 + 2xz^2$

Sol: $2x^3 + 2xy^2 + 2xz^2$

$= 2x \times x^2 + 2x \times y^2 + 2x \times z^2$

$= 2x(x^2 + y^2 + z^2)$

$(iv)\boldsymbol{am^2 + bm^2 + bn^2 + an^2}$

Sol: $am^2 + bm^2 + bn^2 + an^2$

$= m^2(a + b) + n^2(a + b)$

$= (a + b)(m^2 + n^2)$

$(v)\ \boldsymbol{(lm + l) + m + 1}$

Sol: $(lm + l) + m + 1$

$= l \times (m + 1) + 1 \times (m + 1)$

$= (m + 1)(l + 1)$

$(vi)\ \boldsymbol{y(y + z) + 9(y + z)}$

Sol: $y(y + z) + 9(y + z)$

$= (y + z)(y + 9)$

$(vii)\ \boldsymbol{5y^2 - 20y - 8z + 2yz}$

Sol: $5y^2 - 20y + 2yz - 8z$

$= (5y \times y - 5y \times 4) + (2z \times y - 2z \times 4)$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

$(viii)\boldsymbol{10ab + 4a + 5b + 2}$

Sol: $10ab + 4a + 5b + 2$

$= 2a \times 5b + 2a \times 2 + 1 \times 5b + 1 \times 2$

$= 2a(5b + 2) + 1(5b + 2)$

$= (5b + 2)(2a + 1)$

$(ix)\boldsymbol{6xy - 4y + 6 - 9x}$

Sol: $6xy - 4y + 6 - 9x$

$= 6xy - 9x - 4y + 6$

$= 3x \times 2y - 3x \times 3 - 2 \times 2y + 2 \times 3$

$= 3x(2y - 3) - 2(2y - 3)$

$= (2y - 3)(3x - 2)$

4. Factorise.

$(i)\ a^4 - b^4$

Sol: $a^4 - b^4 = (a^2)^2 - (b^2)^2$

$= (a^2 + b^2)(a^2 - b^2)$

$= (a^2+b^2)(a+b)(a-b)$

$(ii)\ \boldsymbol{p^4 - 81}$

Sol: $p^4 - 81 = (p^2)^2 - (9)^2$

$= (p^2+9)(p^2-9)$

$= (p^2+9)(p^2-3^2)$

$= (p^2+9)(p+3)(p-3)$

$(iii)\boldsymbol{x^4 - (y+z)^4}$

Sol: $x^4 - (y+z)^4$

$= (x^2)^2 - [(y+z)^2]^2$

$= [x^2+(y+z)^2][x^2-(y+z)^2]$

$= [x^2+(y+z)^2][x+(y+z)][x-(y+z)]$

$= [x^2+y^2+z^2+2yz](x+y+z)(x-y-z)$

$(v)\ \boldsymbol{a^4 - 2a^2b^2 + b^4}$

Sol: $a^4 - 2a^2b^2 + b^4$

$= (a^2)^2 + 2\times a^2\times b^2 + (b^2)^2$

$= (a^2-b^2)^2$

$= [(a+b)(a-b)]^2$

$= (a+b)^2(a-b)^2$

$= (a+b)(a+b)(a-b)(a-b)$

5. Factorise the following expressions.

$(i)\ \boldsymbol{p^2 + 6p + 8}$

Sol: $p^2 + 6p + 8$

$= p^2 + (2+4)p + 2\times 4$

$= (p+2)(p+4)$

$x^2 + (a+b)x + ab$ $= (x+a)(x+b)$

$(ii)\ \boldsymbol{q^2 - 10q + 21}$

Sol: $q^2 - 10q + 21$

$= q^2 + (-3-7)q + (-3)(-7)$

$= (q-3)(q-7)$

$x^2 + (a+b)x + ab$ $= (x+a)(x+b)$

$(iii) p^2 + 6p - 16$

Sol: $p^2 + 6p - 16$

$= p^2 + (8-2)p + (8)(-2)$

$= (p+8)(p-2)$

$x^2 + (a+b)x + ab$ $= (x+a)(x+b)$

Division of Algebraic Expressions

Example 13: Do the following divisions.

$(i) -20x^4 \div 10x^2$

Sol: $\frac{-20x^4}{10x^2} = \frac{-2 \times 2 \times 5 \times x \times x \times x \times x}{2 \times 5 \times x \times x}$

$= -2 \times x \times x = -2x^2$

$(ii) 7x^2\ y^2\ z^2 \div 14xyz$

Sol: $\frac{7x^2\ y^2\ z^2}{14xyz} = \frac{7 \times x \times x \times y \times y \times z \times z}{2 \times 7 \times x \times y \times z}$

$= \frac{x \times y \times z}{2} = \frac{xyz}{2}$

TRY THESE

(i)Divide $24xy^2\ z^3$ by $6yz^2$

Sol: $\frac{24xy^2\ z^3}{6yz^2} = \frac{2 \times 2 \times 2 \times 3 \times x \times y \times y \times z \times z \times z}{2 \times 3 \times y \times z \times z}$

$= 2 \times 2 \times x \times y \times z = 4xyz$

(ii)Divide $63a^2\ b^4\ c^6$ by $7a^2\ b^2\ c^3$

Sol: $\frac{63a^2\ b^4\ c^6}{7a^2\ b^2\ c^3} = \frac{7 \times 9 \times a^2 \times b^2 \times b^2 \times c^3 \times c^3}{7 \times a^2 \times b^2 \times c^3}$

$= 9 \times b^2 \times c^3 = 9b^2c^3$

Example 14: Divide $24(x^2yz + xy^2z + xyz^2)$ by $8xyz$ *using both the methods*

Sol: $24(x^2yz + xy^2z + xyz^2)$

$= 3 \times 8 \times (xyz \times x + xyz \times y + xyz \times z)$

$= 3 \times 8 \times xyz \times (x + y + z)$

$\frac{24(x^2yz + xy^2z + xyz^2)}{8xyz}$

$= \frac{3 \times 8 \times xyz \times (x + y + z)}{8 \times xyz}$

$= 3(x + y + z)$

Method-II

$\frac{24(x^2yz + xy^2z + xyz^2)}{8xyz}$

$= \frac{24x^2yz}{8xyz} + \frac{24xy^2z}{8xyz} + \frac{24xyz^2}{8xyz}$

$= 3x + 3y + 3z = 3(x + y + z)$

Example 15: Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x - 8)$

Sol: $44(x^4 - 5x^3 - 24x^2) = 44[x^2 \times x^2 - 5x \times x^2 - 24 \times x^2]$

$= 44 \times x^2[x^2 - 5x - 24]$

$= 44 \times x^2[x^2 + (-8 + 3)x + (-8)(3)]$

$= 44 \times x^2[(x - 8)(x + 3)]$

$= 4 \times 11 \times x \times x \times (x - 8) \times (x + 3)$

$$\frac{44(x^4 - 5x^3 - 24x^2)}{11x(x - 8)}$$

$$= \frac{4 \times 11 \times x \times x \times (x - 8) \times (x + 3)}{11 \times x \times (x - 8)} = 4x(x + 3)$$

Example 16: Divide $z(5z^2 - 80)$ by $5z(z + 4)$

Sol: $z(5z^2 - 80) = z \times [5 \times z^2 - 5 \times 16]$

$= z \times 5 \times [z^2 - 16]$

$= z \times 5 \times [z^2 - 4^2]$

$= z \times 5 \times (z + 4) \times (z - 4)$

$$\frac{z(5z^2 - 80)}{5z(z + 4)} = \frac{z \times 5 \times (z + 4) \times (z - 4)}{5 \times z \times (z + 4)} = (z - 4)$$

EXERCISE 12.3

1. Carry out the following divisions.

(i) $28x^4 \div 56x$

Sol: $\frac{28x^4}{56x} = \frac{28 \times x \times x \times x \times x}{2 \times 28 \times x} = \frac{x \times x}{2} = \frac{x^2}{2}$

(ii) $-36y^3 \div 9y^2$

Sol: $\frac{-36y^3}{9y^2} = \frac{-4 \times 9 \times y \times y \times y}{9 \times y \times y} = -4y$

(iii) $66pq^2r^3 \div 11qr^2$

Sol: $\frac{66pq^2r^3}{11qr^2} = \frac{6 \times 11 \times p \times q \times q \times r \times r \times r}{11 \times q \times r \times r}$

$= 6 \times p \times q \times r = 6pqr$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

Sol: $\frac{34x^3y^3z^3}{51xy^2z^3} = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times}{3 \times 17 \times x \times y \times y \times z \times z \times z} = \frac{2x^2y}{3}$

$(v) \mathbf{12a^8 b^8 \div (-6a^6 b^4)}$

Sol: $\frac{12a^8 b^8}{-6a^6 b^4} = \frac{2 \times 6 \times a^6 \times a^2 \times b^4 \times b^4}{-6 \times a^6 \times b^4} = -2a^2b^4$

2. Divide the given polynomial by the given monomial.

$(i) \mathbf{(5x^2 - 6x) \div 3x}$

Sol: $\frac{5x^2 - 6x}{3x} = \frac{5x^2}{3x} - \frac{6x}{3x} = \frac{5x}{3} - 2$

$(ii) \mathbf{(3y^8 - 4y^6 + 5y^4) \div y^4}$

Sol: $\frac{3y^8 - 4y^6 + 5y^4}{y^4} = \frac{3y^8}{y^4} - \frac{4y^6}{y^4} + \frac{5y^4}{y^4}$

$= \frac{3y^4 \times y^4}{y^4} - \frac{4y^2 \times y^4}{y^4} + \frac{5 \times y^4}{y^4}$

$= 3y^4 - 4y^2 + 5$

$(iii) \mathbf{8(x^3y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3) \div 4x^2y^2 z^2}$

Sol: $\frac{8(x^3y^2 z^2 + x^2 y^3 z^2 + x^2 y^2 z^3)}{4x^2y^2 z^2}$

$= \frac{8x^3y^2 z^2}{4x^2y^2 z^2} + \frac{8x^2 y^3 z^2}{4x^2y^2 z^2} + \frac{8x^2 y^2 z^3}{4x^2y^2 z^2}$

$= 2x + 2y + 2z = 2(x + y + z)$

$(iv) \mathbf{(x^3 + 2x^2 + 3x) \div 2x}$

Sol: $\frac{x^3 + 2x^2 + 3x}{2x} = \frac{x^3}{2x} + \frac{2x^2}{2x} + \frac{3x}{2x} = \frac{x^2}{2} + x + \frac{3}{2}$

$(v) \mathbf{(p^3 q^6 - p^6 q^3) \div p^3 q^3}$

Sol: $\frac{p^3 q^6 - p^6 q^3}{p^3 q^3} = \frac{p^3 q^3 \times q^3}{p^3 q^3} - \frac{p^3 q^3 \times p^3}{p^3 q^3} = q^3 - p^3$

3. Work out the following divisions.

$(i) \mathbf{(10x - 25) \div 5}$

Sol: $\frac{10x - 25}{5} = \frac{10x}{5} - \frac{25}{5} = 2x - 5$

$(ii) \mathbf{(10x - 25) \div (2x - 5)}$

Sol: $(10x - 25) = (5 \times 2x - 5 \times 5) = 5 \times (2x - 5)$

$\frac{(10x - 25)}{(2x - 5)} = \frac{5 \times (2x - 5)}{(2x - 5)} = 5$

$(iii) \mathbf{10y(6y + 21) \div 5(2y + 7)}$

Sol: $10y(6y + 21) = 2 \times 5 \times y \times [3 \times 2y + 3 \times 7]$

$= 2 \times 5 \times y \times 3 \times (2y + 7)$

$$\frac{10y(6y + 21)}{5(2y + 7)} = \frac{2 \times 5 \times y \times 3 \times (2y + 7)}{5 \times (2y + 7)} = 2 \times y \times 3 = 6y$$

(iv) $9x^2\ y^2\ (3z - 24) \div 27xy(z - 8)$

Sol: $9x^2\ y^2\ (3z - 24) = : 9x^2\ y^2 \times [3 \times z - 3 \times 8] = 9 \times x \times x \times y \times y \times 3 \times (z - 8)$

$$\frac{9x^2\ y^2\ (3z - 24)}{27xy(z - 8)} = \frac{9 \times x \times x \times y \times y \times 3 \times (z - 8)}{3 \times 9 \times x \times y \times (z - 8)} = xy$$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Sol: $(3a - 12)(5b - 30) = [3 \times a - 3 \times 4][5 \times b - 5 \times 6] = 3 \times (a - 4) \times 5 \times (b - 6)$

$$\frac{96abc(3a - 12)(5b - 30)}{144(a - 4)(b - 6)} = \frac{2 \times 48 \times abc \times 3 \times (a - 4) \times 5 \times (b - 6)}{3 \times 48 \times (a - 4) \times (b - 6)}$$

$= 2 \times 5 \times abc = 10abc$

4. Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$

Sol: $\frac{5(2x + 1)(3x + 5)}{(2x + 1)} = 5(3x + 5)$

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

Sol: $\frac{26xy(x + 5)(y - 4)}{13x(y - 4)} = \frac{2 \times 13 \times x \times y \times (x + 5) \times (y - 4)}{13 \times x \times (y - 4)}$

$= 2 \times y \times (x + 5)$

$= 2y(x + 5)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

Sol: $\frac{52pqr(p + q)(q + r)(r + p)}{104pq(q + r)(r + p)}$

$= \frac{52 \times pq \times r \times (p + q)(q + r)(r + p)}{2 \times 52 \times pq \times (q + r)(r + p)}$

$= \frac{r(p + q)}{2} = \frac{1}{2} r(p + q)$

$(\boldsymbol{iv})$ $\mathbf{20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)}$

Sol: $\dfrac{20(y + 4)(y^2 + 5y + 3)}{5(y + 4)}$

$= \dfrac{4 \times 5 \times (y + 4)(y^2 + 5y + 3)}{5 \times (y + 4)}$

$= 4(y^2 + 5y + 3)$

$(\boldsymbol{v})$ $\mathbf{x(x + 1)(x + 2)(x + 3) \div x(x + 1)}$

Sol: $\dfrac{x(x + 1)(x + 2)(x + 3)}{x(x + 1)} = (x + 2)(x + 3)$

5. Factorise the expressions and divide them as directed.

$(\boldsymbol{i})$ $\mathbf{(y^2 + 7y + 10) \div (y + 5)}$

Sol: $y^2 + 7y + 10 = y^2 + (2 + 5)y + 2 \times 5$

$= (y + 2)(y + 5)$

$\dfrac{(y^2 + 7y + 10)}{(y + 5)} = \dfrac{(y + 2)(y + 5)}{(y + 5)} = (y + 2)$

$(\boldsymbol{ii})$ $\mathbf{(m^2 - 14m - 32) \div (m + 2)}$

Sol: $m^2 - 14m - 32 = m^2 + (-16 + 2)m + (-16) \times (2)$

$= (m - 16)(m + 2)$

$\dfrac{(m^2 - 14m - 32)}{(m + 2)} = \dfrac{(m - 16)(m + 2)}{(m + 2)} = (m - 16)$

$(\boldsymbol{iii})$ $\mathbf{(5p^2 - 25p + 20) \div (p - 1)}$

Sol: $5p^2 - 25p + 20 = 5[p^2 - 5p + 4]$

$= 5[p^2 + (-1 - 4)p + (-1)(-4)]$

$= 5(p - 1)(p - 4)$

$\dfrac{(5p^2 - 25p + 20)}{(p - 1)} = \dfrac{5(p - 1)(p - 4)}{(p - 1)} = 5(p - 4)$

$(\boldsymbol{iv})$ $\mathbf{4yz(z^2 + 6z - 16) \div 2y(z + 8)}$

Sol: $z^2 + 6z - 16 = z^2 + (8 - 2)z + 8 \times (-2)$

$= (z + 8)(z - 2)$

$\dfrac{4yz(z^2 + 6z - 16)}{2y(z + 8)} = \dfrac{2y \times 2z \times (z + 8)(z - 2)}{2y \times (z + 8)} = 2z(z - 2)$

$(\boldsymbol{v})$ $\mathbf{5pq(p^2 - q^2) \div 2p(p + q)}$

Sol: $\frac{5pq(p^2 - q^2\ )}{2p(p\ +\ q)} = \frac{5 \times p \times q \times (p+q)(p-q)}{2 \times p \times (p\ +\ q)} = \frac{5q(p-q)}{2} = \frac{5}{2}q(p-q)$

$(\boldsymbol{vi})$ $\mathbf{12}\boldsymbol{xy}(\mathbf{9}\boldsymbol{x}^{\mathbf{2}} - \mathbf{16}\boldsymbol{y}^{\mathbf{2}}\ ) \div \mathbf{4}\boldsymbol{xy}(\mathbf{3}\boldsymbol{x}\ +\ \mathbf{4}\boldsymbol{y})$

Sol: $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x+4y)(3x-4y)$

$$\frac{12xy(9x^2 - 16y^2\ )}{4xy(3x\ +\ 4y)} = \frac{3 \times 4xy \times (3x+4y) \times (3x-4y)}{4xy \times (3x\ +\ 4y)}$$

$= 3(3x-4y)$

$(\boldsymbol{vii})\mathbf{39}\boldsymbol{y}^{\mathbf{3}}\ (\mathbf{50}\boldsymbol{y}^{\mathbf{2}} - \mathbf{98}) \div \mathbf{26}\boldsymbol{y}^{\mathbf{2}}\ (\mathbf{5}\boldsymbol{y}\ +\ \mathbf{7})$

Sol: $50y^2 - 98 = 2 \times 25y^2 - 2 \times 49 = 2[25y^2 - 49]$

$= 2[(5y)^2 - (7)^2] = 2(5y+7)(5y-7)$

$$\frac{39y^3\ (50y^2 - 98)}{26y^2\ (5y\ +\ 7)} = \frac{3 \times 13 \times y^2 \times y \times 2 \times (5y+7) \times (5y-7)}{2 \times 13 \times y^2 \times\ (5y\ +\ 7)}$$

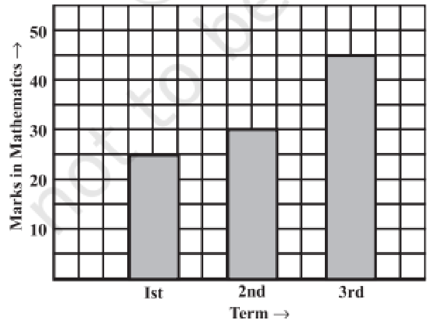
$= 3y(5y-7)$

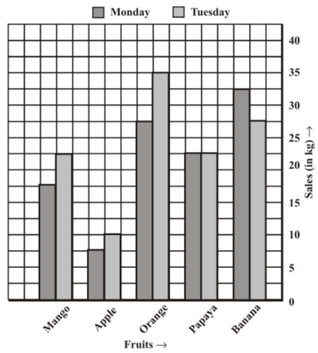
CHAPTER 13

VIII CLASS-NCERT (2023-24)
INTRODUCTION TO GRAPHS (Notes)
PREPARED BY : BALABHADRA SURESH-9866845885
https://sureshmathsmaterial.com

1. A Bar graph:

A bar graph is used to show comparison among categories. It may consist of two or more parallel vertical (or horizontal) bars (rectangles).

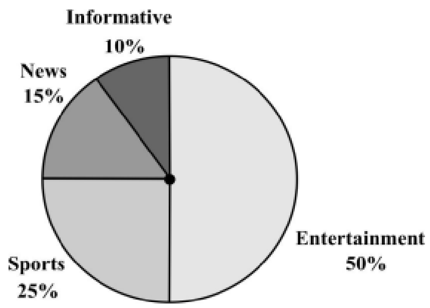




Bar graphs can also have double bars. This graph gives a comparative.

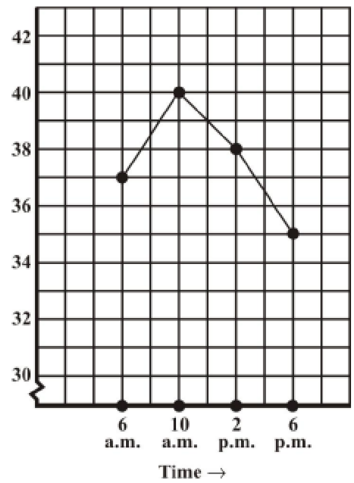
2. A Pie graph (or a circle-graph) :

A pie-graph is used to compare parts of a whole. The circle represents the whole



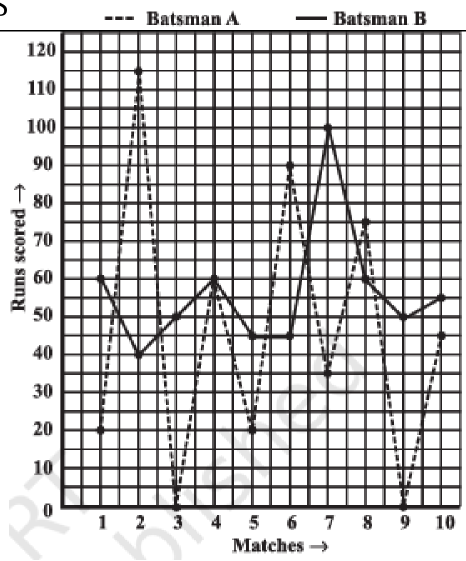
3. A line graph:

A line graph displays data that changes continuously over periods of time.



Example 1: (A graph on "performance") The given graph (Fig 15.7) represents the total runs scored by two batsmen A and B, during each of the ten different matches in the year 2007. Study the graph and answer the following questions.

(i) What information is given on the two axes?



Sol: On X-axis: The matches played during the year 2007

On Y-axis: The total runs scored in each match.

(ii) Which line shows the runs scored by batsman A?

Sol: The dotted line shows the runs scored by Batsman A.

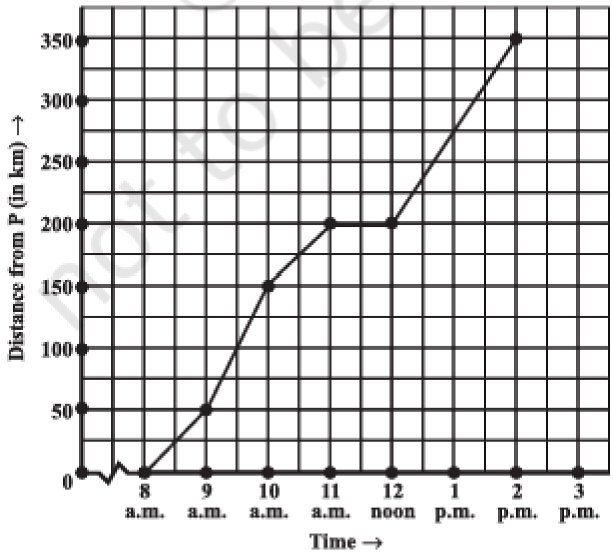
(iii) Were the run scored by them same in any match in 2007? If so, in which match ?

Sol: During the 4th match, both have scored the same number of 60 runs (Graphs intersecting point)

(iii) Among the two batsmen, who is steadier? How do you judge it?

Sol: Batsman A has one great "peak" but many deep "valleys". He does not appear to be consistent. B, on the other hand has never scored below a total of 40 runs, even though his highest score is only 100 in comparison to 115 of A. Also A has scored a zero in two matches and in a total of 5 matches he has scored less than 40 runs. Since A has a lot of ups and downs, B is a more consistent and reliable batsman.

Example 2: The given graph describes the distances of a car from a city P at different times when it is travelling from City P to City Q, which are 350 km apart. Study the graph and answer the following:



(i) What information is given on the two axes?

Sol: On X-axis: Time

On Y axis: The distance of the car from City P.

(ii) From where and when did the car begin its journey?

Sol: The car started from City P at 8 a.m

(iii) How far did the car go in the first hour?

Sol: The car travelled 50 km during the first hour.

(iv) How far did the car go during

(a) The 2nd hour?

Sol: $150 - 50 = 100km$

(ii) The 3rd hour?

Sol: $200 - 150 = 50km$

(v) Was the speed same during the first three hours? How do you know it?

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Sol: Speed in 1st hour $= \frac{50}{1} = 50km/h$

Speed in 2nd hour $= \frac{100}{1} = 100km/h$

Speed in 3rd hour $= \frac{50}{1} = 50km/h$

We find that the speed of the car was not the same all the time.

(vi) Did the car stop for some duration at any place? Justify your answer.

Sol: No distance covered during the period 11a.m to 12 noon .

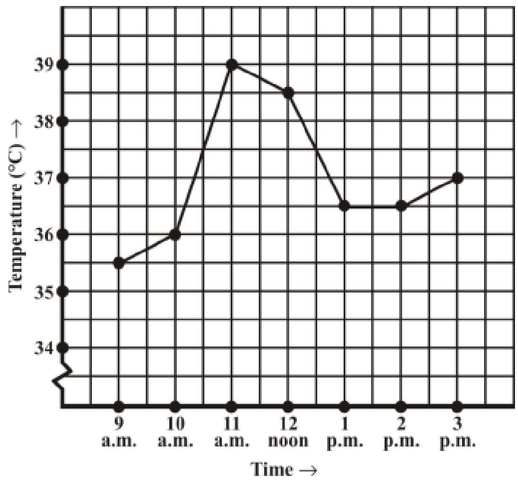
This shows that the car did not travel during the interval 11 a.m. to 12 noon.

(vii) When did the car reach City Q?

Sol: The car reached City Q at 2 p.m.

EXERCISE 13.1

1. The following graph shows the temperature of a patient in a hospital, recorded every hour.



(a) What was the patient's temperature at 1 p.m. ?

Sol: 36.5^0C

(b) When was the patient's temperature 38.5° C?

Sol: 12 noon.

(c) The patient's temperature was the same two times during the period given. What were these two times?

Sol: 1 pm and 2 pm (36.5^0C)

(d) **What was the temperature at 1.30 p.m.? How did you arrive at your answer?**

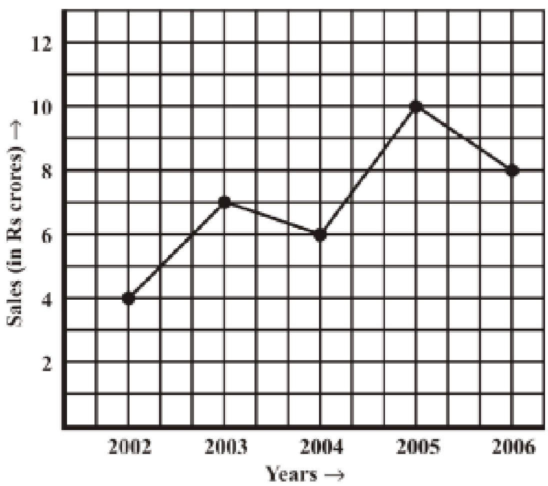
Sol: The temperature from 1 pm to 2 pm is 36.5^0C.

So, the temperature at 1:30 pm is 36.5^0C .

(e) During which periods did the patients' temperature showed an upward trend?

Sol: During 9 am to 10 am, 10 am to 11 am and 2 pm to 3 pm, the patient's temperature showed an upward trend.

2. The following line graph shows the yearly sales figures for a manufacturing company.



(a) What were the sales in (i) 2002 (ii) 2006?

Sol: (i) The sales in 2002 is Rs 4 crores

(ii) The sales in 2006 is Rs 8 crores

(b) What were the sales in (i) 2003 (ii) 2005?

Sol: (i) The sales in 2003 is Rs 7 crores

(ii) The sales in 2005 is Rs 10 crores

(c) Compute the difference between the sales in 2002 and 2006.

Sol: Difference between the sales in 2002 and 2006

=8 crores−4 crores= Rs 4 crores

(d) In which year was there the greatest difference between the sales as compared to its previous

year?

Sol: Difference between the sales of the year

2002 and 2003 = Rs (7 − 4) crores= Rs 3 crores

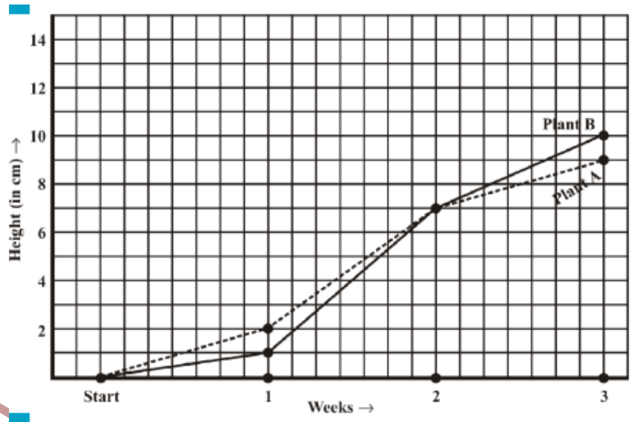
2003 and 2004 = Rs (7 − 6) crores= Rs 1 crore

2004 and 2005 = Rs (10 − 6) crores= Rs 4 crores

2005 and 2006 = Rs (10 − 8) crores= Rs 2 crores

The difference was the maximum in the year 2005 as compared to its previous year 2004 .

3. For an experiment in Botany, two different plants, plant A and plant B were grown under similar laboratory conditions. Their heights were measured at the end of each week for 3 weeks. The results are shown by the following graph.



(a) How high was Plant A after (i) 2 weeks (ii) 3 weeks?

Sol: (i) The height of Plant A after 2 weeks= 7cm

(ii) The height of Plant A after 3 weeks= 9cm

(b) How high was Plant B after (i) 2 weeks (ii) 3 weeks?

Sol: (i) The height of Plant B after 2 weeks= 7cm

(ii) The height of Plant B after 3 weeks= 10cm

(c) How much did Plant A grow during the 3rd week?

Sol: Growth of plant A during the third week=9cm−7cm=2cm.

(d) How much did Plant B grow from the end of the 2nd week to the end of the 3rd week?

Sol: Growth of plant B from the end of the 2nd week to the end of the 3rd week

= 10 cm − 7 cm = 3 cm

(e) During which week did Plant A grow most?

Sol: Growth of plant A during

1^{st} week=2cm−0=2cm

2^{nd} week=7cm−2cm=5cm

3^{rd} week=9cm−7cm=2cm

Therefore, plant A grew the most is 5 cm, during the 2nd week.

(f) During which week did Plant B grow least?

Sol: Growth of plant B during

1^{st} week=1cm−0=1cm

2^{nd} week=7cm−1cm=6cm

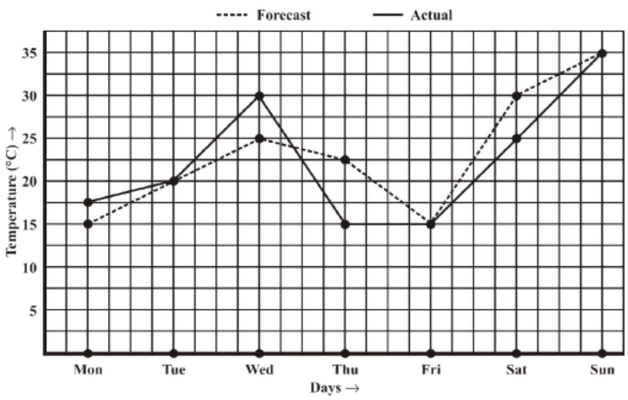
3^{rd} week=10cm−7cm=3cm

Therefore, plant B grew the least is 1 cm, during the 1^{st} week

(g) Were the two plants of the same height during any week shown here? Specify

Sol: At the end of the 2nd week, the heights of both plants were same i.e. 7 cm.

4. The following graph shows the temperature forecast and the actual temperature for each day of a week.



(a) On which days was the forecast temperature the same as the actual temperature?

Sol: Tuesday, Friday and Sunday.

(b) What was the maximum forecast temperature during the week?

Sol: 35^0C

(c) What was the minimum actual temperature during the week?

Sol: 15^0C

(d) On which day did the actual temperature differ the most from the forecast temperature?

Sol: On Thursday.

5. Use the tables below to draw linear graphs.

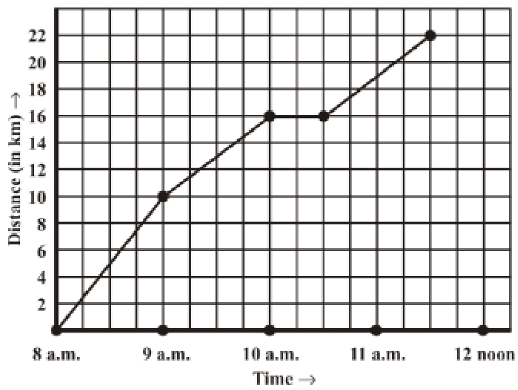
(a) The number of days a hill side city received snow in different years.

Year	2003	2004	2005	2006
Days	8	10	5	12

(b) Population (in thousands) of men and women in a village in different years.

Year	2003	2004	2005	2006	2007
Number of Men	12	12.5	13	13.2	13.5
Number of Women	11.3	11.9	13	13.6	12.8

6. **A courier-person cycles from a town to a neighbouring suburban area to deliver a parcel to a merchant. His distance from the town at different times is shown by the following graph.**



(a) **What is the scale taken for the time axis?**

Sol: On time axis(X-axis) : 4Units=1 hour.

(b) **How much time did the person take for the travel?**

Sol: 8 a.m to 11.30 a.m=$11.30 - 8 = 3.30 = 3\frac{1}{2}$ $hours$

(c) **How far is the place of the merchant from the town?**

Sol: 22 km

(d) **Did the person stop on his way? Explain.**

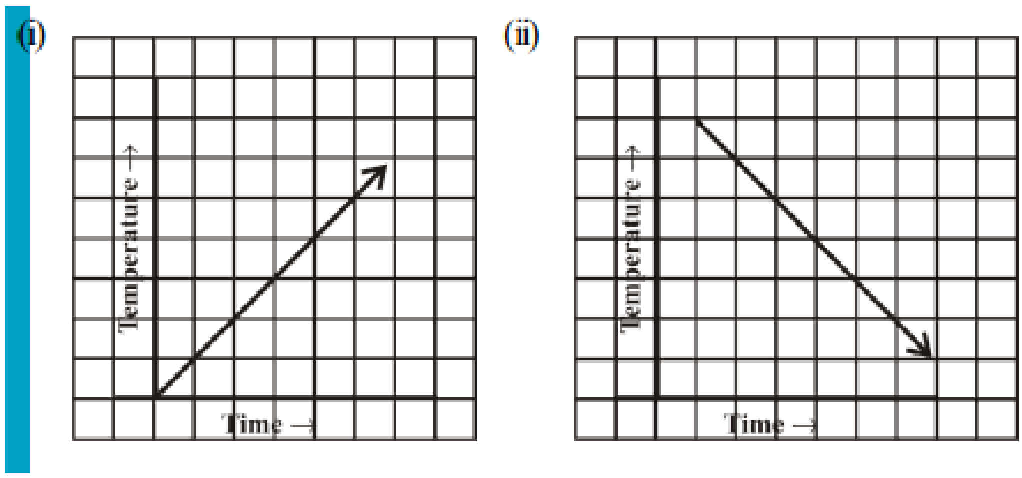
Sol: Yes, the person stopped on his way from 10 a.m. to 10: 30 a.m

(e) **During which period did he ride fastest?**

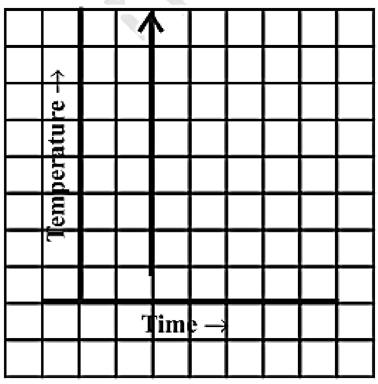
Sol: The person maximum distance travelled in time period 8 a.m to 9 a.m.

Thus, the person's ride was the fastest between 8 a.m. and 9 a.m.

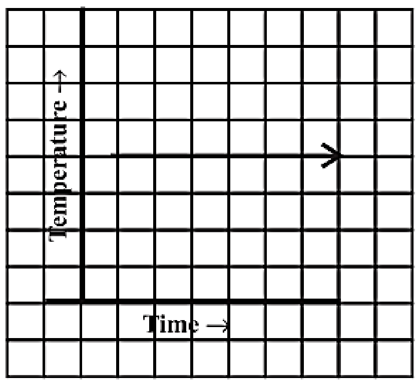
7. **Can there be a time-temperature graph as follows? Justify your answer**



(iii)



(iv)



(i) The temperature can increase with the increase in time. (Direct proportional)

(ii) The temperature can decrease with the decrease in time. (Direct proportional)

(iii) Temperature changes at the same time. The graph is not possible.

(iv) Same temperature in different times. (Temperature is constant)

THINK, DISCUSS AND WRITE:

The number of litres of petrol you buy to fill a car's petrol tank will decide the amount you have to pay. Which is the independent variable here? Think about it

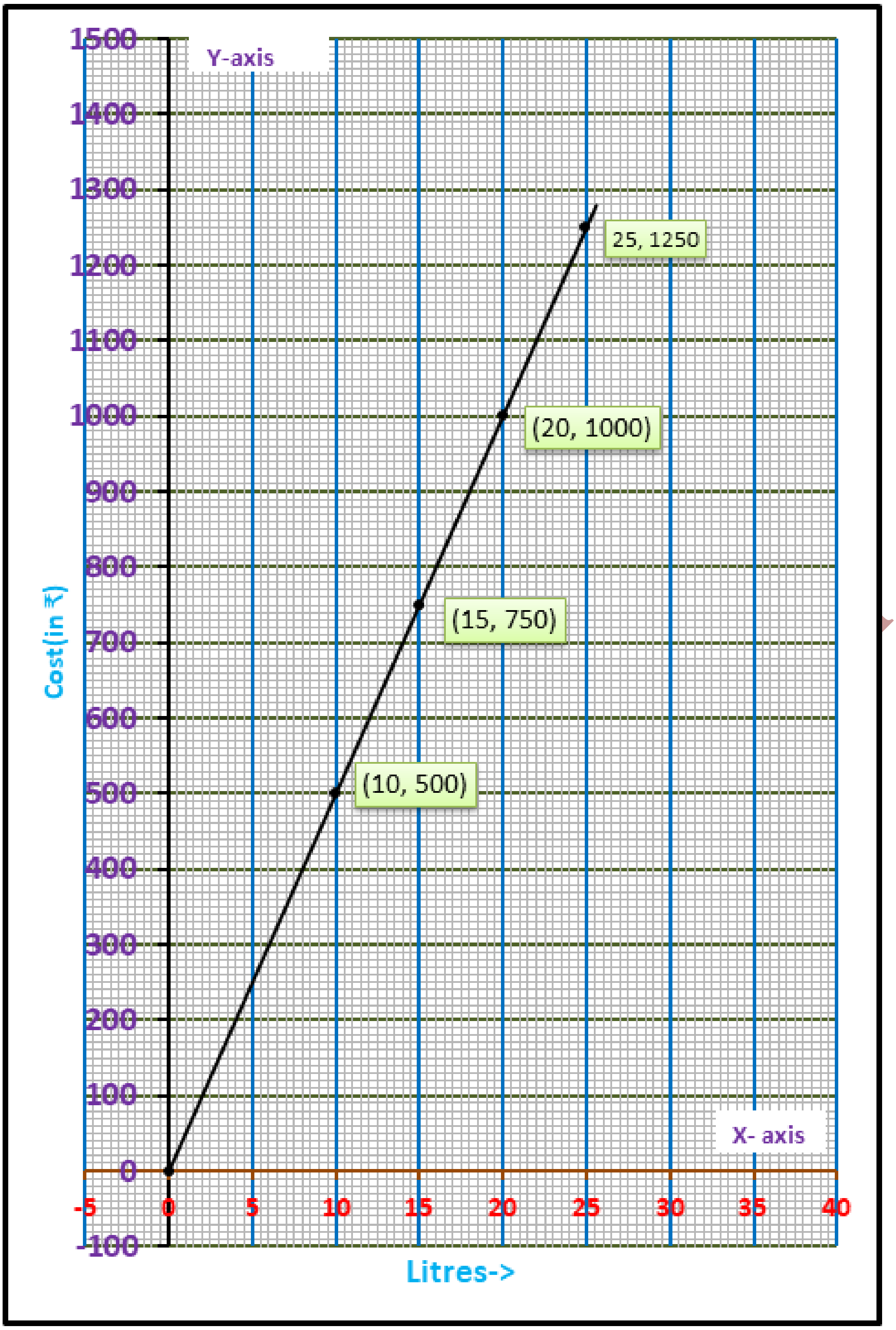
Sol: The amount money linked with the quantity of petrol.

So, the number of litres of petrol is independent variable.

Example 3: (Quantity and Cost) The following table gives the quantity of petrol and its cost.

No. of Litres of petrol	10	15	20	25
Cost of petrol in `	500	750	1000	1250

Plot a graph to show the data.



TRY THESE

In the above example, use the graph to find how much petrol can be purchased for ₹800.

Sol: 16 litres

Example 4: (Principal and Simple Interest)

A bank gives 10% Simple Interest (S.I.) on deposits by senior citizens. Draw a graph to illustrate the relation between the sum deposited and simple interest earned. Find from your graph

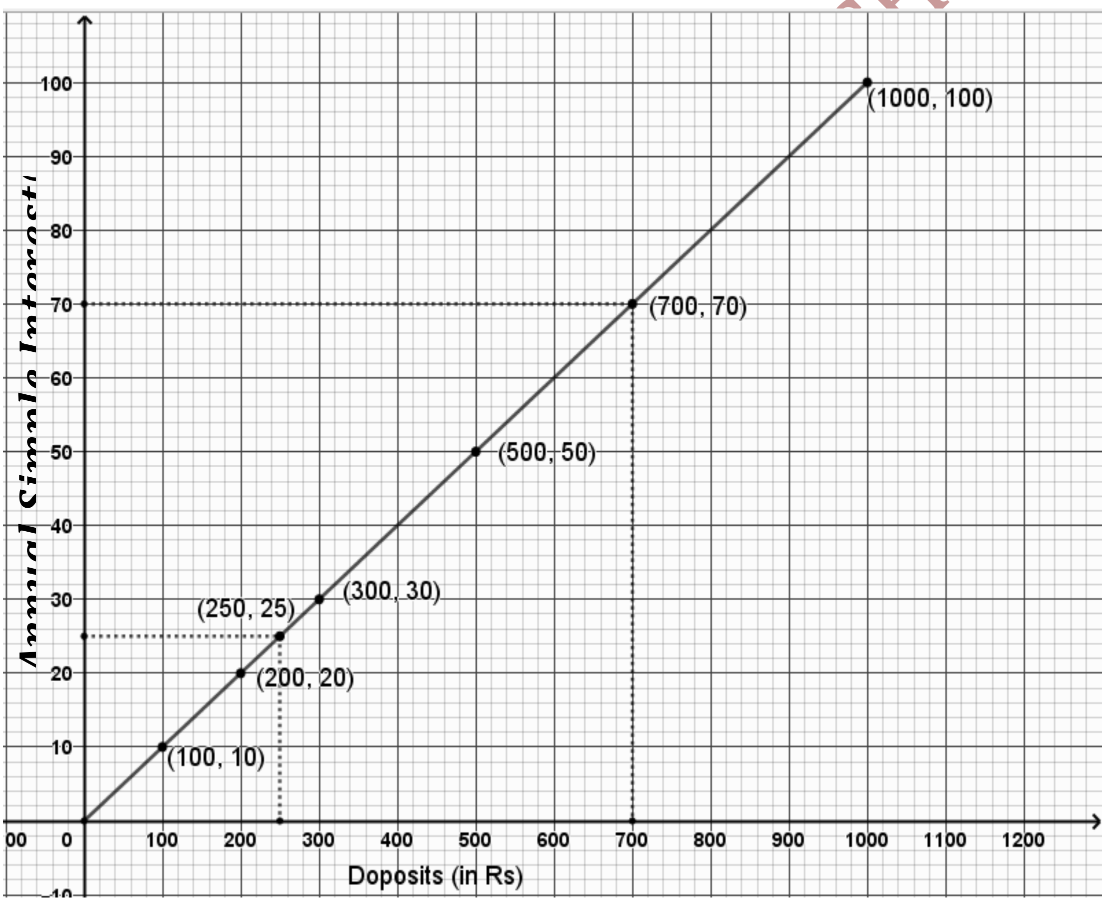
(a) The annual interest obtainable for an investment of ₹ 250.

(b) The investment one has to make to get an annual simple interest of ₹70.

Sol:

Sum deposited(P)	Simple interest for a year($I = \frac{P \times T \times R}{100}$)	Point (P, I)(In ₹)
₹100	$\frac{100 \times 1 \times 10}{100} = 10$	(100,10)
₹ 200	$\frac{200 \times 1 \times 10}{100} = 20$	(200,20)
₹ 300	$\frac{300 \times 1 \times 10}{100} = 30$	(300,30)
₹ 500	$\frac{500 \times 1 \times 10}{100} = 50$	(500,50)
₹ 1000	$\frac{1000 \times 1 \times 10}{100} = 100$	(1000,100)

Scale : 1 unit = ₹ 100 on horizontal axis; 1 unit = ₹ 10 on vertical axis.



(a) Corresponding to₹ 250 on horizontal axis, we get the interest to be ₹ 25 on vertical axis.

(b) Corresponding to ₹ 70 on the vertical axis, we get the sum to be ₹700 on the horizontal axis

Example 5: (Time and Distance) Ajit can ride a scooter constantly at a speed of 30 kms/hour. Draw a time-distance graph for this situation. Use it to find (i) the time taken by Ajit to ride 75 km. (ii) the distance covered by Ajit in 3 $\frac{1}{2}$hours.

Sol:

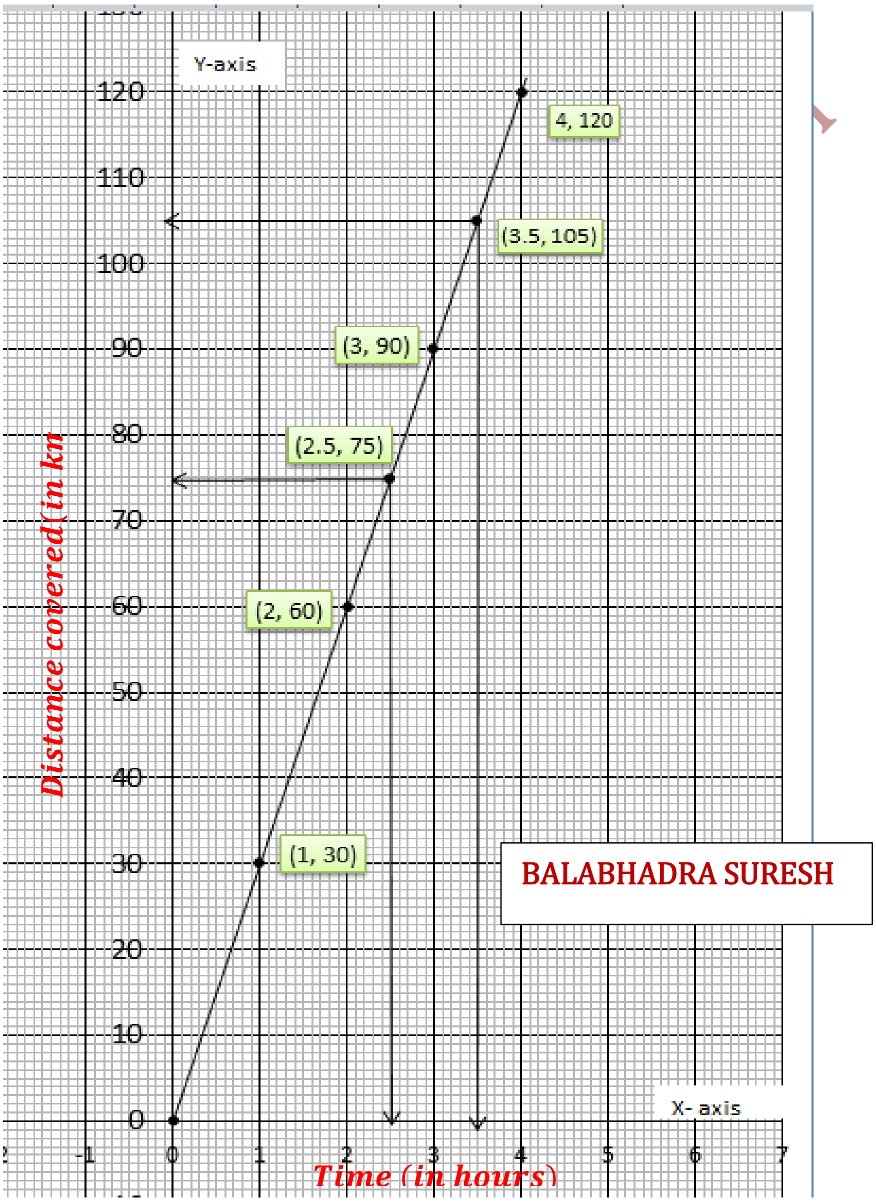
Hours of ride	Distance covered (d=t×s)	Point(t,d)
1 hour	30 km	(1,30)
2 hour	2×30 km=60 km	(2,60)
3 hour	3×30 km=90 km	(3,90)
4 hour	4×30 km=120 km	(4,120)

Scale: On X-axis (Horizontal): 2 units=1 hour

On Y-axis (Vertical): 1 unit=10 km

(a) Corresponding to 75 km on the vertical axis, we get the time to be 2.5 hours on the horizontal axis. Thus 2.5 hours are needed to cover 75 km.

(b) Corresponding to $3\frac{1}{2}$ hours on the horizontal axis, the distance covered is 105 km on the vertical axis.



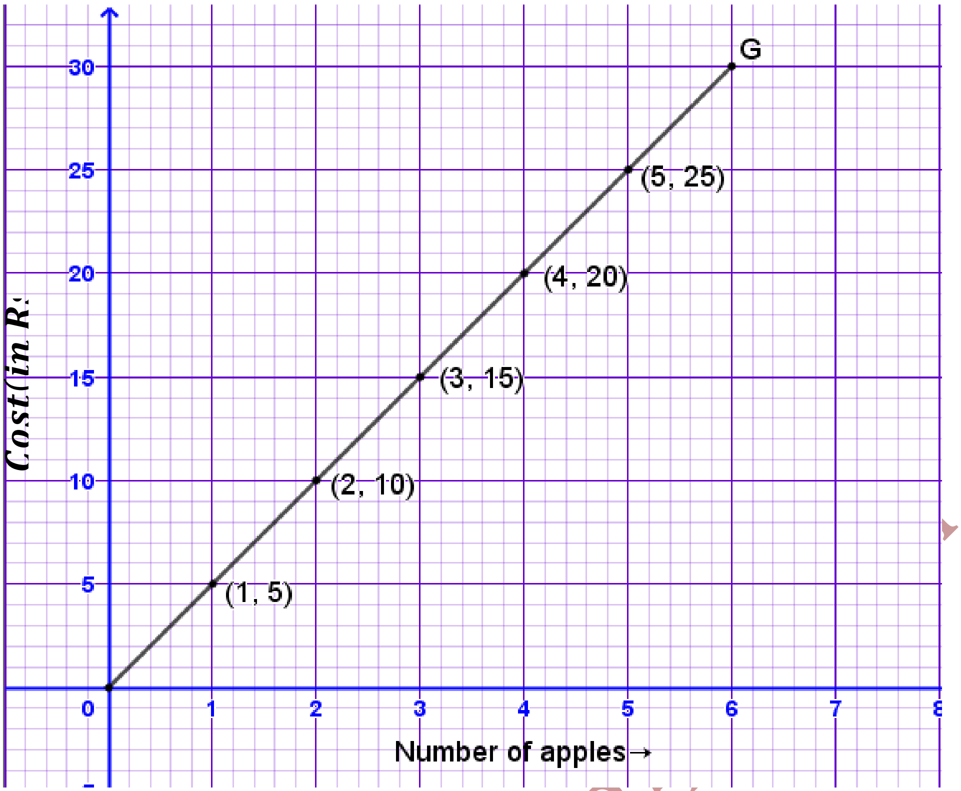
EXERCISE 13.2

1. Draw the graphs for the following tables of values, with suitable scales on the axes

(a) Cost of apples

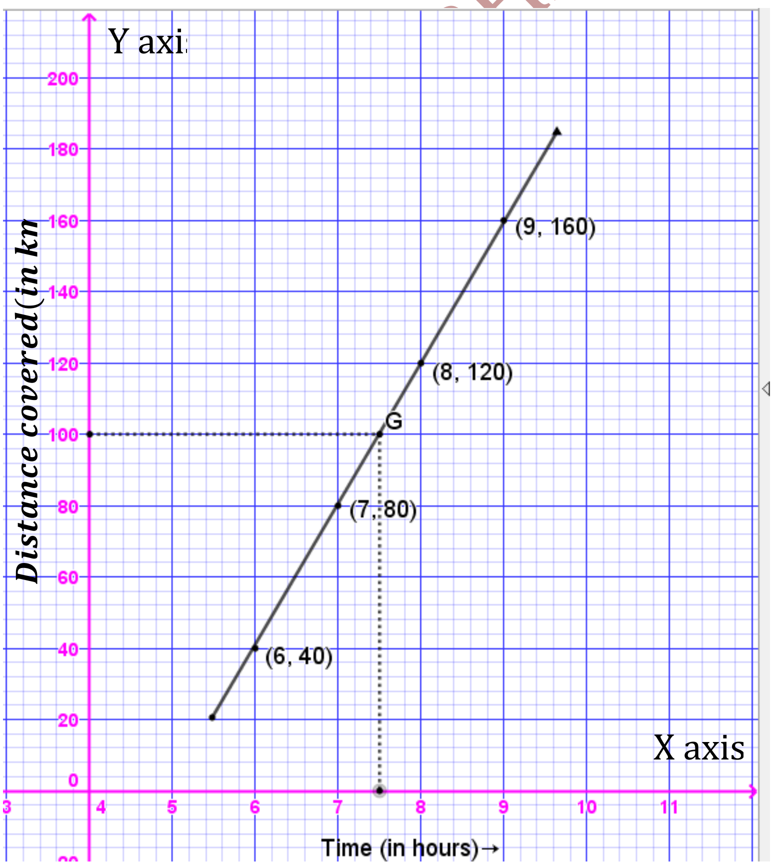
Number of apples	1	2	3	4	5
Cost (in ₹)	5	10	15	20	25

Scale: On X-axis 1 cm=1 apple and on Y-axis 1 cm=₹5



(b) Distance travelled by a car

Time (in hours)	6 a.m	7 a.m	8 a.m	9 a.m
Distances (in km)	40	80	120	160



Scale: (In regular graph 1 unit=1 cm)

On X-axis (Horizontal): 1 units=1 hour

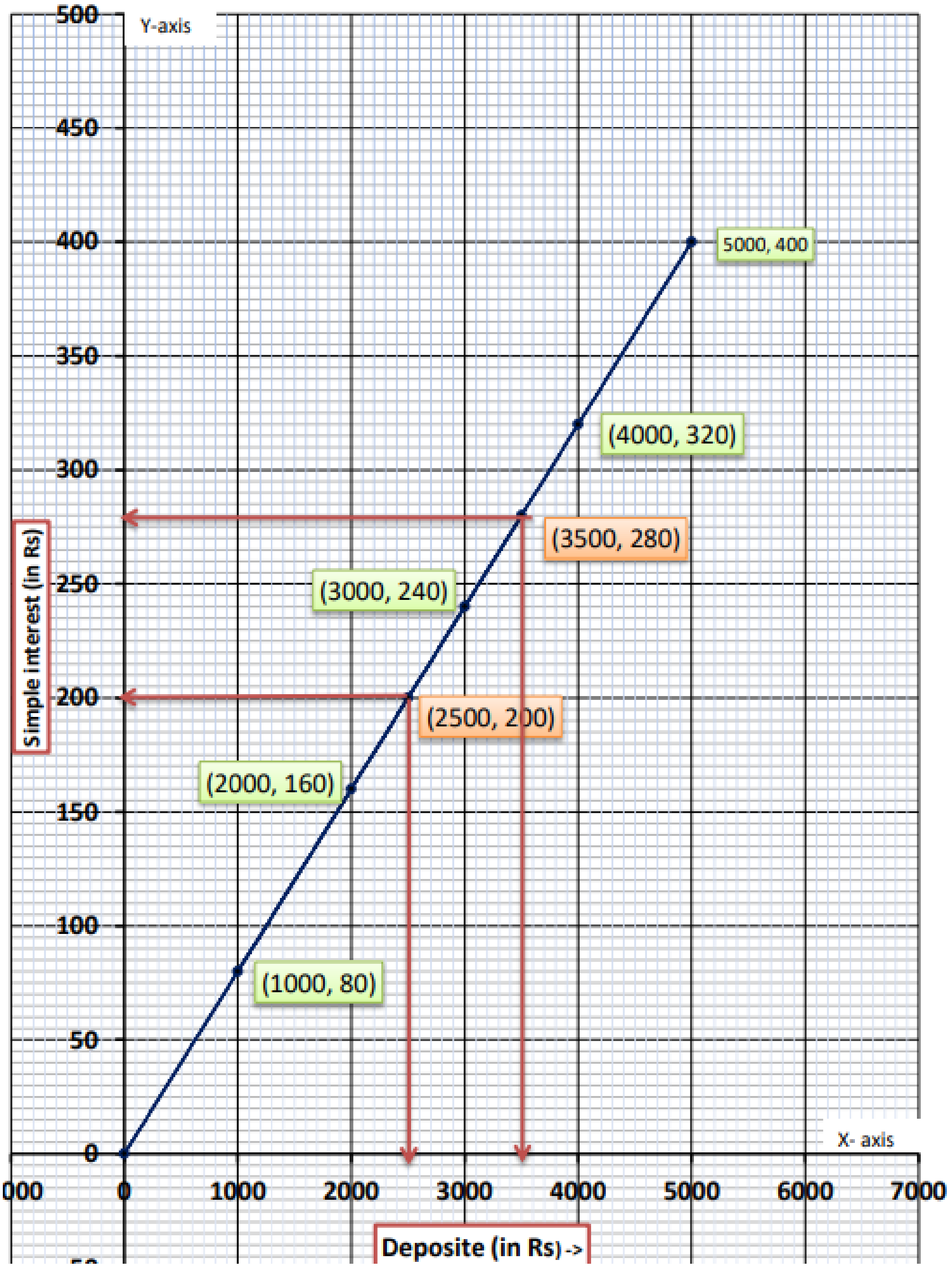
On Y-axis (Vertical): 1 unit=20 km

(a) During the period 7:30 am to 8 am, the car covered a distance of 20 km.(120-100)

(b) Corresponding to 100 km on the vertical axis 7.30 am hours on the horizontal axis .
So, the car covered a distance of 100 km at 7:30 am since its start.

(c) Interest on deposits for a year

Deposit (in `)	1000	2000	3000	4000	5000
Simple Interest (in `)	80	160	240	320	400



Scale :

On X-axis (Horizontal): 1 units= ₹1000

On Y-axis (Vertical): 1 unit= ₹50

(i) Does the graph pass through the origin?

Sol: Yes

(ii) Use the graph to find the interest on ₹ 2500 for a year.

Sol: From the graph the interest on ₹ 2500 for a year is 200

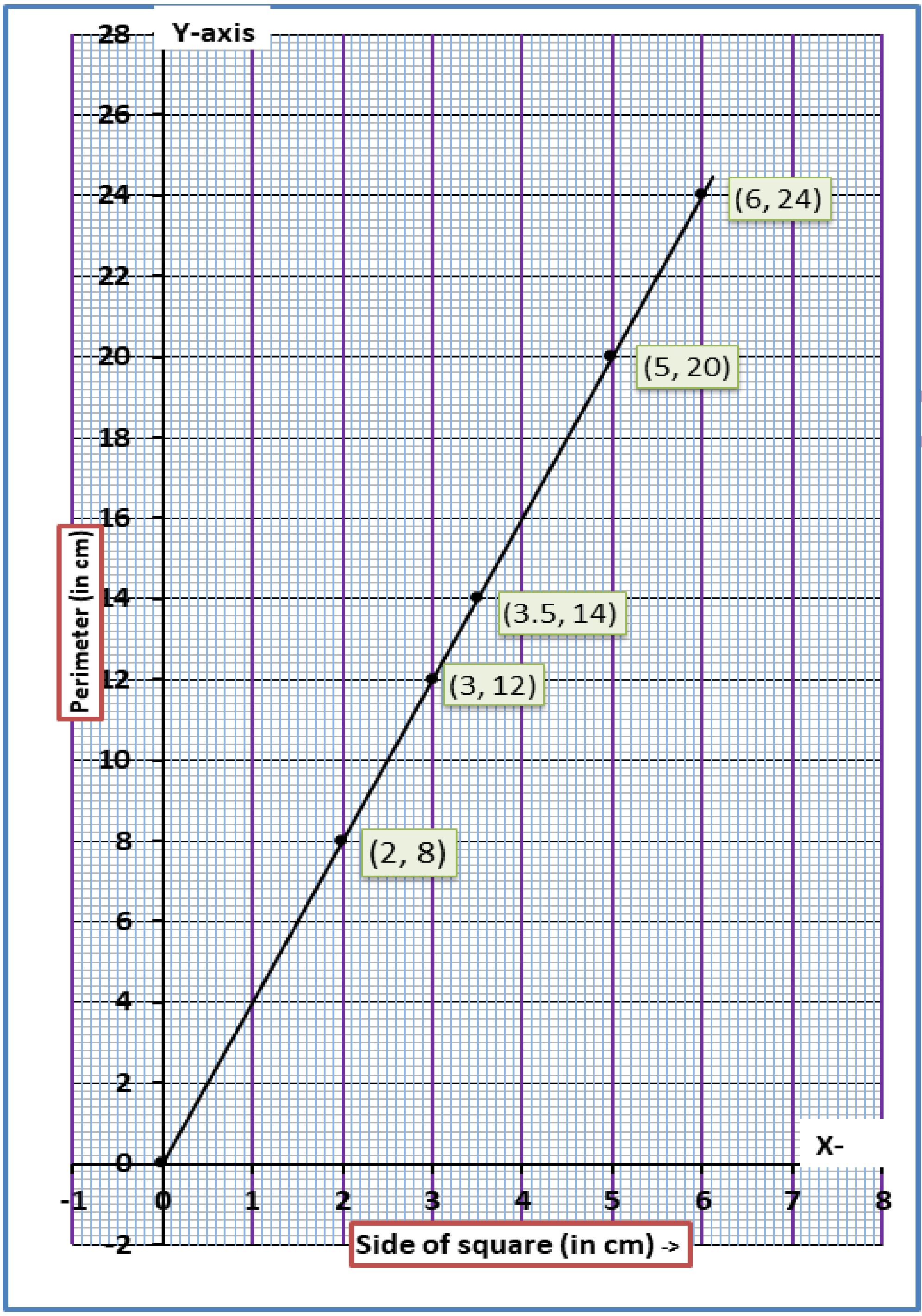
(iii) To get an interest of ₹280 per year, how much money should be deposited?

Sol: To get an interest of ₹ 280 per year, ₹ 3500 should be deposited.

2. Draw a graph for the following

(i)

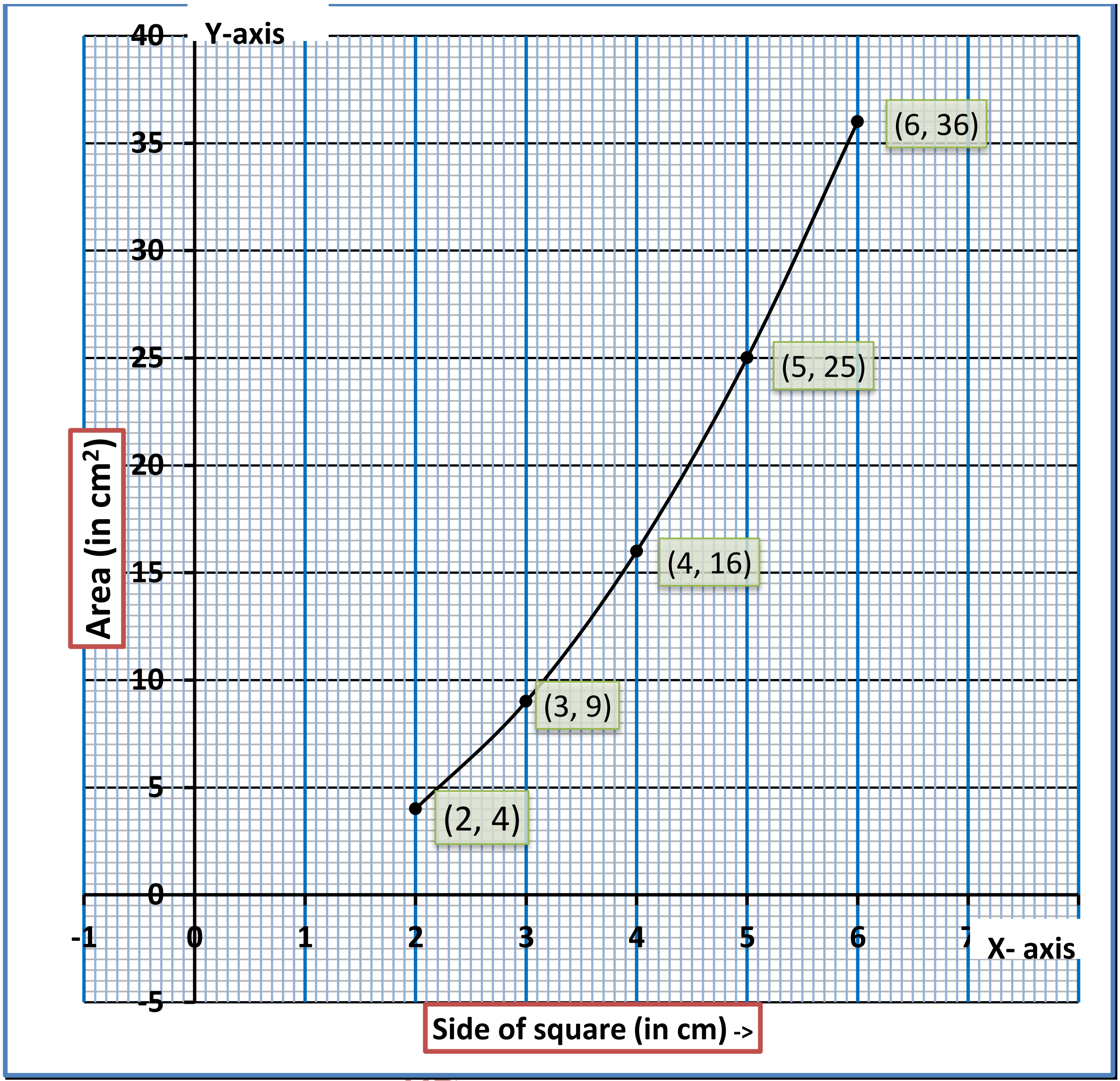
Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24



Given graph is a linear graph.

(ii)Area=side×side

Side of square (in cm)	2	3	4	5	6
Area (in cm^2)	4	9	16	25	36



Given graph not a linear graph.