CHAPTER 1

IX-MATHEMATICS-NCERT

1. NUMBER SYSTEMS (NOTES)

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1. **Natural numbers:** The numbers which are used for counting are called Natural numbers and represented with letter N.
2. Natural numbers N={1,2,3,4,5,……}
3. **Whole numbers**: If '0' is added to Natural numbers then they are called Whole numbers. And is denoted by 'W'
4. Whole numbers W={0,1,2,3,4,5,……}
5. **Integers**: Combination of positive and negative numbers including 0 are called Integers and represented by 'Z' or 'I'.
6. Integers Z=$\{\dots\dots -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\dots\dots\}$
7. Z comes from the German word "zahlen", which means "to count"
8. **Rational numbers:**

   A number which can be written in the form $\frac{p}{q}$, where p and q are integers and q $\neq$ 0 is called a rational number. Example: $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers.

Exp 1 : Are the following statements true or false? Give reasons for your answers.

(i) Every whole number is a natural number.

Sol: False, because zero is a whole number but not a natural number.

(ii) Every integer is a rational number.

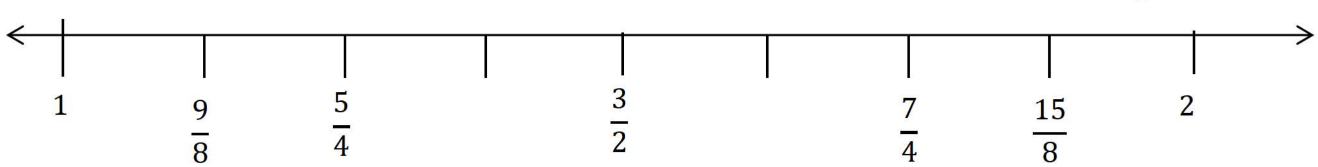
Sol: True, because every integer m can be expressed in the form $\frac{m}{1}$, and so it is a rational number

(iii) Every rational number is an integer.

Sol: False, because $\frac{3}{5}$ is a rational number but not an integer.

Exp 2 : Find five rational numbers between 1 and 2.

Sol 1: If a and b are two rational numbers then a rational number between a and b $= \frac{1}{2}(a+b)$



S.No	Two rational numbers	Between Rational number
1	1 and 2	$\frac{1}{2}(1+2) = \frac{1}{2}(3) = \frac{3}{2}$
2	1 and $\frac{3}{2}$	$\frac{1}{2}\left(1+\frac{3}{2}\right) = \frac{1}{2}\left(\frac{2+3}{2}\right) = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$

3	$\frac{3}{2}$ and 2	$\frac{1}{2}\left(\frac{3}{2}+2\right)=\frac{1}{2}\left(\frac{3+4}{2}\right)=\frac{1}{2}\times\frac{7}{2}=\frac{7}{4}$
4	1 and $\frac{5}{4}$	$\frac{1}{2}\left(1+\frac{5}{4}\right)=\frac{1}{2}\left(\frac{4+5}{4}\right)=\frac{1}{2}\times\frac{9}{4}=\frac{9}{8}$
5	$\frac{7}{4}$ and 2	$\frac{1}{2}\left(\frac{7}{4}+2\right)=\frac{1}{2}\left(\frac{7+8}{4}\right)=\frac{1}{2}\times\frac{15}{4}=\frac{15}{8}$

So, the five rational numbers between 1 and 2 are $\frac{9}{8},\frac{5}{4},\frac{3}{2},\frac{7}{4},\frac{15}{8}$

Sol 2:

$$1 < 2 \Rightarrow \frac{1\times 6}{1\times 6} < \frac{2\times 6}{1\times 6} \Rightarrow \frac{6}{6} < \frac{12}{6}$$

$$\Rightarrow \frac{6}{6} < \frac{7}{6} < \frac{8}{6} < \frac{9}{6} < \frac{10}{6} < \frac{11}{6} < \frac{12}{6}$$

So, the five rational numbers are $\frac{7}{6},\frac{8}{6},\frac{9}{6},\frac{10}{6},\frac{11}{6} \Rightarrow \frac{7}{6},\frac{4}{3},\frac{3}{2},\frac{5}{3},\frac{11}{6}$

There are infinitely many rational numbers between any two given rational numbers

EXERCISE 1.1

1. **Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?**

Sol: yes, zero is a rational number. $0 = \frac{0}{1}$

2. **Find six rational numbers between 3 and 4.**

Sol: $3 < 4 \Rightarrow \frac{3\times7}{1\times7} < \frac{4\times7}{1\times7} \Rightarrow \frac{21}{7} < \frac{28}{7}$

$$\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

So, the six rational numbers are $\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},\frac{27}{7}$

3. **Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.**

Sol: $\frac{3}{5} < \frac{4}{5}$

$$\Rightarrow \frac{3\times 6}{5\times 6} < \frac{4\times 6}{5\times 6}$$

$$\Rightarrow \frac{18}{30} < \frac{24}{30}$$

$$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

So, the five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{19}{30},\frac{20}{30},\frac{21}{30},\frac{22}{30},\frac{23}{30}$

4. **State whether the following statements are true or false. Give reasons for your answers.**

(i) **Every natural number is a whole number.**

Sol: False, because zero is a whole number but nota a natural number.

(ii) Every integer is a whole number.

Sol: False, -5 is an integer but not a whole number

(iii) Every rational number is a whole number.

Sol: False, because $\frac{4}{5}$ is a rational number but not a whole number.

Irrational Numbers

The Pythagoreans in Greece were the first to discover the numbers which were not rationals. These numbers are called irrational numbers

A number cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called irrational.

Examples: $\sqrt{2}, \sqrt{5}, \pi, 0.101001000 \ldots$ *etc*

Real numbers (R) : Collection of both rational (Q) and irrational numbers (Q^1)

Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

Exp 3 : Locate $\sqrt{2}$ on the number line.

Sol: 1.Draw number line. Point O at 0 and Point A at 1.

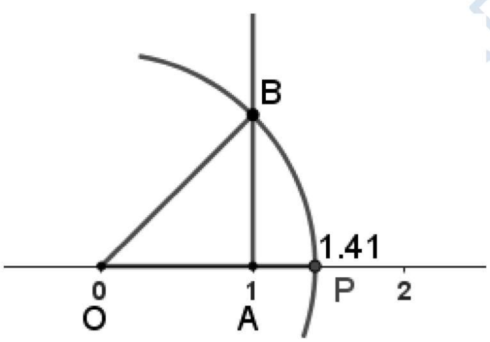
2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

4. Draw an arc with centre O and radius OB, intersects number line at P.

5. The point P corresponds to $\sqrt{2}$ on the number line.



Exp 4 : Locate $\sqrt{3}$ on the number line.

Sol: 1. Draw number line. Point O at 0 and Point A at 1.

2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

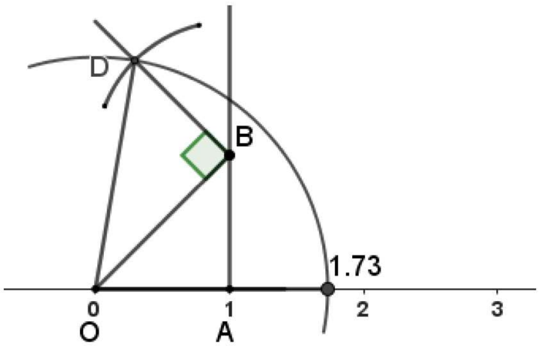
5. Construct BD of unit length perpendicular to OB.

6. Join OD.

7. From Pythagoras theorem OD=$\sqrt{(\sqrt{2})^2+1^2} = \sqrt{2+1} = \sqrt{3}$

8. Draw an arc with centre O and radius OD, intersects number line at Q.

9. The point Q corresponds to $\sqrt{3}$ on the number line.



EXERCISE 1.2

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Sol: yes

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

Sol: False , all negative numbers on the number line but it not express as of the form $\sqrt{m}$, where m is a natural number

(iii) Every real number is an irrational number.

Sol: False, real numbers are Collection of both rational (Q) and irrational numbers (Q^1)

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol: False, because 4 is a positive integer and $\sqrt{4} = \pm 2$ are rational numbers.

3. Show how $\sqrt{5}$ can be represented on the number line.

Sol: 1.Draw number line. Point O at 0 and Point A at 2.

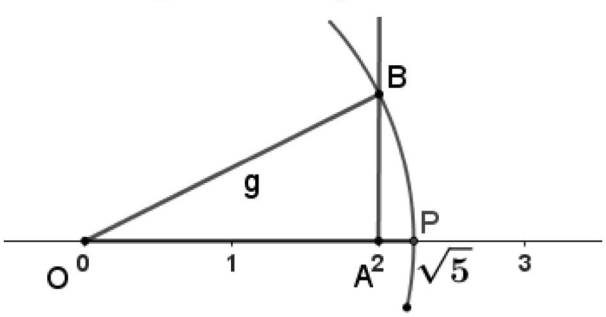
2. Construct AB= 1 unit perpendicular to number line at A

3. Join OB

4. From Pythagoras theorem OB=$\sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5}$

4. Draw an arc with centre O and radius OB, intersects number line at P.

5. The point P corresponds to $\sqrt{5}$ on the number line..



Real Numbers and their Decimal Expansions

Exp 5 : Find the decimal expansions of $\frac{10}{3}, \frac{7}{8}$ *and* $\frac{1}{7}$

$$
\begin{array}{r|l}
 & 3.333\ldots \\
3 & 10.000\ldots \\
 & 9 \\
 & \overline{10} \\
 & 9 \\
 & \overline{10} \\
 & 9 \\
 & \overline{1}
\end{array}
\qquad
\begin{array}{r|l}
 & 0.875 \\
8 & 7.000\ldots \\
 & 64 \\
 & \overline{60} \\
 & 56 \\
 & \overline{40} \\
 & 40 \\
 & \overline{0}
\end{array}
\qquad
\begin{array}{r|l}
 & 0.142857\ldots \\
7 & 1.000000 \\
 & 7 \\
 & \overline{30} \\
 & 28 \\
 & \overline{20} \\
 & 14 \\
 & \overline{60} \\
 & 56 \\
 & \overline{40} \\
 & 35 \\
 & \overline{50} \\
 & 49 \\
 & \overline{1}
\end{array}
$$

$\frac{10}{3} = 3.333.. = 3.\overline{3}$

$\frac{7}{8} = 0.785$

$\frac{1}{7} = 0.142857142\ldots = 0.\overline{142857}$

Terminating decimal: A decimal number that contains a finite number of digits next to the decimal point is called a Terminating decimal

Non terminating recurring decimal: A Non terminating recurring decimal is a decimal in which some digits after the decimal point repeat without terminating.

Example 6 : Show that 3.142678 is a rational number. In other words, express 3.142678 in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Sol: $3.142678 = \frac{3142678}{1000000}$, and hence a rational numbers

Example 7 : Show that $0.3333\ldots = 0.\overline{3}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0

Sol: $Let\ x = 0.\overline{3}$

$x = 0.33333\ldots.$

$10x = 3.333\ldots.$

$10x = 3 + 0.3333\ldots.$

$10x = 3 + x$

$10x - x = 3$

$9x = 3$

$x = \frac{3}{9} = \frac{1}{3} \Rightarrow 0.\overline{3} = \frac{1}{3}$

$Let\ x = 0.\overline{3} = 0.333\ldots\ldots.. \rightarrow (1)$
$10x = 3.333\ldots. \rightarrow (2)$
From (2)-(1)
$10x = 3.333\ldots\ldots.. \rightarrow (2)$
$x = 0.333\ldots\ldots.. \rightarrow (1)$
$9x = 3$
$x = \frac{3}{9} = \frac{1}{3} \Rightarrow 0.\overline{3} = \frac{1}{3}$

Example 8 : Show that $1.272727\ldots = 1.\overline{27}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0.

Sol: $Let\ x = 1.\overline{27}$

$x = 1.272727 ....$

$100x = 127.272727 ....$

$100x = 126 + 1.272727 ...$

$100x = 126 + x$

$100x - x = 126$

$99x = 126$

$x = \frac{126}{99} = \frac{14}{11}$

$1.\overline{27} = \frac{14}{11}$

$Let\ x = 1.\overline{27} = 1.272727 .... \rightarrow (1)$

$100x = 127.272727 .... \rightarrow (2)$

From (2)-(1)

$100x = 127.272727 .... \rightarrow (2)$

$x = 1.272727 .... \rightarrow (1)$

$99x = 126$

$x = \frac{126}{99} = \frac{14}{11} \Rightarrow 1.\overline{27} = \frac{14}{11}$

Example 9 : Show that $0.2353535... = 0.2\overline{35}$. can be expressed in the form p q , where p and q are integers and q ≠ 0.

Sol: $Let\ x = 0.2\overline{35}$

$x = 0.2353535 ...$

$100x = 23.53535 .....$

$100x = 23.3 + 0.23535 ...$

$1000x = 23.3 + x$

$100x - x = 23.3$

$99x = 23.3$

$x = \frac{23.3}{99} = \frac{233}{990}$

$0.2\overline{35} = \frac{233}{990}$

$Let\ x = 0.2\overline{35} = 0.2353535 ... \rightarrow (1)$

$100x = 235.3535 ..... \rightarrow (2)$

From (2)-(1)

$100x = 23.53535 ..... \rightarrow (2)$

$x = 0.2353535 .... \rightarrow (1)$

$99x = 23.3$

$x = \frac{23.3}{99} = \frac{233}{990} \Rightarrow 0.2\overline{35} = \frac{233}{990}$

Irrational: A number whose decimal expansion is non-terminating non-recurring is irrational.

Examples: $\sqrt{2}, \sqrt{5}, \pi, 0.101001000 ...\ \ etc$

Exp10 : Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Sol: $\frac{1}{7} = 0.142857 ... .$

$\frac{2}{7} = 0.285714 ....$

Irrational number is non-terminating non-recurring decimal

$An\ irrational\ number\ between\ \frac{1}{7}\ and\ \frac{2}{7}\ is\ 0.1520002000020000 .....$

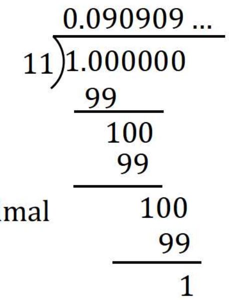
EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has

(i) $\frac{36}{100} = 0.36$

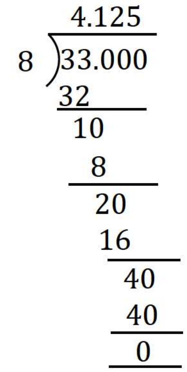
Terminating decimal

(ii) $\frac{1}{11} = 0.090909.. = 0.\overline{09}$



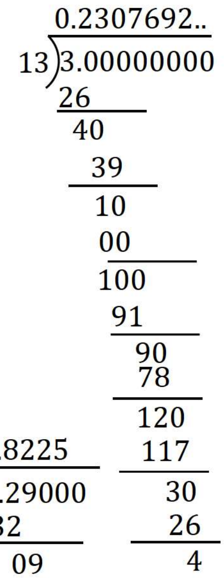
Non terminating recurring decimal

(iii) $4\frac{1}{8} = \frac{33}{8} = 4.125$



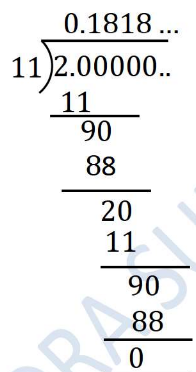
Terminating decimal.

(iv) $\frac{3}{13} = 0.23076923 \ldots = 0.\overline{230769}$



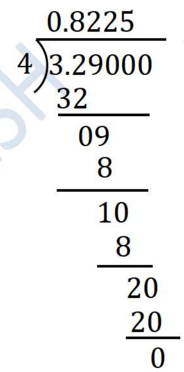
Non terminating recurring decimal

(v) $\frac{2}{11} = 0.1818.. = 0.\overline{18}$



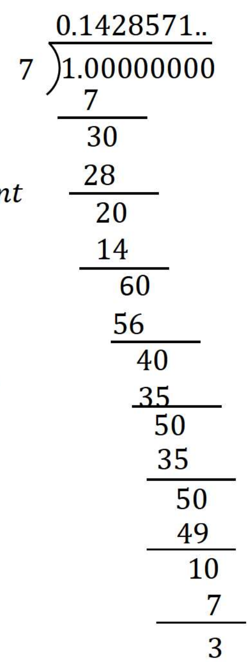
Non terminating recurring decimal

(vi) $\frac{329}{400} = \frac{3.29}{4} = 0.8225$



Terminating decimal

2. You know that $\frac{1}{7} = 0.\overline{142857}$.. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?



Sol: When divided 1 by 7 remainders are 3,2,6,4,5.

$\frac{1}{7} = 0.\overline{142857}$

2 is a remainder after the second step. So, we write the quatient after the second decimal place

$\frac{2}{7} = 0.\overline{285714}$

3 is a remainder after the first step. So, we write the quatient after the first decimal place

$\frac{3}{7} = 0.\overline{428571}$

$\frac{4}{7} = 0.\overline{571428}$

$\frac{5}{7} = 0.\overline{714285}$

$\frac{6}{7} = 0.\overline{857142}$

(OR)

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$$

$$\frac{6}{7} = 6 \times \frac{1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and q ≠ 0

(i) $0.\overline{6}$

Sol: Let $x = 0.\overline{6}$

$x = 0.66666 \ldots..$

$10x = 6.6666 \ldots.$

$10x = 6 + 0.6666 \ldots.$

$10x = 6 + x$

$10x - x = 6$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

$Let\ x = 0.\overline{6} = 0.66666 \ldots.. \rightarrow (1)$

$10x = 6.6666 \ldots. \rightarrow (2)$

From (2)-(1)

$10x = 6.6666 \ldots. \rightarrow (2)$

$x = 0.66666 \ldots.. \rightarrow (1)$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3} \Rightarrow 0.\overline{6} = \frac{2}{3}$

(ii) $0.4\overline{7}$

Sol: $Let\ x = 0.4\overline{7}$

$x = 0.477777 \ldots.$

$10x = 4.777777 \ldots.$

$10x = 4.3 + 0.4777777 \ldots$

$10x = 4.3 + x$

$10x - x = 4.3$

$9x = 4.3$

$x = \frac{4.3}{9} = \frac{43}{90}$

$0.4\overline{7} = \frac{43}{90}$

$Let\ x = 0.4\overline{7} = 0.477777 \ldots. \rightarrow (1)$

$10x = 4.777777 \ldots \rightarrow (2)$

From (2)-(1)

$10x = 4.777777 \ldots \rightarrow (2)$

$x = 0.477777 \ldots.. \rightarrow (1)$

$9x = 4.3$

$x = \frac{4.3}{9} = \frac{43}{90} \Rightarrow 0.4\overline{7} = \frac{43}{90}$

(iii) $0.\overline{001}$

Sol: $Let\ x = 0.\overline{001}$

$x = 0.001001001 \ldots.$

$1000x = 1.001001001 \ldots$

$1000x = 1 + 0.001001001 \ldots$

$1000x = 1 + x$

$1000x - x = 1$

$999x = 1$

$x = \frac{1}{999}$

$0.\overline{001} = \frac{1}{999}$

4. Express 0.99999 .... in the form p q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Sol: $Let\ x = 0.999999 \ldots.$

$10x = 9.9999 \ldots$

$10x = 9 + 0.999 \ldots$

$10x = 9 + x$

$10x - x = 9$

$9x = 9$

$x = \frac{9}{9} = 1$

$0.9999 \ldots = 1$

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

```
      0.0588235294117647..
17 ) 1.00000000
      85
      150
      136
        40
        34
         60
         51
          90
          85
           50
           34
           160
           153
             70
             68
              20
              17
              130
              119
               110
               102
                 80
                 68
                 120
                 119
                   1
```

$$\frac{1}{17} = 0.\overline{0588235294117647}$$

6. **Look at several examples of rational numbers in the form $\frac{p}{q}$ (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?**

Sol: $\frac{1}{2} = 0.5 \quad \frac{1}{10} = 0.1 \quad \frac{32}{5} = 6.4 \quad \frac{5}{8} = 0.625 \quad \frac{27}{25} = 1.08 \quad \frac{3}{50} = 0.06 \quad \frac{7}{20} = 0.35$

$2 = 2^1;\ 10 = 2^1 \times 5^1;\ 8 = 2^3; 25 = 5^2; 50 = 2^1 \times 5^2;\ 20 = 2^2 \times 5^1$

The prime factorisation of q has only powers of 2 or 5 or both

The q (denominator) is in the form of $2^a \times 5^b$ where a, b are whole numbers.

7. **Write three numbers whose decimal expansions are non-terminating non-recurring**

Sol: (i) 0.51250535420062101254.....

(ii) 1.20200200020000....

(iii) 0.2012011201112310....

8. **Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$**

Sol: $\frac{5}{7} = 0.714285 \ldots\ldots..;\ \frac{9}{11} = 0.8181 \ldots..$

Three irrational numbers between the rational numbers $\frac{5}{7}$ *and* $\frac{9}{11}$ *are*

(i) 0.722020020002000 ...

(ii) 0.73030030003000 ....

(iii) 0.7515115111511125 ....

9. **Classify the following numbers as rational or irrational :**

(i) $\sqrt{23}$ → Irrational number

(ii) $\sqrt{225} = 15$ → Rational number

(iii) $0.3796 = \frac{3796}{10000}$ → Rational number

(iv) $7.478478 \ldots = 7.\overline{478}$ → Rational number

(v) $1.101001000100001\ldots$ → Irrational numbe

OPERATIONS ON REAL NUMBERS

Example 11 : Check whether $7\sqrt{5}$, $\frac{7}{\sqrt{5}}$, $\sqrt{2} + 21$, $\pi - 2$ are irrational numbers or not

Sol: $\sqrt{5} = 2.2360679 \ldots$

$$\frac{7}{\sqrt{5}} = \frac{7 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{7\sqrt{5}}{5} = \frac{15.6524753..}{5} = 3.1304 \ldots$$

$\sqrt{2} + 21 = 1.414213.. + 21 = 22.414213 \ldots$

$\pi - 2 = 3.1415 \ldots - 2 = 1.1415 \ldots.$

All these are non-terminating, non-recurring decimals. Thus they are irrational numbers.

If q is rational and s is irrational then $q+s, q-s, qs$ and $\frac{q}{s}(s \neq 0)$ are irrational numbers.

Example 12 : Add $2\sqrt{2}+5\sqrt{3}$ and $\sqrt{2}-3\sqrt{3}$

Sol: $(2\sqrt{2}+5\sqrt{3})+(\sqrt{2}-3\sqrt{3}) = 2\sqrt{2}+\sqrt{2}+5\sqrt{3}-3\sqrt{3} = 3\sqrt{2}+2\sqrt{3}$

Example 13 : Multiply $6\sqrt{5}$ by $2\sqrt{5}$.

Sol: $6\sqrt{5} \times 2\sqrt{5} = 6\times 2\times\sqrt{5}\times\sqrt{5} = 12\times 5 = 60 \quad (\sqrt{a}\times\sqrt{a}=a)$

Example 14 : Divide $8\sqrt{15}$ by $2\sqrt{3}$

Sol: $$\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{4\times 2\times\sqrt{3}\times\sqrt{5}}{2\times\sqrt{3}} = 4\sqrt{5}$$

Note: (i) The sum or difference of a rational number and an irrational number is irrational. (ii) The product or quotient of a non-zero rational number with an irrational number is irrational. (iii) If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.

List some properties relating to square roots

Let a and b be positive real numbers. Then

$(i)\ \sqrt{ab} = \sqrt{a}\times\sqrt{b}$;

$\sqrt{a}\times\sqrt{a} = \sqrt{a^2} = (\sqrt{a})^2 = a$

$(ii)\ \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} : if\ b \neq 0$

$(iii)\ (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$

$(iv)\ (a+\sqrt{b})(a-\sqrt{b}) = a^2-b$

$(v)\ (\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{a}\times\sqrt{c}+\sqrt{a}\times\sqrt{d}+\sqrt{b}\times\sqrt{c}+\sqrt{b}\times\sqrt{d}$

$= \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

$(vi)\ (\sqrt{a}+\sqrt{b})^2 = a+2\sqrt{ab}+b$

$(vii)\ (\sqrt{a}-\sqrt{b})^2 = a-2\sqrt{ab}+b$

Example 15 : Simplify the following expressions:

(i) $(5+\sqrt{7})(2+\sqrt{5})$

Sol: $(5+\sqrt{7})(2+\sqrt{5}) = 5\times 2+5\times\sqrt{5}+\sqrt{7}\times 2+\sqrt{7}\times\sqrt{5}$

$= 10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}$

(ii) $(5+\sqrt{5})(5-\sqrt{5})$

Sol: $(x+y)(x-y)=x^2-y^2$

$$(5+\sqrt{5})(5-\sqrt{5})=5^2-(\sqrt{5})^2=25-5=20$$

(iii) $(\sqrt{3}+\sqrt{7})^2$

Sol: $(x+y)^2=x^2+2xy+y^2$

$$(\sqrt{3}+\sqrt{7})^2=(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{7}+(\sqrt{7})^2$$

$$=3+2\sqrt{21}+7=10+2\sqrt{21}$$

(iv) $(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})$

Sol: $(x-y)(x+y)=x^2-y^2$

$$(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})=(\sqrt{11})^2-(\sqrt{7})^2=11-7=4$$

Example 16: **Rationalise the denominator of $\frac{1}{\sqrt{2}}$**

Sol: Rationalise factor of $\sqrt{2}=\sqrt{2}$

$$\frac{1}{\sqrt{2}}=\frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{\sqrt{2}}{2}$$

Example 17: **Rationalise the denominator of $\frac{1}{2+\sqrt{3}}$**

Sol: Rationalise factor of $2+\sqrt{3}=2-\sqrt{3}$

$$\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=\frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}=\frac{2-\sqrt{3}}{4-3}=\frac{2-\sqrt{3}}{1}=2-\sqrt{3}$$

Example 18 : **Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$**

Sol: Rationalise factor of $\sqrt{3}-\sqrt{5}=\sqrt{3}+\sqrt{5}$

$$\frac{5}{\sqrt{3}-\sqrt{5}}=\frac{5}{\sqrt{3}-\sqrt{5}}\times\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}=\frac{5(\sqrt{3}+\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$$

$$=\frac{5(\sqrt{3}+\sqrt{5})}{3-5}=\frac{5(\sqrt{3}+\sqrt{5})}{-2}=\frac{-5(\sqrt{3}+\sqrt{5})}{2}=\left(\frac{-5}{2}\right)(\sqrt{3}+\sqrt{5})$$

Example 19 : **Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$**

Sol: Rationalise factor of $7+3\sqrt{2}=7-3\sqrt{2}$

$$\frac{1}{7+3\sqrt{2}}=\frac{1}{7+3\sqrt{2}}\times\frac{7-3\sqrt{2}}{7-3\sqrt{2}}=\frac{7-3\sqrt{2}}{(7)^2-(3\sqrt{2})^2}=\frac{7-3\sqrt{2}}{49-9\times2}=\frac{7-3\sqrt{2}}{49-18}=\frac{7-3\sqrt{2}}{31}$$

EXERCISE 1.4

1. **Classify the following numbers as rational or irrational**:

(i) $2-\sqrt{5}$ → Irrational number

(ii) $(3+\sqrt{23})-\sqrt{23}=3+\sqrt{23}-\sqrt{23}=3$ → Rational number

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$ → Rational number

(iv) $\frac{1}{\sqrt{2}}=\frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{\sqrt{2}}{2}$ → *Irrational number*

(v) 2π → *Irrational number*

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Sol: $(3+\sqrt{3})(2+\sqrt{2})=3\times2+3\times\sqrt{2}+\sqrt{3}\times2+\sqrt{3}\times\sqrt{2}$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Sol: $(a+b)(a-b)=a^2-b^2$

$(3+\sqrt{3})(3-\sqrt{3})=3^2-(\sqrt{3})^2$

$=9-3=6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Sol: $(a+b)^2=a^2+2ab+b^2$

$(\sqrt{5}+\sqrt{2})^2=(\sqrt{5})^2+2\times\sqrt{5}\times\sqrt{2}+(\sqrt{2})^2$

$=5+2\sqrt{10}+2$

$=7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Sol: $(x-y)(x+y)=x^2-y^2$

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})=(\sqrt{5})^2-(\sqrt{2})^2=5-2=3$

3. **Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi=\frac{c}{d}\cdot$ This seems to contradict the fact that π is irrational. How will you resolve this contradiction?**

Sol: There is no contradiction .So, you may not realise that either c or d is irrational.

We use $\pi=\frac{22}{7}$ *or* 3.14 these are approximate values .

The actual value of π is 3.141592653589....which is non-terminating non-recurring. Hence π is an irrational number.

4. Represent $\sqrt{9.3}$ on the number line.

Sol: 1. Draw AK=15 cm

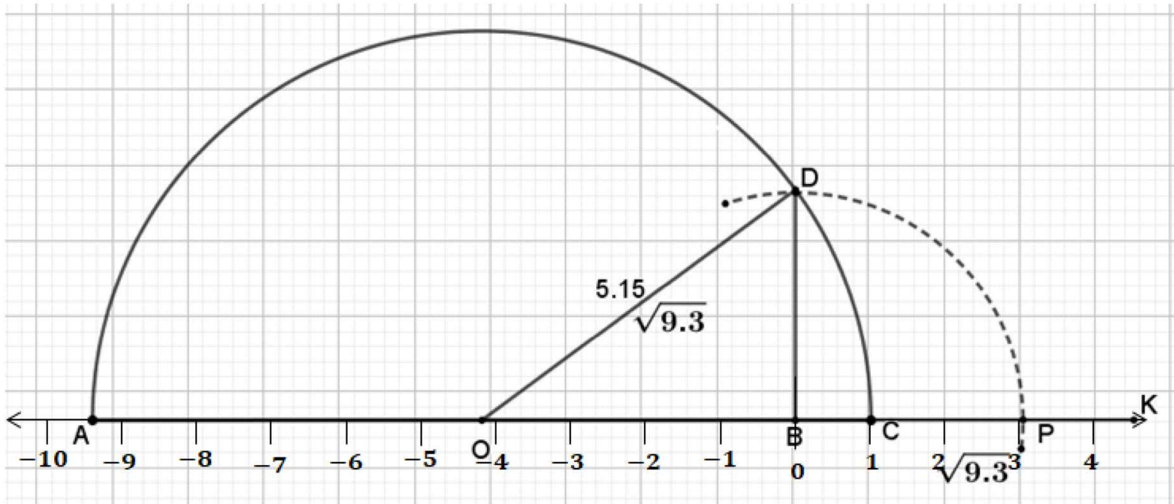
2. Mark B,C on AK such that AB=9.3 and AC=AB+1=10.3

3. Draw perpendicular bisector to AC intersect at O.

4. Draw semicircle on AK with centre O and radius OA.

5. Draw perpendicular line through B to AK intersect semicircle at D.

7. Draw an arc with centre B and radius BD intersect BK at P. P represents $\sqrt{9.3}$



5. **Rationalise the denominators of the following:**

(i) $\frac{1}{\sqrt{7}}$

Sol: Rationalise factor of $\sqrt{7} = \sqrt{7}$

$$\frac{1}{\sqrt{7}} = \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$$

(ii) $\frac{1}{\sqrt{7} - \sqrt{6}}$

Sol: Rationalise factor of $\sqrt{7} - \sqrt{6} = \sqrt{7} + \sqrt{6}$

$$\frac{1}{\sqrt{7} - \sqrt{6}} = \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}}$$

$$= \frac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$$

$$= \frac{\sqrt{7} + \sqrt{6}}{7 - 6} = \frac{\sqrt{7} + \sqrt{6}}{1} = \sqrt{7} + \sqrt{6}$$

(iii) $\frac{1}{\sqrt{5} + \sqrt{2}}$

Sol: Rationalise factor of $\sqrt{5} + \sqrt{2} = \sqrt{5} - \sqrt{2}$

$$\frac{1}{\sqrt{5} + \sqrt{2}} = \frac{1}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

$(iv)\ \frac{1}{\sqrt{7}-2}$

Sol: Rationalise factor of $\sqrt{7}-2 = \sqrt{7}+2$

$$\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$= \frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$$

$$= \frac{\sqrt{7}+2}{7-2} = \frac{\sqrt{7}+2}{5}$$

Laws of Exponents for Real Numbers

(i) $a^m \times a^n = a^{m+n}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^m \times b^m = (ab)^m$

(v) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

(vi) $\frac{1}{a^m} = a^{-m}$

(vii) $\frac{1}{a^{-m}} = a^m$

$(viii)$ $a^0 = 1$

$(ix) a^{-1} = \frac{1}{a}$

Example 20 : Simplify

$(i)\ 2^{\frac{2}{3}}.2^{\frac{1}{3}}$

Sol: $2^{\frac{2}{3}}.2^{\frac{1}{3}} = 2^{\frac{2}{3}+\frac{1}{3}} = 2^{\frac{2+1}{3}} = 2^{\frac{3}{3}} = 2^1 = 2$

$(ii)\ \left(3^{\frac{1}{5}}\right)^4$

Sol: $\left(3^{\frac{1}{5}}\right)^4 = 3^{\frac{1}{5}\times 4} = 3^{\frac{4}{5}}$

$(iii)\ \frac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}$

Sol: $\frac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}} = 7^{\frac{1}{5}-\frac{1}{3}} = 7^{\frac{3-5}{15}} = 7^{\frac{-2}{15}}$

$(iv)\ 13^{\frac{1}{5}}.17^{\frac{1}{5}}$

Sol: $13^{\frac{1}{5}}.17^{\frac{1}{5}} = (13 \times 17)^{\frac{1}{5}} = 221^{\frac{1}{5}}$

EXERCISE 1.5

1. **Find**

(i) $64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}} = 8^{2\times\frac{1}{2}} = 8$

(ii) $32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2^{5\times\frac{1}{5}} = 2$

(iii) $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{3\times\frac{1}{3}} = 5$

2. **Find**

(i) $9^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2\times\frac{3}{2}} = 3^3 = 27$

(ii) $32^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = 2^{5\times\frac{2}{5}} = 2^2 = 4$

(iii) $16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^{4\times\frac{3}{4}} = 2^3 = 8$

(iv) $125^{\frac{-1}{3}} = (5^3)^{\frac{-1}{3}} = 5^{3\times\frac{-1}{3}} = 5^{-1} = \frac{1}{5}$

3. Simplify

(i) $2^{\frac{2}{3}}.2^{\frac{1}{5}} = 2^{\frac{2}{3}+\frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7 = \frac{1^7}{(3^3)^7} = \frac{1}{3^{21}} =$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}} = 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}}.8^{\frac{1}{2}} = (7\times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$

2. POLYNOMIALS(notes)

PREPARED BY: BALABHADRA SURESH

1. An algebraic expression in which the variables involved have only non-negative integral (whole numbers) powers is called a polynomial.

Polynomials	Not polynomials
$2x$	$4x^{\frac{1}{2}}$
$\frac{1}{3}x-4$	$3x^2+4x^{-1}+5$
x^2-2x-1	$4+\frac{1}{x}$

2. If a polynomial contains only one variable then it is called polynomial in one variable.

   Ex: $2x+3$; $5x^2-6x+2$; $5y+6$; $-6y^2+7y-5$

3. In the polynomial $x^2 + 2x$, the expressions x^2 *and* $2x$ are called the terms of the polynomial.

4. Each term of a polynomial has a coefficient. In $-x^3 + 4x^2 + 7x - 2$

   The coefficient of $x^3=-1$

   The coefficient of $x^2=4$

   The coefficient of $x=7$

   constant term $=-2$

5. 2, –5, 7, etc. are examples of **constant polynomials**.

6. The constant polynomial 0 is called the **zero polynomial**.

7. If the variable in a polynomial is x, we may denote the polynomial by p(x), or q(x), or r(x), etc

8. The highest power of the variable in a polynomial as the degree of the polynomial.

   Example: i) $3x^2+7x+5$ → degree=2

   ii) $7x^3+5x^2+2x-6$ → degree=3

Types of polynomials according to degree

1. Constant polynomial: A polynomial of degree 0 is called constant polynomial.

   Ex: 5,-7,120,...

2. Linear polynomial: A polynomial of degree 1 is called a linear polynomial.

   Example: $3x+5, 7x-8, -9x, \ldots$

   The general form a linear polynomial in variable x *is* $ax+b$ $(a,b\in R, a\neq 0)$.

3. Quadratic polynomial : A polynomial of degree 2 is called a quadratic polynomial.

   Example: $x^2-5x+6, 2x^2-5, 7x^2, \ldots$..

   The general form a quadratic polynomial in variable x *is* ax^2+bx+c $(a,b,c\in R, a\neq 0)$.

4. Cubic polynomial : A polynomial of degree 3 is called a cubic polynomial.

Example: $5x^3 - 4x^2 + x - 1, 2x^3 - 3x + 5, -3x^3 - 10, \ldots\ldots$

The general form a cubic polynomial in variable x *is* $ax^3 + bx^2 + cx + d$ $(a, b, c, d \in R, a \neq 0)$.

9. **The general form of nth degree polynomial in one variable x**:

$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots\cdots\cdots + a_1 x + a_0$ is a polynomial of nth degree ,

where $a_0, a_1, a_2, \ldots\ldots\ldots, a_{n-1}, a_n$ are real coefficients and $a_0 \neq 0$.

EXERCISE 2.1

1. **Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.**

(i) $\mathbf{4x^2 - 3x + 7}$ → polynomial in one variable x

(ii) $\mathbf{y^2 + \sqrt{2}}$ → polynomial in one variable y

(iii) $\mathbf{3\sqrt{t} + t\sqrt{2}}$ → not a polynomial

(iv) $\mathbf{y + \frac{2}{y}}$ → not a polynomial

(v) $\mathbf{x^{10} + y^3 + t^{50}}$ → polynomial in three variables $x, y$and t

2. **Write the coefficient of x ^{2} in each of the following**

$(i)\mathbf{2 + x^2 + x}$ → coefficient of $x^2 = 1$

$(ii)\mathbf{2 - x^2 + x^3}$ → coefficient of $x^2 = -1$

$(iii)\mathbf{\frac{\pi}{2}x^2 + x}$ → coefficient of $x^2 = \frac{\pi}{2}$

$(iv)\mathbf{\sqrt{2}x - 1}$ → coefficient of $x^2 = 0$

3. **Give one example each of a binomial of degree 35, and of a monomial of degree 100**.

Sol: A binomial of degree 35 : $x^{35} + x^2$

A monomial of degree 100: $3x^{100}$

4. **Write the degree of each of the following polynomials:**

Polynomial	Degree
$(i)\ 5x^3 + 4x^2 7x$	3
$(ii)\ 4 - y^2$	2
$(iii) 5t - \sqrt{7}$	1
$(iv) 3$	0

5. **Classify the following as linear, quadratic and cubic polynomials:**

Sol: Linear polynomials: $(iv) 1 + x$ $(v) 3t$

Quadratic polynomials: $(i) x^2 + x$ $(iii)\ y + y^2 + 4$ $(vi)\ r^2$

Cubic polynomials: $(ii)\ x - x^3$ $(vii)\ 7x^3$

Example 2 : Find the value of each of the following polynomials at the indicated value of variables

(i) $\boldsymbol{p(x) = 5x^2 - 3x + 7 \ at \ x = 1}$

Sol: $p(1) = 5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 12 - 3 = 9$

(ii) $\boldsymbol{q(y) = 3y^2 - 4y + \sqrt{11} \ at \ y = 2}$

Sol: $q(2) = 3(2)^2 - 4(2) + \sqrt{11} = 12 - 8 + \sqrt{11} = 4 + \sqrt{11}$

(iii) $\boldsymbol{p(t) = 4t^4 + 5t^3 - t^2 \ at \ t = a}$

Sol: $p(a) = 4a^4 + 5a^3 - a^2$

Zeroes of a Polynomial

1. A real number 'c' is a zero of a polynomial $p(x)$ if $p(c)$ = 0. In this case, 'c' is also called a root of the polynomial equation $p(x) = 0$.
2. Every linear polynomial in one variable has a unique zero, a non-zero constant polynomial has no zero.
3. Every real number is a zero of the **zero polynomial**.

Linear Polynomial	Zero of the polynomial
$x + a$	$-a$
$x - a$	a
$ax + b$	$\frac{-b}{a}$
$ax - b$	$\frac{b}{a}$

Example 3 : Check whether –2 and 2 are zeroes of the polynomial x + 2.

Solu : Let $p(x) = x + 2$

Then $p(2) = 2 + 2 = 4, p(-2) = -2 + 2 = 0$

Therefore, –2 is a zero of the polynomial x + 2, but 2 is not.

Example 4 : Find a zero of the polynomial p(x) = 2x + 1.

Sol: Let $p(x) = 0$

$2x + 1 = 0$

$2x = -1$

$x = \frac{-1}{2}$

$So, \frac{-1}{2}$ is a zero of the polynomial 2x + 1

Example 5 : Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$

Sol: $Let \ \ p(x) = x^2 - 2x$

$p(2) = (2)^2 - 2(2) = 4 - 4 = 0$

$p(0) = (0)^2 - 2(0) = 0 - 0 = 0$

Hence, 2 and 0 are both zeroes of the polynomial $x^2 - 2x$.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at (i) x = 0 (ii) x = −1 (iii) x = 2

Sol: *Let* $p(x) = 5x - 4x^2 + 3$

(i) $p(0) = 5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

(ii) $p(-1) = 5(-1) - 4(-1)^2 + 3 = -5 - 4 + 3 = -9 + 3 = -6$

(iii) $p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3 = 13 - 16 = -3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

$(i)\ \ \boldsymbol{p(y) = y^2 - y + 1}$

Sol: $p(0) = (0)^2 - 0 + 1 = 0 - 0 + 1 = 1$

$p(1) = (1)^2 - 1 + 1 = 1 - 1 + 1 = 1$

$p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 5 - 2 = 3$

$(ii)\ \boldsymbol{p(t) = 2 + t + 2t^2 - t^3}$

Sol: $p(0) = 2 + 0 + 2 \times (0)^2 - (0)^3 = 2 + 0 + 0 - 0 = 2$

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3 = 2 + 1 + 2 - 1 = 4$

$p(2) = 2 + 2 + 2 \times 2^2 - 2^3 = 4 + 8 - 8 = 4$

$(iii)\ \ \boldsymbol{p(x) = x^3}$

Sol: $p(0) = 0^3 = 0$

$p(1) = 1^3 = 1$

$p(2) = 2^3 = 8$

$(iv)\ \ \boldsymbol{p(x) = (x - 1)\,(x + 1)}$

Sol: $p(0) = (0 - 1)\,(0 + 1) = (-1) \times 1 = -1$

$p(1) = (1 - 1)(1 + 1) = 0 \times 2 = 0$

$p(2) = (2 - 1)\,(2 + 1) = 1 \times 3 = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them

$(i)\ \ \boldsymbol{p(x) = 3x + 1;\ x = -\frac{1}{3}}$

Sol: $p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

$p\left(-\frac{1}{3}\right) = 0$

$So,\left(-\frac{1}{3}\right)\ is\ a\ zero\ of\ the\ polynomial\ 3x + 1$

$(ii)\ \ \boldsymbol{p(x) = 5x - \pi;\ x = \frac{4}{5}}$

Sol: $p(\frac{4}{5}) = 5 \times \left(\frac{4}{5}\right) - \pi = 4 - \pi$

$p\left(\frac{4}{5}\right) \neq 0$

So $\frac{4}{5}$ is not a zero of the polynomial $5x - \pi$.

(iii) $\boldsymbol{p(x) = x^2 - 1;\ x = 1, -1}$

Sol: $p(1) = 1^2 - 1 = 1 - 1 = 0$

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

$p(1) = 0\ \ and\ \ p(-1) = 0$

So, $1, -1$ *are the zeroes of the polynomial* $x^2 - 1$.

(iv) $\boldsymbol{p(x) = (x + 1)(x - 2), x = -1, 2}$

sol: $p(-1) = (-1 + 1)(-1 - 2) = 0 \times (-3) = 0$

$p(2) = (2 + 1)(2 - 2) = 3 \times 0 = 0$

So, $-1, 2$ *are the zeroes of the polynomial* $(x + 1)(x - 2)$

(v) $\boldsymbol{p(x) = x^2 ;\ x = 0}$

Sol: $p(0) = 0^2 = 0$

So, 0 is a zero of the polynomial x^2

(vi) $\boldsymbol{p(x) = lx + m, x = -\frac{m}{l}}$

Sol: $p(x) = lx + m$

$p\left(-\frac{m}{l}\right) = l\left(-\frac{m}{l}\right) + m = -m + m = 0$

(vii) $\boldsymbol{p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}}$

Sol: $p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{4}{3} - 1 = 4 - 1 = 3$

$-\frac{1}{\sqrt{3}}$ is the zero of the polynomial $3x^2 - 1$, but $\frac{2}{\sqrt{3}}$ is not.

(viii) $\boldsymbol{p(x) = 2x + 1, x = \frac{1}{2}}$

Sol: $\mathrm{p}\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 \ \ 2 + 1 = 3$

$\frac{1}{2}$ is not a zero of the polynomial $2\mathrm{x} + 1$

4. Find the zero of the polynomial in each of the following cases

(i) $\boldsymbol{p(x) = x + 5}$

Sol: $Let\ p(x) = 0$

$x + 5 = 0$

$x = -5$

$\therefore$ -5 is the zero of the polynomial $x + 5$

(ii) $\boldsymbol{p(x) = x - 5}$

Sol: $Let\ p(x) = 0$

$x - 5 = 0$

$x = 5$

$\therefore\ 5\ is\ the\ zero\ of\ the\ polynomial\ x - 5$

(iii) $\boldsymbol{p(x) = 2x + 5}$

Sol: $Let\ p(x) = 0$

$2x + 5 = 0$

$2x = -5 \Rightarrow x = \frac{-5}{2}$

$\therefore\ \frac{-5}{2}\ is\ the\ zero\ of\ the\ polynomial\ 2x + 5$

(vi) $\boldsymbol{p(x) = 3x - 2}$

Sol: $Let\ p(x) = 0$

$3x - 2 = 0$

$3x = 2 \Rightarrow x = \frac{2}{3}$

$\therefore\ \frac{2}{3}\ is\ the\ zero\ of\ the\ polynomial\ 3x - 2$

(v) $\boldsymbol{p(x) = 3x}$

Sol: $Let\ p(x) = 0$

$3x = 0$

$x = 0$

$\therefore\ 0\ is\ the\ zero\ of\ the\ polynomial\ 3x$

(vi) $\boldsymbol{p(x) = ax\ , a \neq 0}$

Sol: $Let\ p(x) = 0$

$ax = 0$

$x = 0$

$\therefore\ 0\ is\ the\ zero\ of\ the\ polynomial\ ax$

(vii) $\boldsymbol{p(x) = cx + d, c \neq 0, c, d\ are\ real\ numbers.}$

Sol: $Let\ p(x) = 0$

$cx + d = 0$

$cx = -d \Rightarrow x = \frac{-d}{c}$

$\therefore \frac{-d}{c}$ *is the zero of the polynomial* $cx + d$

Remainder Theorem: Let $p(x)$ be any polynomial of degree greater than or equal to one and let 'a' be any real number. If $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$.

Factor Theorem : $p(x)$ is a polynomial of degree n ≥ 1 and 'a' is any real number

(i) $If\ p(a) = 0\ then\ (x - a)\ is\ a\ factor\ of\ p(x)$. and

(ii) $If\ (x - a)\ is\ a\ factor\ of\ polynomial\ p(x)\ then\ p(a) = 0$.

Example 6 : Examine whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$ and of $2x + 4$.

Sol: Let $p(x) = x^3 + 3x^2 + 5x + 6$

$p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6$

$= -8 + 12 - 10 + 6 = 18 - 18 = 0$

$\therefore x + 2\ is\ a\ factor\ of\ x^3 + 3x^2 + 5x + 6$

$since\ 2x + 4 = 2(x + 2)\ but\ 2\ is\ not\ a\ factor\ of\ p(x)$

$So, 2x + 4\ is\ not\ a\ factor\ of\ p(x)$

Example 7 : Find the value of k, if $x - 1$ is a factor of $4x^3 + 3x^2 - 4x + k$

Sol: $Let\ p(x) = 4x^3 + 3x^2 - 4x + k$

$If\ x - 1\ is\ a\ factor\ of\ p(x)\ then\ p(1) = 0$

$4(1)^3 + 3(1)^2 - 4(1) + k = 0$

$4 + 3 - 4 + k = 0$

$3 + k = 0$

$k = -3$

Factorisation of the polynomial $ax^2 + bx + c$ by splitting the middle term.

Let its factors be $(px + q)$ and $(rx + s)$.

$ax^2 + bx + c = (px + q)(rx + s)$

$ax^2 + bx + c = prx^2 + (ps + qr)x + qs$

$ps \times qr = a \times c\ and\ \ ps + qr = b$

Example 8 : Factorise $6x^2 + 17x + 5$ by splitting the middle term, and by using the Factor Theorem

Sol: $6x^2 + 17x + 5 = 6x^2 + 2x + 15x + 5$

$= 2x(3x + 1) + 5(3x + 1)$

$= (3x + 1)(2x + 5)$

Example 9 : Factorise $y^2 - 5y + 6$ by using the Factor Theorem.

Sol: $p(y) = y^2 - 5y + 6$

$p(1) = (1)^2 - 5(1) + 6 = 1 - 5 + 6 = 7 - 5 = 2$

$p(2) = (2)^2 - 5(2) + 6 = 4 - 10 + 6 = 10 - 10 = 0$

$(y - 2)$ *is a factor of* $p(y)$

$p(3) = (3)^2 - 5(3) + 6 = 9 - 15 + 6 = 15 - 15 = 0$

$(y - 3)$ *is a factor of* $p(y)$

$\therefore y^2 - 5y + 6 = (y - 2)(y - 3)$

Example 10 : Factorise $\boldsymbol{x^3 - 23x^2 + 142x - 120}$.

Sol: $p(x) = x^3 - 23x^2 + 142x - 120$

$p(1) = (1)^3 - 23(1)^2 + 142(1) - 120 = 1 - 23 + 142 - 120 = 143 - 143 = 0$

$\therefore (x - 1)$ *is a factor of* $p(x)$

$x^3 - 23x^2 + 142x - 120 = x^3 - x^2 - 22x^2 + 22x + 120x - 120$

$= x^2(x - 1) - 22x(x - 1) + 120(x - 1)$

$= (x - 1)(x^2 - 22x + 120)$

$(x^2 - 22x + 120) = (x^2 - 12x - 10x + 120)$

$= x(x - 12) - 10(x - 12)$

$= (x - 12)(x - 10)$

$x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 12)(x - 10)$

EXERCISE 2.3

1. Determine which of the following polynomials has (x + 1) a factor

(*i*) $\mathbf{x^3 + x^2 + x + 1}$

Sol: $p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 2 - 2 = 0$

$(x + 1)$ is a factor *of* $x^3 + x^2 + x + 1$

(*ii*) $\mathbf{x^4 + x^3 + x^2 + x + 1}$

Sol: $p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 3 - 2 = 1$

$(x + 1)$ is not a factor *of* $x^4 + x^3 + x^2 + x + 1$

(*iii*) $\boldsymbol{x^4 + 3x^3 + 3x^2 + x + 1}$

Sol: $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3 \times (-1)^3 + 3 \times (-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

$(x + 1)$ is not a factor *of* $x^4 + 3x^3 + 3x^2 + x + 1$

(*iv*) $\mathbf{x^3 - x^2 - (2 + \sqrt{2})\,x + \sqrt{2}}$

Sol: $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2}) \times (-1) + \sqrt{2}$

$= -1 - 1 + 2 - \sqrt{2} + \sqrt{2} = 0$

$(x + 1)$ is a factor *of* $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

$(\boldsymbol{i})\ \boldsymbol{p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1}$

Sol: $p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

$\therefore\ g(x)\ is\ a\ factor\ of\ p(x)$

$(ii)\ p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

Sol: $p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= 14 - 14 = 0$

$\therefore\ g(x)\ is\ a\ factor\ of\ p(x)$

$(iii) p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Sol: $p(x) = x^3 - 4x^2 + x + 6$

$p(3) = 3^3 - 4 \times 3^2 + 2 + 6$

$= 27 - 36 + 8$

$= 36 - 36 = 0$

$\therefore\ g(x)\ is\ a\ factor\ of p(x)$

3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

$(\boldsymbol{i})\ \boldsymbol{p(x) = x^2 + x + k}$

If $x - 1$ is a factor of p(x) then $p(1) = 0$

$1^2 + 1 + k = 0$

$2 + k = 0$

$k = -2$

(ii) $\boldsymbol{p(x) = 2x^2 + kx + 2}$

If $x - 1$ is a factor of $p(x)$ then $p(1) = 0$

$2 \times 1^2 + k \times 1 + 2 = 0$

$2 + k + 2 = 0$

$k + 4 = 0$

$k = -4$

(iii) $\boldsymbol{p(x) = kx^2 - \sqrt{2}\,x + 1}$

If $x - 1$ is a factor of $p(x)$ then $p(1) = 0$

$k \times (1)^2 - \sqrt{2} \times 1 + 1 = 0$

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

$(iv) p(x) = kx^2 - 3x + k$

If $x - 1$ is a factor of $p(x)$ then $p(1) = 0$

$k(1)^2 - 3(1) + k = 0$

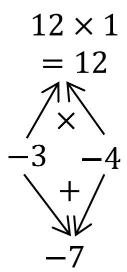
$k - 3 + k = 0$

$2k - 3 = 0$

$k = \frac{3}{2}$

4. Factorise :

(i) $\boldsymbol{12x^2 - 7x + 1}$

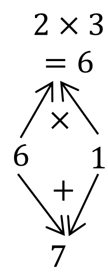


$Sol: 12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$= 3x(4x - 1) - 1(4x - 1)$

$= (4x - 1)(3x - 1)$

(ii) $\boldsymbol{2x^2 + 7x + 3}$

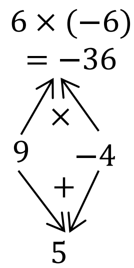


$Sol: 2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (x + 3)(2x + 1)$

(iii) $\boldsymbol{6x^2 + 5x - 6}$

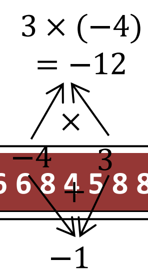


$Sol: 6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

(iv) $\boldsymbol{3x^2 - x - 4}$



$Sol: 3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

5. Factorise :

(i) $x^3 - 2x^2 - x + 2$

Sol: *Let* $p(x) = x^3 - 2x^2 - x + 2$

$p(x) = x^3 - 2x^2 - x + 2$

$p(1) = 1^3 - 2 \times 1^2 - 1 + 2$

$= 1 - 2 - 1 + 2 = 0$

So $(x - 1)$ is a factor of $p(x)$

$x^3 - 2x^2 - x + 2$

$= x^3 - x^2 - x^2 + x - 2x + 2$

$= x^2(x - 1) - x(x - 1) - 2(x - 1)$

$= (x - 1)(x^2 - x - 2)$

$= (x - 1)(x + 1)(x - 2)$

$x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x - 2) + 1(x - 2)$

$-1 \times 2 = -2$

$-1 + 2 = 1$

(ii) $x^3 - 3x^2 - 9x - 5$

Sol: $p(x) = x^3 - 3x^2 - 9x - 5$

$p(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 + 9 - 5$

$= 9 - 9 = 0$

So $(x + 1)$ is a factor of $p(x)$

$x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$

$= x^2(x + 1) - 4x(x + 1) - 5(x + 1)$

$= (x + 1)(x^2 - 4x - 5)$

$= (x + 1)(x + 1)(x - 5)$

$= (x + 1)^2(x - 5)$

$x^2 - 4x - 5$

$= x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

$-5 \times 1 = -5$

$-5 + 1 = -4$

$(iii) x^3 + 13x^2 + 32x + 20$

Sol: $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13 \times (-1)^2 + 32 \times (-1) + 20$

$= -1 + 13 - 32 + 20$

$= 33 - 33 = 0$

So $(x + 1)$ is a factor of $p(x)$

$x^3 + 13x^2 + 32x + 20$

$= x^3 + x^2 + 12x^2 + 12x + 20x + 20$

$= x^2(x+1) + 12x(x+1) + 20(x+1)$

$= (x+1)(x^2 + 12x + 20)$

$= (x+1)(x+2)(x+10)$

$x^2 + 12x + 20$	
$= x^2 + 2x + 10x + 20$	$2 \times 10 = 20$
$= x(x+2) + 10(x+2)$	$2 + 10 = 12$

$(iv) 2y^3 + y^2 - 2y - 1$

Sol: $p(y) = y^3 + y^2 - y - 1$

$p(-1) = (-1)^3 + (-1)^2 - (-1) - 1$

$= -1 + 1 + 1 - 1$

$= 2 - 2 = 0$

So $(y+1)$ is a factor of $p(y)$

$y^3 + y^2 - y - 1 = y^3 + y^2 - y - 1$

$= y^2(y+1) - 1(y+1)$

$= (y+1)(y^2 - 1)$

$= (y+1)(y+1)(y-1)$

Algebraic Identities

(i) $(x+y)^2 \equiv x^2 + 2xy + y^2$

(ii) $(x-y)^2 \equiv x^2 - 2xy + y^2$

(iii) $(x+y)(x-y) \equiv x^2 - y^2$

(iv) $(x+a)(x+b) \equiv x^2 + (a+b)x + ab.$

Example 11 : Find the following products using appropriate identities:

$(i)(x+3)(x+3)$

Sol: $(x+y)^2 = x^2 + 2xy + y^2$

$(x+3)(x+3) = (x+3)^2 = x^2 + 2 \times x \times 3 + 3^2$

$= x^2 + 6x + 9$

$(ii)(x-3)(x+5)$

Sol: $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(x-3)(x+5) = x^2 + (-3+5)x + (-3) \times 5$

$= x^2 + 2x - 15$

Example 12 : Evaluate 105 × 106 without multiplying directly

Sol: $(x+a)(x+b) \equiv x^2 + (a+b)x + ab$

$105 \times 106 = (100+5)(100+6)$

$= (100)^2 + (5+6) \times 100 + 5 \times 6$

$= 10000 + 1100 + 30 = 11130$

Example 13 : Factorise:

$(i) 49a^2 + 70ab + 25b^2$

$Sol: x^2 + 2xy + y^2 = (x + y)^2$

$49a^2 + 70ab + 25b^2 = (7a)^2 + 2 \times 7a \times 5b + (5b)^2$

$= (7a + 5b)^2 = (7a + 5b)(7a + 5b)$

$(ii) \frac{25}{4}x^2 - \frac{y^2}{9}$

$Sol: x^2 - y^2 = (x + y)(x - y)$

$\frac{25}{4}x^2 - \frac{y^2}{9} = \left(\frac{5}{2}x\right)^2 - \left(\frac{y}{3}\right)^2 = \left(\frac{5}{2}x + \frac{y}{3}\right)\left(\frac{5}{2}x - \frac{y}{3}\right)$

Identity V : $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$Sol: (x + y + z)^2 = [(x + y) + z]^2 = (x + y)^2 + 2(x + y)z + z^2$

$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Example 14 : Write $(3a + 4b + 5c)^2$ in expanded form

Sol: $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(3a + 4b + 5c)^2 = (3a)^2 + (4b)^2 + (5c)^2 + 2(3a)(4b) + 2(4b)(5c) + 2(3a)(5c)$

$= 9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac$

Example 16 : Factorise $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

Sol: $x^2 + y^2 + z^2 + 2xy + 2yz + 2zx = (x + y + z)^2$

$4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= (2x - y + z)^2 = (2x - y + z)(2x - y + z)$

Identity VI : $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$

$Sol: (x + y)^3 = (x + y)(x + y)^2 = (x + y)(x^2 + 2xy + y^2)$

$= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 = x^3 + y^3 + 3x^2y + 3xy^2 = x^3 + y^3 + 3xy(x + y)$

Identity VII : $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - y^3 - 3xy(x - y)$

$Sol: (x - y)^3 = (x - y)(x - y)^2 = (x - y)(x^2 - 2xy + y^2)$

$= x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3 = x^3 - y^3 - 3x^2y + 3xy^2 = x^3 - y^3 - 3xy(x - y)$

Example 17 : Write the following cubes in the expanded form:

$(i)(3a + 4b)^3$

$Sol: (x + y)^3 = x^3 + y^3 + 3xy(x + y)$

$(3a + 4b)^3 = (3a)^3 + (4b)^3 + 3(3a)(4b)(3a + 4b)$

$= 27a^3 + 64b^4 + 36ab(3a + 4b) = 27a^3 + 64b^4 + 108a^2b + 144ab^2$

$(ii)\ (5p - 3q)^3$

$Sol: (x - y)^3 = x^3 - y^3 - 3xy(x - y)$

$(5p - 3q)^3 = (5p)^3 - (3q)^3 - 3(5p)(3q)(5p - 3q)$

$= 125p^3 - 27q^3 - 45pq(5p - 3q)$

$= 125p^3 - 27q^3 - 225p^2q + 135pq^2$

Example 18 : Evaluate each of the following using suitable identities:

$(\boldsymbol{i})\ (\mathbf{104})^3$

$Sol: (x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$

$(104)^3 = (100 + 4)^3 = (100)^3 + (4)^3 + 3(100)(4)(100 + 4)$

$= 1000000 + 64 + 1200 \times 104$

$= 1000000 + 64 + 124800 = 1124864$

$(\boldsymbol{ii})(\mathbf{999})^3$

$Sol: (x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$

$(999)^3 = (1000 - 1)^3 = (100)^3 - (1)^3 - 3(100)(1)(1000 - 1)$

$= 1000000 - 1 - 300 \times 999$

$= 1000000 - 1 - 2997000 = 997002999$

Example 19 : $\boldsymbol{Factorise}\ \mathbf{8}\boldsymbol{x}^3 + \mathbf{27}\boldsymbol{y}^3 + \mathbf{36}\boldsymbol{x}^2\boldsymbol{y} + \mathbf{54}\boldsymbol{xy}^2$

$Sol: x^3 + y^3 + 3x^2y + 3xy^2 = (x + y)^3$

$8x^3 + 27y^3 + 36x^2y + 54xy^2 = (2x)^3 + (3y)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 = (2x + 3y)^3$

Identity VIII : $\boldsymbol{x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)}$

$\boldsymbol{Example}\ \mathbf{20} : \boldsymbol{Factorise} : \mathbf{8}\boldsymbol{x}^3 + \boldsymbol{y}^3 + \mathbf{27}\boldsymbol{z}^3 - \mathbf{18}\boldsymbol{xyz}$

$Sol: x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$8x^3 + y^3 + 27z^3 - 18xyz = (2x)^3 + (y)^3 + (3z)^3 - 3(2x)(y)(3z)$

$= (2x + y + 3z)[(2x)^2 + (y)^2 + (3z)^2 - (2x)(y) - (y)(3z) - (2x)(3z)]$

$= (2x + y + 3z)(4x^2 + y^2 + 9z^2 - 2xy - 3yz - 6xz)$

EXERCISE 2.4

1. Use suitable identities to find the following products

$(\boldsymbol{i})(\boldsymbol{x} + \mathbf{4})(\boldsymbol{x} + \mathbf{10})$

$Sol: (x + a)(x + b) \equiv x^2 + (a + b)x + ab\ ; a = 4, b = 10$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + 4 \times 10 = x^2 + 14x + 40$

$(\boldsymbol{ii})(\boldsymbol{x} + \mathbf{8})(\boldsymbol{x} - \mathbf{10})$

$Sol: (x + a)(x + b) \equiv x^2 + (a + b)x + ab; a = 8, b = -10$

$(x + 8)(x - 10) = x^2 + (8 - 10)x + 8 \times (-10) = x^2 - 2x - 80$

$(\boldsymbol{iii})(\mathbf{3x} + \mathbf{4})(\mathbf{3x} - \mathbf{5})$

$Sol: (x + a)(x + b) \equiv x^2 + (a + b)x + ab;\ x = 3x, a = 4, b = -5$

$(3x + 4)(3x - 5) = (3x)^2 + (4 - 5)(3x) + 4 \times (-5)$

$= 9x^2 - 3x - 20$

$(iv)\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$Sol: (a+b)(a-b)=a^2-b^2;\ a=y^2, b=\frac{3}{2}$

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}$$

$(v)(3-2x)(3+2x)$

$Sol: (a+b)(a-b)=a^2-b^2; a=3, b=2x$

$$(3-2x)(3+2x)=(3)^2-(2x)^2=9-4x^2$$

2. Evaluate the following products without multiplying directly

$(i)103\times 107$

$Sol: (x+a)(x+b)\equiv x^2+(a+b)x+ab; x=100, a=3, b=7$

$$103\times 107=(100+3)(100+7)=(100)^2+(3+7)(100)+3\times 7$$
$$=10000+1000+21=11021$$

$(ii)\ 95\times 96$

$Sol: (x+a)(x+b)\equiv x^2+(a+b)x+ab; x=90, a=5, b=6$

$$95\times 96=(90+5)(90+6)=(90)^2+(5+6)(90)+5\times 6$$
$$=8100+990+30=9120$$

$(iii)104\times 96$

$Sol: (a+b)(a-b)=a^2-b^2; a=100, b=4$

$$104\times 96=(100+4)(100-4)=(100)^2-(4)^2=10000-16=9984$$

3. Factorise the following using appropriate identities

$(i)\ 9x^2+6xy+y^2$

$Sol: a^2+2ab+b^2=(a+b)^2;\ a=3x, b=y$

$$9x^2+6xy+y^2=(3x)^2+2(3x)(y)+(y)^2=(3x+y)^2$$

$(ii)\ 4y^2-4y+1$

$Sol: a^2-2ab+b^2=(a-b)^2;\ a=2y, b=1$

$$4y^2-4y+1=(2y)^2+2(2y)(1)+(1)^2=(2y+1)^2$$

$(iii)\ x^2-\frac{y^2}{100}$

$Sol: a^2-b^2=(a+b)(a-b); a=x, b=\frac{y}{10}$

$$x^2-\frac{y^2}{100}=x^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right)$$

4. Expand each of the following, using suitable identities:

$(i)(x+2y+4z)^2$

$Sol: (a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$

$(x+2y+4z)^2 = (x)^2+(2y)^2+(4z)^2+2(x)(2y)+2(2y)(4z)+2(x)(4z)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

$(ii)(\mathbf{2x-y+z})^2$

$Sol: (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(\mathbf{2x-y+z})^2 = (2x)^2+(-y)^2+(z)^2+2(2x)(-y)+2(-y)(z)+2(2x)(z)$

$= 4x^2+y^2+z^2-4xy-2yz+4xz$

$(iii)(\mathbf{-2x+3y+2z})^2$

$Sol: (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(-2x+3y+2z)^2 = (-2x)^2+(3y)^2+(2z)^2+2(-2x)(3y)+2(3y)(2z)+2(-2x)(2z)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

$(iv)(\mathbf{3a-7b-c})^2$

$Sol: (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2xz$

$(3a-7b-c)^2 = (3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(3a)(-c)$

$= 9a^2+49b^2+c^2-42ab+14bc-6ac$

$(v)(\mathbf{-2x+5y-3z})^2$

$Sol: (a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$

$(-2x+5y-3z)^2 = (-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-2x)(-3z)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12xz$

$(vi)\left[\mathbf{\frac{1}{4}a-\frac{1}{2}b+1}\right]^2$

$Sol: (x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2xz$

$\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2\left(\frac{1}{4}a\right)(1)$

$= \frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a = \frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}$

5. Factorise:

$(i)\mathbf{4x^2+9y^2+16z^2+12xy-24yz-16xz}$

$Sol: a^2+b^2+c^2+2ab+2bc+2ac = (a+b+c)^2$

$4x^2+9y^2+16z^2+12xy-24yz-16xz$

$= (2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$

$= (2x+3y-4z)^2$

$(ii)\mathbf{2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz}$

$Sol: a^2+b^2+c^2+2ab+2bc+2ac = (a+b+c)^2$

$2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$= (-\sqrt{2}x)^2+(y)^2+(2\sqrt{2}z)^2+2(-\sqrt{2}x)(y)+2(y)(2\sqrt{2}z)+2(-\sqrt{2}x)(2\sqrt{2}z)$

$= \left(-\sqrt{2}x + y2\sqrt{2}z\right)^2$

6. Write the following cubes in expanded form

$(\boldsymbol{i})(\boldsymbol{2x+1})^{\boldsymbol{3}}$

$Sol: (a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

$(2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

$(ii)(2a-3b)^3$

$Sol: (x + y)^3 = x^3 + y^3 + 3xy\,(x + y)$

$(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$(\boldsymbol{iii})\left[\frac{\boldsymbol{3}}{\boldsymbol{2}}\boldsymbol{x}+\boldsymbol{1}\right]^{\boldsymbol{3}}$

$Sol: (a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

$\left[\frac{3}{2}x+1\right]^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$= \frac{9}{4}x^2 + 1 + \frac{9}{4}x^2 + \frac{9}{2}x$

$(\boldsymbol{iv})\left[\boldsymbol{x}-\frac{\boldsymbol{2}}{\boldsymbol{3}}\boldsymbol{y}\right]^{\boldsymbol{3}}$

$Sol: (a - b)^3 = a^3 - b^3 - 3ab\,(a - b)$

$\left[x-\frac{2}{3}y\right]^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

7. Evaluate the following using suitable identities:

$(i)(99)^3$

$Sol: (x - y)^3 \equiv x^3 - y^3 - 3xy\,(x - y)$

$(99)^3 = (100-1)^3$

$= 100^3 - 1^3 - 3(100)(1)[100-1]$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 9{,}70{,}299$

$\boldsymbol{(ii)(102)^3}$

$Sol: (x + y)^3 \equiv x^3 + y^3 + 3xy\,(x + y)$

$(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)[100 + 2]$

$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 10{,}61{,}208$

$\boldsymbol{(iii)(998)^3}$

$Sol: (x - y)^3 \equiv x^3 - y^3 - 3xy\,(x - y)$

$(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)[1000 - 2]$

$= 1000000000 - 8 - 6000(998)$

$= 1000000000 - 8 - 5988000$

$= 99{,}40{,}11{,}992$

8. **Factorise each of the following**:

$\boldsymbol{(i)\ 8a^3 + b^3 + 12a^2b + 6ab^2}$

$Sol: :\ x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$

$8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)^2b + 3(2a)(b)^2$

$= (2a + b)^3$

$\boldsymbol{(ii) 8a^3 - b^3 - 12a^2b + 6ab^2}$

$Sol: x^3 - y^3 - 3x^2y + 3xy^2 \equiv (x - y)^3$

$8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)^2b + 3(2a)(b)^2$

$= (2a - b)^3$

$(iii)\ 27 - 125a^3 - 135a + 225a^2$

$Sol: x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3 \times 3^2 \times 5a + 3 \times 3 \times (5a)^2$

$= (3 - 5a)^3$

$(vi)\ \ 64a^3 - 27b^3 - 144a^2b + 108ab^2$

$Sol:\ x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$= (4a - 3b)^3$

$(v) 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$Sol: x^3 - y^3 - 3x^2y + 3xy^2 = (x - y)^3$

$$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p = (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)^2\left(\frac{1}{6}\right) + 3(3p)\left(\frac{1}{6}\right)^2 = \left(3p - \frac{1}{6}\right)^3$$

9. Verify $(i)\ x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ $(ii)\ x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol: (i) $\text{R.H.S} = (x + y)(x^2 - xy + y^2) = x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3 = \text{L.H.S}$

(i) $\text{R.H.S} = (x - y)(x^2 + xy + y^2) = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3 = x^3 - y^3 = \text{L.H.S}$

10. (i)Factorise $27y^3 + 125z^3$

Sol: $(i)\ a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$27y^3 + 125z^3 = (3y)^3 + (5z)^3 = (3y + 5z)[(3y)^2 - (3y)(5z) + (5z)^2]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(ii) ***Factorise*** : $64m^3 - 343n^3$

$Sol: x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$64m^3 - 343n^3 = (4m)^3 - (7n)^3 = (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise : $27x^{33} + y^3 + z^3 - 9xyz$

Sol: $x^3 + y^3 + z^3 - 3xyz \equiv (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - (z)(3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol: $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$= (x + y + z)\frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$= (x + y + z)\frac{1}{2}[(x^2 - 2xy + y^2) + (y^2 - 2yz + z^2) + (z^2 - 2xz + x^2)]$

$= (x + y + z)\frac{1}{2}[(x - y)^2 + (y - z)^2 + (z - x)^2]$

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol: We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

If $x + y + z = 0$ then

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$x^3 + y^3 + z^3 - 3xyz = 0$

$x^3 + y^3 + z^3 = 3xyz$

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-\mathbf{12})^3 + (\mathbf{7})^3 + (\mathbf{5})^3$

Sol: Let $x = -12, y = 7, z = 5$

$x + y + z = -12 + 7 + 5 = 0$

We know that if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5)$

$\Rightarrow (-12)^3 + (7)^3 + (5)^3 = -1260$

$(\boldsymbol{ii})$ $(\mathbf{28})^3 + (-\mathbf{15})^3 + (-\mathbf{13})^3$

$Sol: Let\ x = 28, y = -15, z = -13$

$x + y + z = 28 - 15 - 13 = 0$

We know that if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$\Rightarrow (28)^3 + (-15)^3 + (-13)^3 = 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i)Area: $25a^2 - 35a + 12$

Sol: $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$= 5a(5a - 3) - 4(5a - 3)$

$= (5a - 3)(5a - 4)$

Length=$5a - 3$ and Breadth= $5a - 4$

(ii)**Area**: $\mathbf{35y^2 + 13y - 12}$

Sol: $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

Length=$5y + 4$ and Breadth= $7y - 3$

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i)**Volume** : $\mathbf{3x^2 - 12x}$

Sol: $3x^2 - 12x = 3x(x - 4)$

$length = 3, \quad breadth = x, \quad height = x - 4$

(ii) $\mathbf{12ky^2 + 8ky - 20k}$

Sol: $12ky^2 + 8ky - 20k =$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 - 3y + 5y - 5)$

$= 4k[3y(y - 1) + 5(y - 1)]$

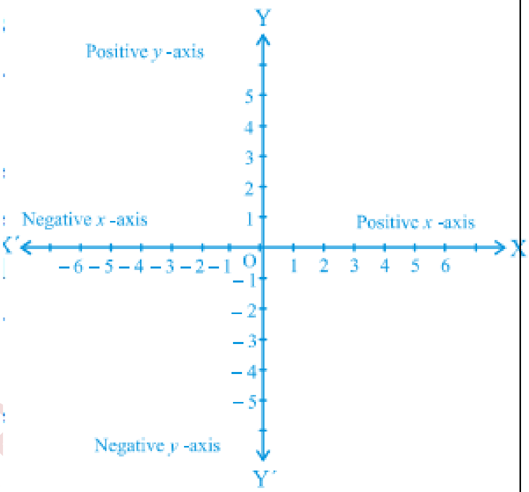
$= 4k(3y + 5)(y - 1) = l \times b \times h$

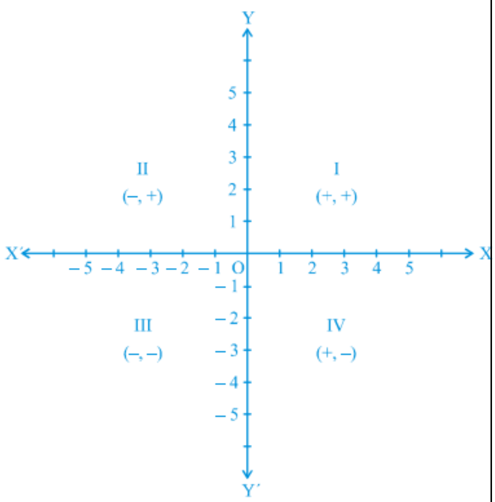
$length = 4k, \quad breadth = (3y + 5), \quad height = (y - 1)$

3.COORDINATE GEOMETRY(NOTES)

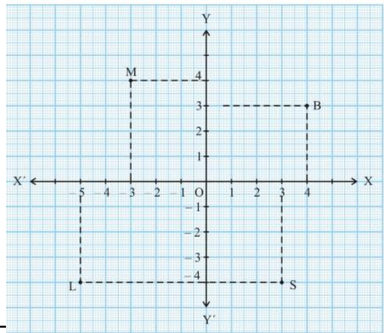
PREPARED BY: BALABHADRA SURESH

1. COORDINATE GEOMETRY was initially developed by the French philosopher and mathematician **René Déscartes.**
2. In honour of Déscartes, the system used for describing the position of a point in a plane is also known as the Cartesian system
3. The horizontal line X′X is called the x - axis and the vertical line YY′ is called the y – axis.
4. The point of intersection of the axes is called the origin, and is denoted by O
5. The positive numbers lie on the directions OX and OY are called the positive directions of the x - axis and the y - axis
6. The negative numbers lie on the directions OX′ and OY′ are called the negative directions of the x - axis and the y - axis
7. The coordinate axes divide the plane into four parts called **quadrants.**
8. The distance of a point from the y - axis is called its x-coordinate, or abscissa, and the distance of the point from the x-axis is called its y-coordinate, or ordinate
9. If the abscissa of a point is x and the ordinate is y, then (x, y) are called the coordinates of the point.
10. The coordinates of a point on the x-axis are of the form (x, 0) and that of the point on the y-axis are (0, y)
11. The coordinates of the origin are (0, 0).
12. The coordinates of a point are of the form (+ , +) in the first quadrant, (–, +) in the second quadrant, (–, –) in the third quadrant and (+, –) in the fourth quadrant, where + denotes a positive real number and – denotes a negative real number.
13. If $x \neq y$, then $(x, y) \neq (y, x)$, and $(x, y) = (y, x)$, if $x = y$.
14.





Example 1 : See Fig. 3.11 and complete the following statements:



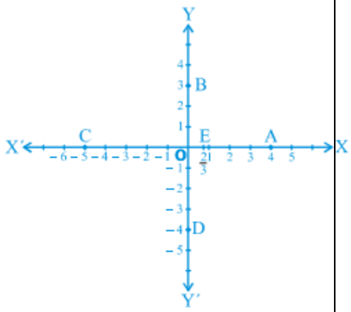
(i) The abscissa and the ordinate of the point B are **4** and **3** respectively. Hence, the coordinates of B are (**4,3**).

(ii) The x-coordinate and the y-coordinate of the point M are **-3** and **4** respectively. Hence, the coordinates of M are (**-3,4**).

(iii) The x-coordinate and the y-coordinate of the point L are **-5** and **-4** respectively. Hence, the coordinates of L are (**-5,-4**).

(iv) The x-coordinate and the y-coordinate of the point S are **3** and **-4** respectively. Hence, the coordinates of S are (**3,-4**).

Example 2 : Write the coordinates of the points marked on the axes .



Sol: $A = (4,0)$; $B = (0,3)$; $C = (-5,0)$; $D = (0,-4)$; $E = \left(\frac{2}{3},0\right)$

EXERCISE 3.2

1. Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

Sol: The horizontal line is called the x-axis and vertical line is called the y-axis.

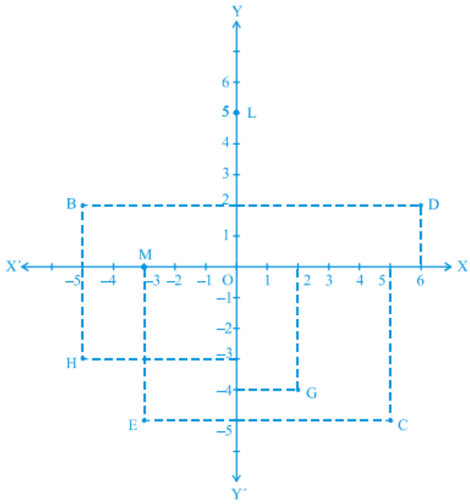
(ii) What is the name of each part of the plane formed by these two lines?

Sol: Quadrant

(iii) Write the name of the point where these two lines intersect.

Sol: Origin(O)

2. See Fig.3.14, and write the following:



(i) The coordinates of B.

Sol: $B = (-5,2)$

(ii) The coordinates of C.

Sol: $C = (5,-5)$

(iii) The point identified by the coordinates (–3, –5).

Sol: $(-3,-5) = E$

(iv) The point identified by the coordinates (2, – 4).

Sol: $(2,-4) = G$

(v) The abscissa of the point D.

Sol: 6

(vi) The ordinate of the point H.

Sol: -3

(vii) The coordinates of the point L.

Sol: $L = (0,5)$

(viii) The coordinates of the point M

Sol: $M = (-3,0)$

4. LINEAR EQUATIONS IN TWO VARIABLES (Notes)

PREPARED BY: BALABHADRA SURESH

1. If a linear equation has two variables then it is called a linear equation in two variables.
2. The general form of linear equation in two variables x, y is $ax + by + c = 0$. Where a, b, c are real numbers, and a, b are not both zero.
3. The process of finding solution(s) is called solving an equation .
4. A linear equation in two variables has infinitely many solutions. Every solution of the linear equation can be represented by a unique point on the graph of the equation.
5. The graphs of x = a and y = a are lines parallel to the y-axis and x-axis, respectively

Example 1 : Write each of the following equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:

(i) $\mathbf{2x + 3y = 4.37}$

Sol: $2x + 3y = 4.37 \Rightarrow 2x + 3y - 4.37 = 0$

$a = 2, b = 3, c = -4.37$

(ii) $\mathbf{x - 4 = \sqrt{3}y}$

$Sol: x - 4 = \sqrt{3}y \Rightarrow x - \sqrt{3}y - 4 = 0$

$a = 1, b = -\sqrt{3}, c = -4$

(iii) $\mathbf{4 = 5x - 3y}$

Sol: $5x - 3y - 4 = 0$

$a = 5, b = -3, c = -4$

(iv) $\mathbf{2x = y}$

Sol: $2x - y = 0$

$a = 2, b = -1, c = 0$

Example 2 : Write each of the following as an equation in two variables:

(i) $x = -5$

$Sol: 1.x + 0.y + 5 = 0$

(ii) y = 2

Sol: $0.x + 1.y - 2 = 0$

(iii) $2x = 3$

Sol: $2.x + 0.y - 3 = 0$

(iv) $5y = 2$

Sol: $0.x + 5.y - 2 = 0$

EXERCISE 4.1

1. The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

Sol: Let the cost of a notebook = ₹ x and cost of a pen = ₹ y

The cost of a notebook = 2× the cost of a pen

$x = 2y$

$x - 2y = 0$

2. Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case

(i) $\mathbf{2x + 3y = 9.3\overline{5}}$

Sol: $2x + 3y - 9.3\overline{5} = 0$

$a = 2, b = 3, c = -9.3\overline{5}$

(ii) $\mathbf{x - 5y - 10 = 0}$

Sol: $1.x - 5.y - 10 = 0$

$a = 1, b = -5, c = -10$

(iii) $\mathbf{-2x + 3y = 6}$

Sol: $-2.x + 3.y - 6 = 0$

$a = -2, b = 3, c = 6$

(iv) $\mathbf{x = 3y}$

Sol: $x - 3y = 0$

$a = 1, b = -3, c = 0$

(v) $\mathbf{2x = -5y}$

Sol: $2x + 5y + 0 = 0$

$a = 2, b = 5, c = 0$

(vi) $\mathbf{3x + 2 = 0}$

Sol: $3x + 0.y + 2 = 0$

$a = 3, b = 0, c = 2$

(vii) $\mathbf{y - 2 = 0}$

Sol: $0.x + 1.y - 2 = 0$

$a = 0, b = 1, c = -2$

(viii) $\mathbf{5 = 2x}$

Sol: $2x + 0.y - 5 = 0$

$a = 2, b = 0, c = -5$

Solution of a Linear Equation

(i) Any pair of values of 'x' and 'y' which satisfy the linear equation in two variables $ax + by + c = 0$ is called its solution.

(ii) A linear equation in two variables has infinitely many solutions.

Example 3 : Find four different solutions of the equation x + 2y = 6.

Sol: Given equation $x + 2y = 6$.

(i) $Let\ x = 0 \Rightarrow 0 + 2y = 6$

$\Rightarrow 2y = 6$

$\Rightarrow y = \frac{6}{2} = 3$

$Solution: (0,3)$

(ii) $Let\ x = 2 \Rightarrow 2 + 2y = 6$

$\Rightarrow 2y = 6 - 2$

$\Rightarrow 2y = 4$

$\Rightarrow y = \frac{4}{2} = 2$

$Solution: (2,2)$

(iii) $Let\ x = 4\ \Rightarrow 4 + 2y = 6$

$\Rightarrow 2y = 6 - 4$

$\Rightarrow 2y = 2$

$\Rightarrow y = \frac{2}{2} = 1$

$Solution: (4,1)$

(vi) $Let\ x = 6\ \Rightarrow 6 + 2y = 6$

$\Rightarrow 2y = 6 - 6$

$\Rightarrow 2y = 0$

$\Rightarrow y = \frac{0}{2} = 0$

$Solution: (6,0)$

Example 4 : Find two solutions for each of the following equations:

$(\boldsymbol{i})$ $\boldsymbol{4x + 3y = 12}$

$Sol: Let\ x = o \Rightarrow 4 \times 0 + 3y = 12$

$\Rightarrow 3y = 12$

$\Rightarrow y = \frac{12}{3} = 4$

$Solution: (0,4)$

$Let\ y = 0 \Rightarrow 4x + 3 \times 0 = 12$

$\Rightarrow 4x = 12$

$\Rightarrow x = \frac{12}{4} = 3$

$Solution: (3,0)$

$(\boldsymbol{ii})$ $\boldsymbol{2x + 5y = 0}$

$Sol: Let\ x = o \Rightarrow 2 \times 0 + 5y = 0$

$\Rightarrow 5y = 0$

$\Rightarrow y = 0$

$Solution: (0,0)$

$Let\ x = 1 \Rightarrow 2 \times 1 + 5y = 0$

$\Rightarrow 2 + 5y = 0$

$\Rightarrow 5y = -2$

$\Rightarrow y = \frac{-2}{5}$

$Solution: \left(1, \frac{-2}{5}\right)$

$(\boldsymbol{iii})\quad \boldsymbol{3y + 4 = 0}$

$Sol: 3y + 4 = 0$

$\Rightarrow 3y = -4$

$\Rightarrow y = \frac{-4}{3}$

$Solutions: \left(0, \frac{-4}{3}\right), \left(1, \frac{-4}{3}\right)$

EXERCISE 4.2

1. Which one of the following options is true, and why? y = 3x + 5 has

(i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions

Sol: (iii) infinitely many solutions is true

2. Write four solutions for each of the following equations:

$(\boldsymbol{i})\boldsymbol{2x + y = 7}$

$(a)\ Let\ x = 0 \ \Rightarrow 2 \times 0 + y = 7$

$\Rightarrow y = 7$

$Solution: (0,7)$

$(b)\ Let\ x = 2 \ \Rightarrow 2 \times 2 + y = 7$

$\Rightarrow 4 + y = 7$

$\Rightarrow y = 7 - 4$

$\Rightarrow y = 3$

$Solution: (2,3)$

$(c)\ Let\ x = 4 \ \Rightarrow 2 \times 4 + y = 7$

$\Rightarrow 8 + y = 7$

$\Rightarrow y = 7 - 8$

$\Rightarrow y = -1$

$Solution: (4, -1)$

$(d)\ Let\ y = 0 \ \Rightarrow 2x + 0 = 7$

$\Rightarrow 2x = 7$

$\Rightarrow x = \frac{7}{2}$

$Solution: \left(\frac{7}{2}, 0\right)$

$(\boldsymbol{ii})\boldsymbol{\pi x + y = 9}$

$(a)\ Let\ x = 0 \ \Rightarrow \pi \times 0 + y = 9$

$\Rightarrow 0 + y = 9$

$\Rightarrow y = 9$

$Solution: (0,9)$

$(b)\ Let\ x = 1 \Rightarrow \pi \times 1 + y = 9$

$\Rightarrow \pi + y = 69$

$\Rightarrow y = 9 - \pi$

$Solution: (1,9 - \pi)$

$(c)\ Let\ x = -1\ \Rightarrow \pi \times (-1) + y = 9$

$\Rightarrow -\pi + y = 69$

$\Rightarrow y = 9 + \pi$

$Solution: (-1,9 + \pi)$

$(d)\ Let\ \ y = 0\ \Rightarrow \pi x + 0 = 9$

$\Rightarrow \pi x = 9$

$\Rightarrow x = \frac{9}{\pi}$

$Solution: \left(\frac{9}{\pi}, 0\right)$

$(\boldsymbol{iii})\boldsymbol{x}\ =\ \boldsymbol{4y}$

$(a)\ Let\ x = 0\ \Rightarrow 0 - 4y = 0$

$\Rightarrow -4y = 0$

$\Rightarrow y = 0$

$Solution: (0,0)$

$(b)\ Let\ x = 4\ \Rightarrow 4 = 4y$

$\Rightarrow y = \frac{4}{4} = 1$

$Solution: (4,1)$

$(c)\ Let\ x = 2\ \Rightarrow 2 = 4y$

$\Rightarrow y = \frac{2}{4}$

$\Rightarrow y = \frac{1}{2}$

$Solution: \left(2, \frac{1}{2}\right)$

$(d)\ Let\ y = -1\ \Rightarrow x = 4 \times (-1)$

$\Rightarrow x = -4$

$Solution: (-4, -1)$

3. Check which of the following are solutions of the equation x – 2y = 4 and which are not:

$(i)(0,2)(ii)(2,0)(iii)(4,0)(iv)(\sqrt{2}, 4\sqrt{2})(v)(1,1)$

Sol: $(i)(0,2)$

$\text{LHS} = x - 2y = 0 - 2 \times 2 = 0 - 4 = -4 \neq \text{RHS}$

$\therefore$ $(0,2)$ is not a solution to the equation.

(ii) $(2,0)$

$\text{LHS} = x - 2y = 2 - 2 \times 0 = 2 - 0 = 2 \neq \text{RHS}$

$\therefore$ $(2,0)$ is not a solution to the equation.

(iii) $(4,0)$

$\text{LHS} = x - 2y = 4 - 2 \times 0 = 4 - 0 = 4 = \text{RHS}$

$\therefore$ $(4,0)$ is a solution to the equation

(iv) $(\sqrt{2}, 4\sqrt{2})$

$\text{LHS} = x - 2y = \sqrt{2} - 2 \times 4\sqrt{2} = \sqrt{2} + 8\sqrt{2} = 9\sqrt{2} \neq \text{RHS}$

$\therefore$ $(\sqrt{2}, 4\sqrt{2})$ is not a solution to the equation.

(v) $(1,1)$

$\text{LHS} = x - 2y = 1 - 2 \times 1 = 1 - 2 = -1 \neq \text{RHS}$

$\therefore$ $(1,1)$ is not a solution to the equation.

4. **Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$.**

Sol: Given equation: $2x + 3y = k$

if $x = 2, y = 1$ is a solution of the given equation then

$2 \times 2 + 3 \times 1 = k$

$4 + 3 = k$

$k = 7$

CHAPTER 5

IX-MATHEMATICS-NCERT(2023-24)

5. INTRODUCTION TO EUCLID'S GEOMETRY (NOTES)

PREPARED BY: BALABHADRA SURESH

1. The word 'geometry' comes from the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure'
2. In the Indian subcontinent, the excavations at Harappa and Mohenjo-Daro, etc. show that the Indus Valley Civilisation (about 3000 BCE) made extensive use of geometry
3. In ancient India, the **Sulbasutras** (800 BCE to 500 BCE) were the manuals of geometrical constructions
4. The **sriyantra** (given in the Atharvaveda) consists of nine interwoven isosceles triangles.
5. Euclid, a teacher of mathematics at Alexandria in Egypt, collected all the known work and arranged it in his famous treatise MATHEMATICS called 'Elements'.
6. Euclid divided the 'Elements' into thirteen chapters, each called a book
7. Euclid listing 23 definitions in Book 1 of the 'Elements'
8. Though Euclid defined a point, a line, and a plane, these definitions are not accepted by mathematicians. Therefore, these terms are now taken as undefined
9. A point is that which has no part.
10. A line is breadthless length.
11. The ends of a line are points.
12. A straight line is a line which lies evenly with the points on itself.
13. A surface is that which has length and breadth only.
14. The edges of a surface are lines.
15. A plane surface is a surface which lies evenly with the straight lines on itself.
16. A system of axioms is called consistent.
17. The statements that were proved are called propositions or theorems.
18. Euclid deduced 465 propositions.
19. Axioms or postulates are the assumptions which are obvious universal truths. They are not proved
20. Theorems are statements which are proved, using definitions, axioms, previously proved statements and deductive reasoning.
21. **Euclid's axioms**
    (1) Things which are equal to the same thing are equal to one another.
    (2) If equals are added to equals, the wholes are equal.
    (3) If equals are subtracted from equals, the remainders are equal.
    (4) Things which coincide with one another are equal to one another.
    (5) The whole is greater than the part.
    (6) Things which are double of the same things are equal to one another.
    (7) Things which are halves of the same things are equal to one another.
22. **Euclid's five postulates**
    (i) A straight line may be drawn from any one point to any other point
    (ii) A terminated line can be produced indefinitely.
    (iii) A circle can be drawn with any centre and any radius.
    (iv) All right angles are equal to one another.

(v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.

Example 1 : If A, B and C are three points on a line, and B lies between A and C (see Fig. 5.7), then prove that AB + BC = AC.

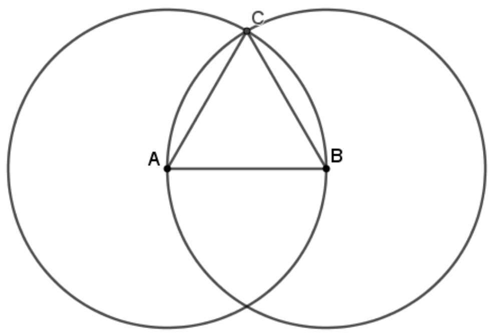
Sol: AC coincides with AB + BC

From Euclid's Axiom (4) : things which coincide with one another are equal to one another

∴AB + BC = AC

Example 2 : Prove that an equilateral triangle can be constructed on any given line segment.

Sol: 1. Using Euclid's Postulate 3, you can draw a circle with point A as the centre and AB as the radius.



2. Draw another circle with point B as the centre and BA as the radius.

3. The two circles meet at a point, say C. Now, draw the line segments AC and BC to form ΔABC.

Proof: AB = AC(radii of the same circle)→(1)

AB = BC (Radii of the same circle)→(2)

Euclid's axiom that things which are equal to the same thing are equal to one another,

From (1) and (2) : AB = BC = AC

So, Δ ABC is an equilateral triangle.

Theorem 5.1 : Two distinct lines cannot have more than one point in common.

Sol: let us suppose that the two lines l and m intersect in two distinct points, say P and Q .

Two lines passing through two distinct points P and Q

But this assumption clashes with the axiom that only one line can pass through two distinct points.

Our assumption is wrong

So, we conclude that two distinct lines cannot have more than one point in common.

EXERCISE 5.1

1. Which of the following statements are true and which are false? Give reasons for your answers.

(i) Only one line can pass through a single point.

Sol: False. We can draw infinite number of lines passing through a single point.

(ii) There are an infinite number of lines which pass through two distinct points.

Sol: False. We can draw only one line pass through two distinct points .

(iii) A terminated line can be produced indefinitely on both the sides.

Sol: True. According to Postulate 2 ,A terminated line can be produced indefinitely.

(iv) If two circles are equal, then their radii are equal.

Sol: True.

(v) In Fig. 5.9, if AB = PQ and PQ = XY, then AB = XY.

Sol: True. Things which are equal to the same thing are equal to one another.

2. **Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?**
   (i) parallel lines (ii) perpendicular lines (iii) line segment (iv) radius of a circle (v) square

Sol: First we define (a) Point (b) Ray (c) Line

(a) Point: A small dot has no dimensions.

(b) Line: A line is breadthless length.

(c) Ray: A part of a line it has one end point.

(i) Parallel lines: If two lines have no common points, they are called parallel lines.

(ii) Perpendicular lines: If the angle between two lines is equal to 90º, then these lines are perpendicular to each other.

(iii) Line segment: A terminated line is called a line segment. It has two endpoints.

(iv)Radius of a circle: The distance from the centre to any point on the circle is called the radius of the circle.

(v) Square: A square is a regular quadrilateral which means that it has four equal sides and four right angles

3. **Consider two 'postulates' given below: Are these postulates consistent? Do they follow from Euclid's postulates? Explain**
   (i) Given any two distinct points A and B, there exists a third point C which is in between A and B.
   (ii)There exist at least three points that are not on the same line. Do these postulates contain any undefined terms?.

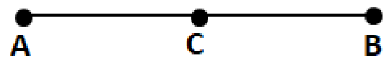
Sol: They are consistent, because they deal with two different situations –

(i) says that given two points A and B, there is a point C lying on the line in between them;

(ii) says that given A and B, you can take C not lying on the line through A and B. These 'postulates' do not follow from Euclid's postulates.

4. **If a point C lies between two points A and B such that AC = BC, then prove that AC = $\frac{1}{2}$AB. Explain by drawing the figure.**

vSol: AC = BC (Given)



AC + AC = BC + AC(Equals are added to equals)

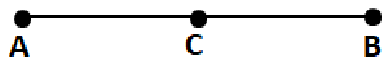
2AC = AB (BC + AC coincides with AB)

$AC = \frac{1}{2}AB$

5. In Question 4, point C is called a mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

Sol: point C is mid-point of line segment AB

AC=BC



AC+AC=BC+AC(Equals are added to equals)

2AC=AB

$AC = \frac{1}{2}AB \rightarrow (1)$

Let's assume that D is another mid-point of AB

AD=BD→(1)

AD+AD=BD+AD(Equals are added to equals)

2AD=AB

$AD = \frac{1}{2}AB \rightarrow (2)$

From (1) and (2)

AC=AD (Things which are equal to the same thing are equal to one another)

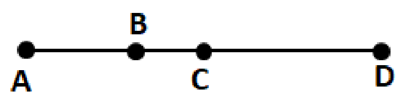
C coincide with D

Thus, a line segment has only one midpoint.

6. **In Fig. 5.10, if AC = BD, then prove that AB = CD.**

Sol: If AC=BD then

AB+BC=BC+CD (B,C are lies between AC and BD)



AB+BC-BC=BC+CD-BC (Subtracting both sides BC)

AB=CD

7. Why is Axiom 5, in the list of Euclid's axioms, considered a 'universal truth'? (Note that the question is not about the fifth postulate.)

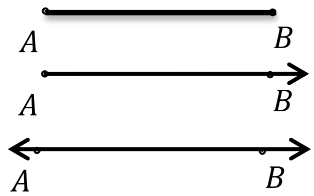
Sol: Since this is true for anything in any part of the world, this is a universal truth.

IX-MATHEMATICS-NCERT

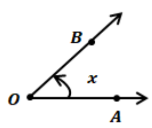
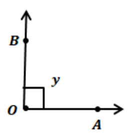
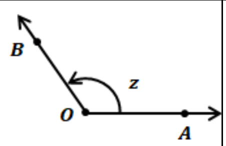
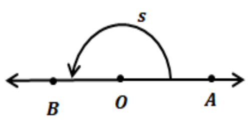
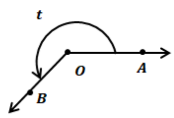
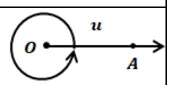
6. LINES AND ANGLES (NOTES)

PREPARED BY: BALABHADRA SURESH

1. A part (or portion) of a line with two end points is called a line-segment
2. The line segment AB is denoted by $\overline{AB}$
3. A part of a line with one end point is called a ray.
4. The ray AB is denoted by $\overrightarrow{AB}$.
5. The line AB is denoted by $\overleftrightarrow{AB}$



6. Sometimes small letters l, m, n, etc. will be used to denote lines.
7. If three or more points lie on the same line, they are called collinear points; otherwise they are called non-collinear points.
8. An angle is formed when two rays originate from the same end point. The rays making an angle are called the arms of the angle and the end point is called the vertex of the angle
9. The angle between 0^0 and 90^0 is called acute angle.
10. The angle 90^0 is called right angle
11. The angle between 90^0 and 180^0 is called obtuse angle.
12. The angle 180^0 is called straight angle.
13. The angle between 180^0 and 360^0 is called reflex angle.

Name	Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle	Complete angle
Measure	$0^0 < x < 90^0$	$y = 90^0$	$90^0 < z < 180^0$	$s = 180^0$	$180^0 < t < 360^0$	$u = 360^0$
Illustration						

14. **Complementary angles**: Two angles whose sum is 90° are called complementary angles.
15. **Supplementary angles**: Two angles whose sum is 180° are called supplementary angles.
16. **Adjacent angles**: Two angles are adjacent, if they have a common vertex, a common arm and their non-common arms are on different sides of the common arm.

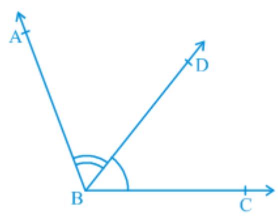


Fig. 6.2 : Adjacent angles

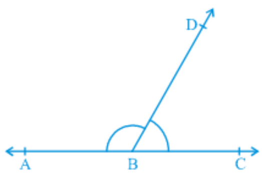


Fig. 6.3 : Linear pair of angles

17. **Linear pair of angles**: the sum of two adjacent angles is 180°, then they are called a linear pair of angles.

18. **vertically opposite angles** : If $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$ intersect at O then

$\angle AOC$ is vertically opposite to $\angle BOD$ *and*

$\angle AOD$ is vertically opposite to $\angle BOC$

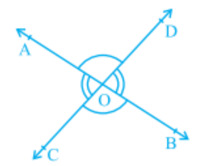
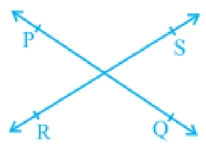
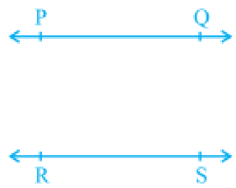


Fig. 6.4 : Vertically opposite angles

19. Intersecting Lines and Non-intersecting Lines:



(i) Intersecting lines



(ii) Non-intersecting (parallel) lines

20. Axiom 6.1 : If a ray stands on a line, then the sum of two adjacent angles so formed is 180°

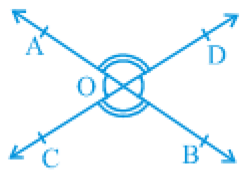
21. Axiom 6.2 : If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

Theorem 6.1 : If two lines intersect each other, then the vertically opposite angles are equal

Proof: let AB and CD be two lines intersecting at O.

Two pairs of vertically opposite angles are

(i) ∠ AOC and ∠ BOD (ii) ∠ AOD and ∠ BOC.



Now, ray OA stands on line CD

∠ AOC + ∠ AOD = 180°(Linear pair axiom)→ (1)

Now, ray OD stands on line AB

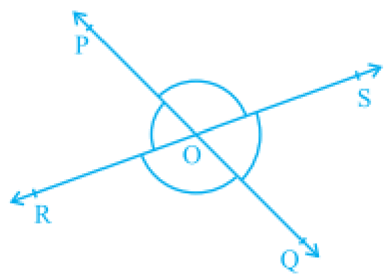
∠ AOD + ∠ BOD = 180° (Linear pair axiom))→ (2)

From (1) and (2): ∠ AOC + ∠ AOD=∠ AOD + ∠ BOD

∠ AOC=∠ BOD

Similarly, we can prove ∠AOD = ∠BOC.

Example 1 : In Fig. 6.9, lines PQ and RS intersect each other at point O. If ∠ POR : ∠ ROQ = 5 : 7, find all the angles



Sol: Given ∠ POR: ∠ ROQ = 5 : 7

$Let\ \angle POR = 5x\ \ and\ \ \angle ROQ = 7x$

∠ POR + ∠ ROQ = 180°(Linear pair of angles)

5x+7x=180°

12x=180°

$$x = \frac{180^0}{12} = 15^0$$

$$\angle POR = 5x = 5 \times 15^0 = 75^0$$

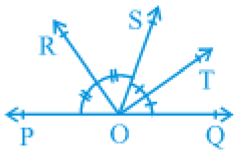
$\angle ROQ \; = 7x = 7 \times 15^0 = 105^0$

Now ∠ POS = ∠ROQ = 105° (Vertically opposite angles)

∠ SOQ = ∠POR = 75° (Vertically opposite angles)

Example 2 : In Fig. 6.10, ray OS stands on a line POQ. Ray OR and ray OT are angle bisectors of ∠ POS and ∠ SOQ, respectively. If ∠ POS = x, find ∠ ROT

Sol: Given ∠ POS = x



∠ POS + ∠ SOQ = 180° (Linear pair of angles)

x + ∠ SOQ = 180°

∠ SOQ = 180° – x

Ray OR and ray OT are angle bisectors of ∠ POS and ∠ SOQ

$\angle ROS \;=\; \frac{1}{2} \times \angle POS$ and $\angle SOT \;=\; \frac{1}{2} \times \angle SOQ$

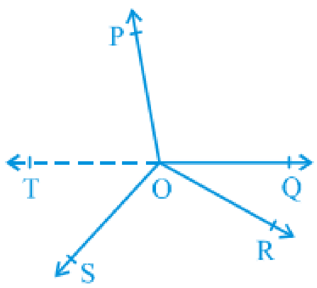
$\angle ROS \;=\; \frac{1}{2} \times x$ and $\angle SOT \;=\; \frac{1}{2} \times (180° - x)$

$\angle ROS \;=\; \frac{x}{2}$ and $\angle SOT \;=\; 90^0 - \frac{x}{2}$

$\angle ROT \;=\; \angle ROS \;+\; \angle SOT = \frac{x}{2} + 90^0 - \frac{x}{2} = 90^0$

Example 3 : In Fig. 6.11, OP, OQ, OR and OS are four rays. Prove that ∠ POQ + ∠ QOR + ∠ SOR + ∠ POS = 360°

Sol: Let us produce ray OQ backwards to a point T



TOQ is a line. So, ∠ TOP + ∠ POQ = 180° (Linear pair axiom)→ (1)

Similarly ∠ TOS + ∠ SOQ = 180° (Linear pair axiom)→ (2)

But ∠ SOQ = ∠ SOR + ∠ QOR

So, (2) becomes ∠TOS + ∠SOR + ∠QOR = 180° → (3)

(1)+(3) we get

∠TOP + ∠POQ+∠TOS + ∠SOR + ∠QOR=180°+180° =360°→ (4)

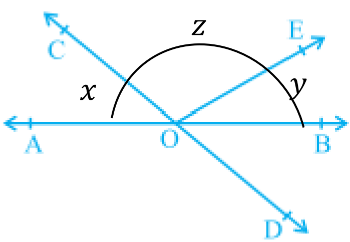
But ∠ TOP + ∠ TOS = ∠ POS

Therefore, (4) becomes ∠ POQ + ∠ QOR + ∠ SOR + ∠ POS = 360°

EXERCISE 6.1

1. **In Fig. 6.13, lines AB and CD intersect at O. If ∠ AOC + ∠ BOE = 70° and ∠ BOD = 40°, find ∠ BOE and reflex ∠ COE.**

Sol: $\angle AOC \;= \angle BOD$ (Vertically opposite angles)



$x = 40^0$

∠ AOC + ∠ BOE = 70°

$x + y = 70^0$

$40^0 + y = 70^0$

$y = 70^0 - 40^0$

$y = 30^0$

$x + y + z = 180^0$ (Linear angles)

$70^0 + z = 180^0$

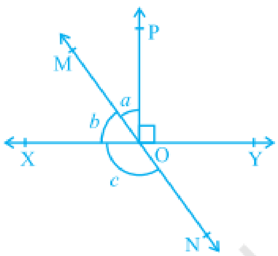
$z = 180^0 - 70^0$

$z = 110^0 \Rightarrow \angle COE = 110^0$

Reflex $\angle$ COE $= 360^0 - \angle$COE $= 360^0 - 110^0 = 250^0$

2. In Fig. 6.14, lines XY and MN intersect at O. If ∠ POY = 90° and a : b = 2 : 3, find c.

Sol: a : b = 2 : 3



$Let\ a = 2x\ and\ b = 3x$

$a + b = 90^0 \Rightarrow 2x + 3x = 90^0 \Rightarrow 5x = 90^0 \Rightarrow x = \frac{90^0}{5} = 18^0$

$a = 2x = 2 \times 18^0 = 36^0\ and\ b = 3x = 3 \times 18^0 = 54^0$

$b + c = 180^0\ (Linear\ pair)$

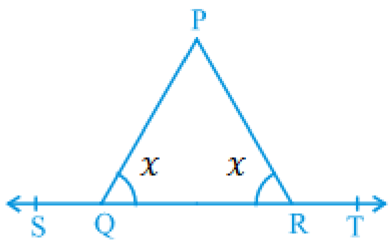
$54^0 + c = 180^0$

$c = 180^0 - 54^0$

$c = 126^0$

3. In Fig. 6.15, ∠ PQR = ∠ PRQ, then prove that ∠ PQS = ∠ PRT.

Sol: $\angle$ PQR $= \angle$ PRQ $= x$



$\angle$ PQS $+ \angle$ PQR $= 180^0$($Linear\ pair$)

$\angle$ PQS $+ x = 180^0 \rightarrow (1)$

$\angle PRT + \angle$ PRQ $= 180^0$($Linear\ pair$)

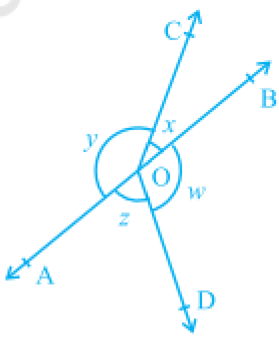
$\angle PRT + x = 180^0 \rightarrow (2)$

From (1) and (2)

$\angle$ PQS + x $= \angle PRT$ + x

$\angle$ PQS $= \angle$ PRT

4. In Fig. 6.16, if x + y = w + z, then prove that AOB is a line



Sol: $(x + y) + (z + w) = 360^0 (Complete\ angle)$

$if\ x + y = w + z\ then$

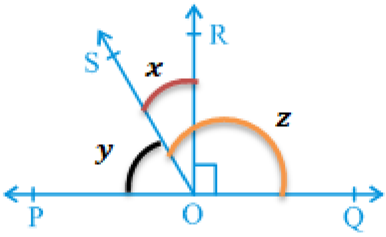
$(x + y) + (x + y) = 360^0$

$2(x + y) = 360^0$

$$(x+y)=\frac{360^0}{2}=180^0$$

$\therefore$ AOB is a line

5. In Fig. 6.17, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ ROS = $\frac{1}{2}$(∠ QOS – ∠ POS).



Sol: $\angle ROQ + \angle ROP = 180^0 (Linear\ pair)$

But $\angle ROQ = 90^0$

So, $\angle ROP = 90^0$

$x + y = 90^0$

$2(x+y) = 2\times 90^0$

$2x + 2y = 180^0 \rightarrow (1)$

$y + z = 180^0 \rightarrow (2)(Linear\ pair)$

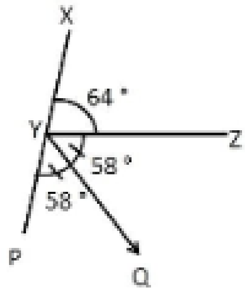
$2x + 2y = y + z$

$2x = y + z - 2y$

$2x = z - y \Rightarrow x = \frac{1}{2}(z-y)$

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

6. **It is given that ∠ XYZ = 64° and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠ ZYP, find ∠ XYQ and reflex ∠ QYP.**



Sol: $\angle XYZ + \angle ZYP = 180^0$ (linear pair)

$64^0 + \angle ZYP = 180^0$

$\angle ZYP = 180^0 - 64^0$

$\angle ZYP = 116^0$

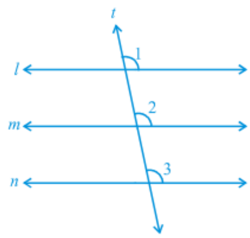
If ray YQ bisects ∠ ZYP then

$\angle ZYQ = \angle QYP = \frac{1}{2}\times\angle ZYP = \frac{1}{2}\times 116^0 = 58^0$

$\angle XYQ = \angle XYZ + \angle ZYQ = 64^0 + 58^0 = 122^0$

Reflex $\angle QYP = 360^0 - \angle QYP = 360^0 - 58^0 = 302^0$

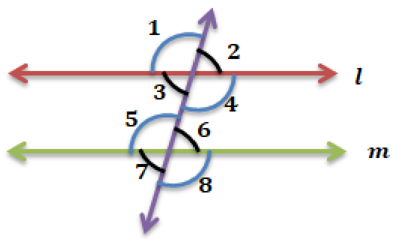
Lines Parallel to the Same Line



Lines which are parallel to the same line are parallel to each other.

If $l \parallel m$ *and* $l \parallel n$ *then* $m \parallel n$

If f two parallel lines l *and* m are cut by a transversal then



(i) Each pair of corresponding angles are equal in measure.

$\angle 1 = \angle 5;\ \ \angle 2 = \angle 6;\ \ \angle 3 = \angle 7;\ \ \angle 4 = \angle 8$

(ii) Each pair of alternate interior angles are equal.

$\angle 3 = \angle 6$; $\angle 4 = \angle 5$

(iii) Each pair of interior angles on the same side of the transversal are supplementary.

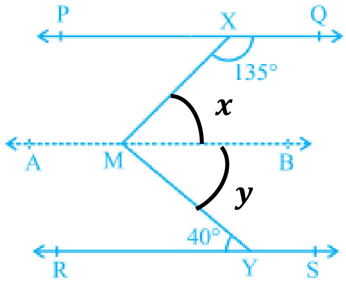
$\angle 3 + \angle 5 = 180^0$; $\angle 4 + \angle 6 = 180^0$

(iv) Each pair of exterior angles on the same side of the transversal are supplementary

$\angle 1 + \angle 7 = 180^0$; $\angle 2 + \angle 8 = 180^0$

Example 4 : In Fig. 6.19, if PQ || RS, ∠ MXQ = 135° and ∠ MYR = 40°, find ∠ XMY

Sol: Draw a line AB parallel to line PQ



$PQ \parallel AB \text{ and } XM \text{ is transversal}$

$x + 135^0 = 180^0 (Co - interior\ angles\ are\ supplementary)$

$x = 180^0 - 135^0$

$x = 45^0$

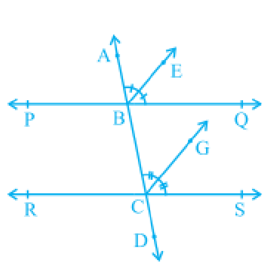
$AB \parallel RS \text{ and } MY \text{ is transversal}$

$y = 40^0 (Alternate\ interior\ angles)$

$\angle XMY = x + y = 45^0 + 40^0 = 85^0$

Example 5 : If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Sol: A transversal AD intersects PQ and RS at points B and C respectively.



$\overrightarrow{BE}$ is the bisector of ∠ ABQ and $\overrightarrow{CG}$ is the bisector of ∠ BCS; and

$\overrightarrow{BE} \parallel \overrightarrow{CG}$.

$\angle ABE = \frac{1}{2} \angle ABQ$ and $\angle BCG = \frac{1}{2} \angle BCS$

But BE || CG and AD is the transversal.

Therefore, ∠ ABE = ∠ BCG (Corresponding angles axiom)

$\frac{1}{2} \angle ABQ = \frac{1}{2} \angle BCS$

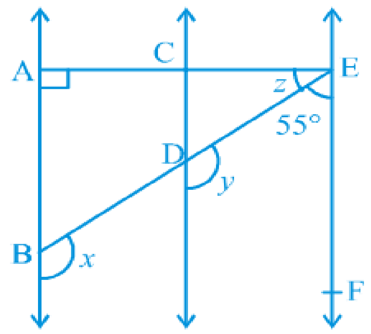
$\angle ABQ = \angle BCS$

Corresponding angles are equal. From Converse of corresponding angles axiom

PQ || RS

Example 6 : In Fig. 6.22, AB || CD and CD || EF. Also EA ⊥ AB. If ∠ BEF = 55°, find the values of x, y and z.

Sol: CD || EF and DE is transversal



y + 55° = 180° (Co-interior angles are supplementary)

y = 180º – 55º = 125º

AB || CD and BD is transversal

x = y (corresponding angles)

$x = 125^{\circ}$

AB || EF and AE is transversal.

∠ EAB + ∠ FEA = 180°(Co-interior angles are supplementary)

$90° + z + 55° = 180°$

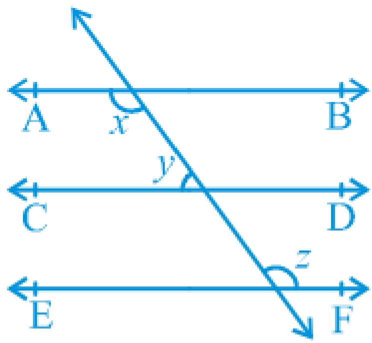
$z + 145° = 180°$

$z = 180° - 145°$

$z = 35°$

EXERCISE 6.2

1. **In Fig. 6.23, if AB || CD, CD || EF and y : z = 3 : 7, find x.**



$Sol: y : z = 3 : 7$

Let $y = 3a$ and $z = 7a$

$But\ x = z$ (Alternate interior angles)

$x = 7a$

$x + y = 180^0$(co – interior angles are supplementary)

$7a + 3a = 180^0$

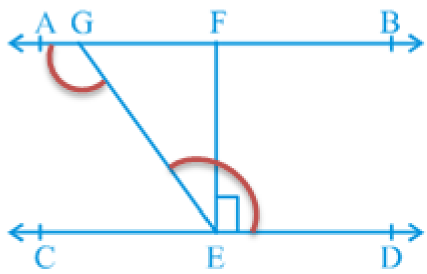
$10a = 180^0$

$$a = \frac{180^0}{10} = 18^0$$

$x = z = 7a = 7 \times 18^0 = 126^0$

$y = 3a = 3 \times 18^0 = 54^0$

2. **In Fig. 6.24, if AB || CD, EF ⊥ CD and ∠ GED = 126°, find ∠ AGE, ∠ GEF and ∠ FGE.**



Sol: Given EF ⊥ CD

$\angle FED = \angle FEC = 90^0$

AB || CD and GE transversal

$\angle AGE = \angle GED$ (*Alternate interior angles*)

But ∠ GED = 126°

$\angle AGE = 126°$

$\angle GEF = \angle GED - \angle FED = 126° - 90° = 36°$

$\angle FGE + \angle AGE = 180^0$(*Linear pair*)

$\angle FGE + 126° = 180^0$

$\angle FGE = 180^0 - 126° = 54°$

3. **In Fig. 6.25, if PQ || ST, ∠ PQR = 110° and ∠ RST = 130°, find ∠ QRS.**

Sol: PQ || ST and QR is transversal

$x + 110^0 = 180^0$(*Co interior angles are supplementary*)

$x = 180^0 - 110^0 = 70^0$

$similarly\ \ y + 130^0 = 180^0$

$y + 130^0 = 180^0$

$y = 180^0 - 130^0 = 50^0$

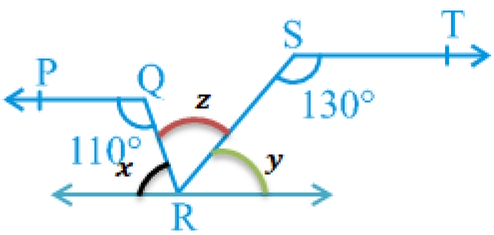
$x + y + z = 180^0 (\ Linear\ angles)$

$70^0 + 50^0 + z = 180^0$

$120^0 + z = 180^0$

$z = 180^0 - 120^0 = 60^0$

$\angle QRS = 60^0$



4. In Fig. 6.26, if AB || CD, ∠ APQ = 50° and ∠ PRD = 127°, find x and y.

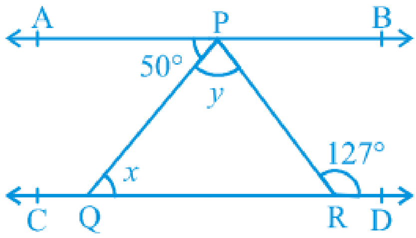
Sol: AB || CD and PQ is transversal

$x = 50^0$(Alternate interior angles)

AB || CD and PR is transversal

$y + 50^0 = 127^0$(Alternate interior angles)

$y = 127^0 - 50^0 = 77^0$



5. In Fig. 6.27, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD

Sol: Draw BL and CM perpendicular to PQ

$BL \parallel CM\ and\ BC$ is transversal

$\angle LBC = \angle MCB \rightarrow (1)$(Alternate interior angles)

But Angle of incidence = Angle of reflection

$\angle ABL = \angle LBC\ and\ \angle MCB = \angle MCD \rightarrow (2)$

From (1) and (2)

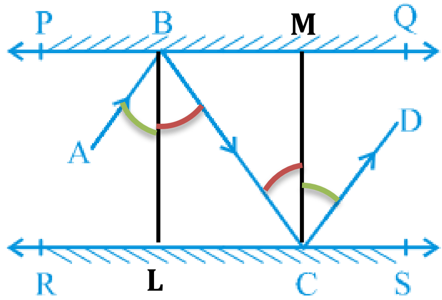
$\angle ABL = \angle MCD \rightarrow (3)$

$(1) + (3) \Rightarrow \angle LBC + \angle ABL = \angle MCB + \angle MCD$

$\angle ABC = \angle BCD$

Pair of alternate interior angles are equal

$\therefore AB \parallel CD$

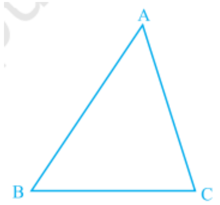


CHAPTER 7

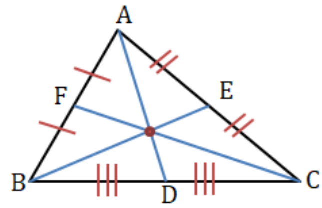
IX-MATHEMATICS-NCERT-2023-24

7. TRIANGLES (notes)

PREPARED BY: BALABHADRA SURESH



1. A closed figure formed by three intersecting lines is called a triangle.
2. A triangle has three sides, three angles and three vertices.
3. AB, BC, CA are the three sides, ∠ A, ∠ B, ∠ C are the three angles and A, B, C are three vertices
4. Triangle ABC, denoted as Δ ABC



5. **Median**:A median connects a vertex of a triangle to the mid-point of the opposite side.
6. **congruent figures**: The figures that have the same shape and size are called congruent figures

   Ex: (i) Two circles of the same radii are congruent

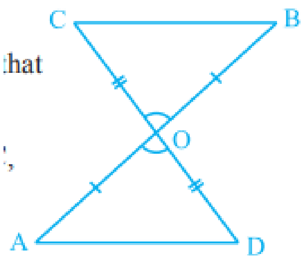
   (ii) Two squares of the same sides are congruent.
7. The two triangles are congruent If the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.
8. If Δ PQR is congruent to Δ ABC, we write Δ PQR ≅ Δ ABC.
9. FD ↔ AB, DE ↔ BC and EF ↔ CA and F ↔ A, D ↔ B and E ↔ C .So, Δ FDE ≅ Δ ABC.
10. Congruent triangles corresponding parts are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Criteria for Congruence of Triangles

(SAS congruence rule) : Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle

SAS congruence rule holds but not ASS or SSA rule.

Exp 1 : In Fig. 7.8, OA = OB and OD = OC. Show that (i) Δ AOD ≅ Δ BOC and (ii) AD || BC



Sol: (i) In Δ AOD and Δ BOC

OA = OB (Given)

OD = OC (Given)

∠ AOD = ∠ BOC (Vertically opposite angles)

Δ AOD ≅ Δ BOC (by the SAS congruence rule)

(ii) Since Δ AOD ≅ Δ BOC

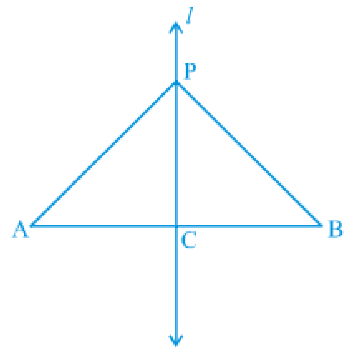
∠ OAD = ∠ OBC (CPCT)

Alternate interior angles are equal

∴ AD || BC

Example 2 : AB is a line segment and line l is its perpendicular bisector. If a point P lies on l, show that P is equidistant from A and B.

Sol: l is perpendicular bisector of AB



Δ PCA and Δ PCB.

AC = BC (C is midpoint of AB)

∠ PCA = ∠ PCB = 90° (l ⊥ AB)

PC = PC (Common)

$So, \Delta\, PCA \cong \Delta\, PCB\ (SAS\ congruence\ rule)$

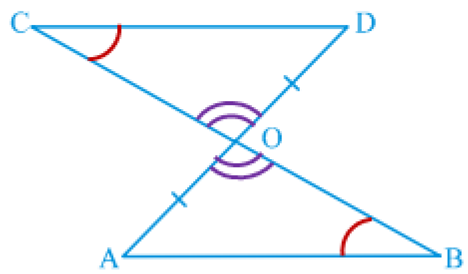
$PA = PB\ (CPCT)$

ASA congruence rule : Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

AAS and SAA are same as ASA congruence rule.

Example 3 : Line-segment AB is parallel to another line-segment CD. O is the mid-point of AD (see Fig. 7.15). Show that (i) ΔAOB ≅ ΔDOC (ii) O is also the mid-point of BC.

Sol: In Δ AOB and Δ DOC.



∠ ABO = ∠ DCO (Alternate interior angles)

OA = OD (Given)

∠ AOB = ∠ DOC (Vertically opposite angles)

∴ ΔAOB ≅ ΔDOC (ASA rule)

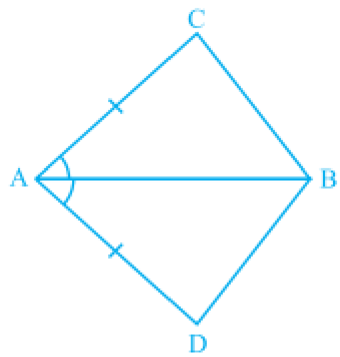
OB = OC (CPCT)

So, O is the mid-point of BC.

EXERCISE 7.1

1. **In quadrilateral ACBD, AC = AD and AB bisects ∠ A (see Fig. 7.16). Show that Δ ABC ≅ Δ ABD. What can you say about BC and BD?**

Sol: In ΔABC and ΔABD



AC = AD (Given)

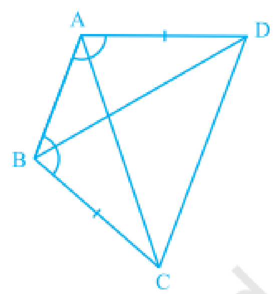
∠ BAC = ∠ BAD (AB bisects ∠ A)

AB=AB (Common)

Δ ABC ≅ Δ ABD (SAS congruency rule)

BC=BD(CPCT)

2. **ABCD is a quadrilateral in which AD = BC and ∠ DAB = ∠ CBA (see Fig. 7.17). Prove that**
   (i) Δ ABD ≅ Δ BAC (ii) BD = AC (iii) ∠ ABD = ∠ BAC.



Sol: (i) In Δ ABD and Δ BAC

AD = BC (Given)

$\angle$ DAB = $\angle$ CBA (Given)

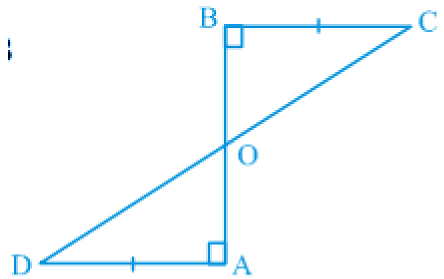
AB=AB (Common)

Δ ABD $\cong$ Δ BAC (SAS congruence rule)

(ii) Δ ABD $\cong$ Δ BAC $\Rightarrow$ BD = AC (CPCT)

(iii) Δ ABD $\cong$ Δ BAC $\Rightarrow$ $\angle$ ABD = $\angle$ BAC (CPCT)

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.



Sol: In ΔOAD, ΔOBC

$\angle OAD = \angle OBC = 90^0$ (*Given*)

$\angle AOD = \angle BOC$ (*Vertically opposite angles*)

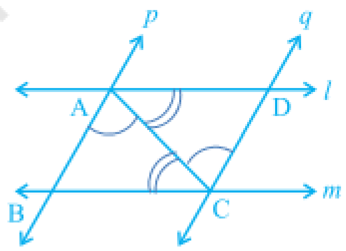
AD=BC (Given)

ΔOAD$\cong$$\Delta$OBC (AAS congruence rule)

OA=OB

$\therefore$ CD bisects AB.

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that Δ ABC $\cong$ Δ CDA.



Sol: In ΔABC and ΔCDA

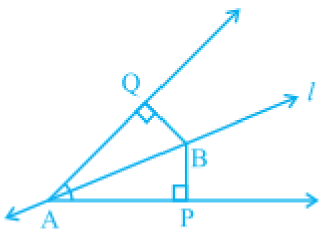
$\angle BAC = \angle DCA$ ($p||q$, *Alternate interior angles*)

AC=AC(Common)

$\angle BCA = \angle DAC$ ($l||m$, *Alternate interior angles*)

ΔABC $\cong$$\Delta$CDA (By ASA congruence rule)

5. Line *l* is the bisector of an angle $\angle$ A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of $\angle$ A (see Fig. 7.20). Show that: (i) Δ APB $\cong$ Δ AQB (ii) BP = BQ or B is equidistant from the arms of $\angle$ A.



Sol: (i) In ΔAPB and ΔAQB

$\angle BAP = \angle BAQ$ (*l is the angle bisector of* $\angle A$)

AB=AB(Common)

$\angle APB = \angle AQB = 90^0$

Δ APB $\cong$$\Delta$ AQB (By ASA congruence rule)

(ii) Δ APB $\cong$$\Delta$ AQB

BP=BQ (By CPCT)

B is equidistant from the arms of $\angle$ A

6. In Fig. 7.21, AC = AE, AB = AD and $\angle$ BAD = $\angle$ EAC. Show that BC = DE.

Sol: $\angle$ BAD = $\angle$ EAC (Given)

$\angle$BAD + $\angle$DAC = $\angle$EAC +$\angle$$\angle$DAC

∠ BAC = ∠ DAE→(1)

In ΔBAC and ΔDAE

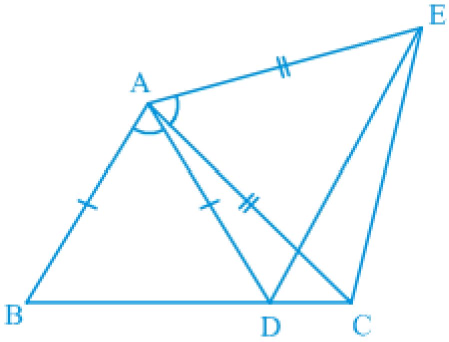
AB = AD (Given)

∠ BAC = ∠ DAE (From (1))

AC = AE (Given)

$\Delta BAC \cong \Delta DAE$ (*By SAS congruence rule*)

∴ BC = DE (By CPCT)



7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠ BAD = ∠ ABE and ∠ EPA = ∠ DPB (see Fig. 7.22). Show that (i) Δ DAP ≅ Δ EBP (ii) AD = BE

Sol: ∠ EPA = ∠ DPB (given)

∠ EPA+∠DPE = ∠ DPB+∠DPE

∴∠APD=∠BPE →(1)

In ΔAPD and ΔBPE

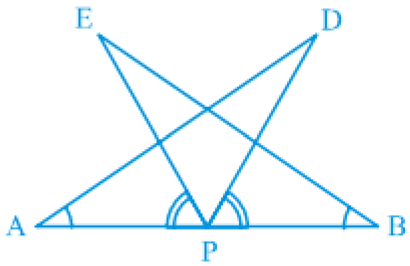
∠ BAD = ∠ ABE (Given)

AP=BP (P is midpoint of AB)

∠APD=∠BPE (From (1))

Δ APD ≅ ΔBPE (*By ASA congruence rule*)

∴ AD = BE (By CPCT)



8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that: (i) Δ AMC ≅ Δ BMD (ii) ∠ DBC is a right angle. (*iii*) $\Delta\, DBC \cong \Delta\, ACB$ (*iv*) $CM = \frac{1}{2}AB$

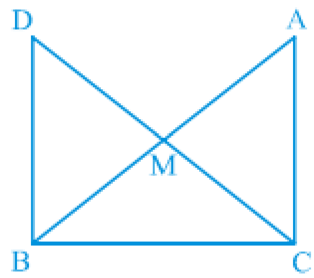
Sol: (i) In Δ AMC and Δ BMD

AM=BM (M is midpoint of AB)

∠ AMC=∠ BMD (Vertically opposite angles)

DM = CM (Given)

∴Δ AMC ≅ Δ BMD (By SAS congruence rule)



(ii) Δ AMC ≅ Δ BMD

∠ACM=∠BDM (By CPCT)

Alternate interior angles are equal

∴ DB || AC

∠DBC +∠ACB=180^0 (co-interior angles are supplementary)

∠DBC+90^0 =180^0(Given ∠ACB=90^0)

∴∠DBC=90^0

(iii) *In Δ DBC and Δ ACB*

DB=AC (Δ AMC ≅ Δ BMD)

$\angle DBC = \angle ACB = 90^0$

BC=CB(common)

$\Delta\, DBC \;\cong\; \Delta\, ACB\ (By\ SAS\ congruence\ rule)$

(iv) $\Delta\, DBC \;\cong\; \Delta\, ACB$

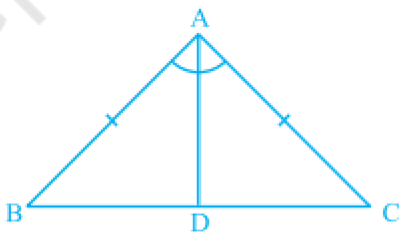
AB=DC (by CPCT)

AB=2 CM (CM=DM)

$$CM = \frac{1}{2}AB$$

Some Properties of a Triangle

Theorem 7.2 : Angles opposite to equal sides of an isosceles triangle are equal.



Sol: ΔABC is an isosceles triangle in which AB=AC

Draw AD is angle bisector of ∠A

In Δ BAD and Δ CAD

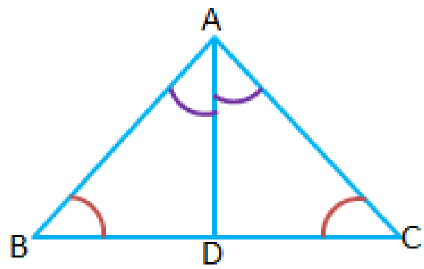
AB = AC (Given)

∠ BAD = ∠ CAD (By construction)

AD = AD (Common)

So, Δ BAD ≅ Δ CAD (By SAS rule)

∠B = ∠C (CPCT)



Theorem 7.3 : The sides opposite to equal angles of a triangle are equal.

Proof: In ΔABC, ∠B=∠C

Draw AD is angle bisector of ∠A

In Δ BAD and Δ CAD

∠B=∠C (given)

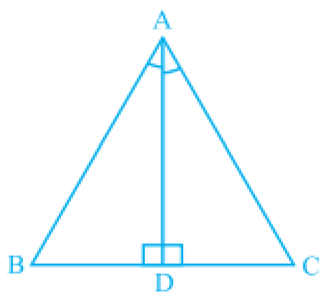
∠ BAD = ∠ CAD (By construction)

AD = AD (Common)

So, Δ BAD ≅ Δ CAD (By AAS congruence rule)

AB = AC (by CPCT)

Example 4 : In Δ ABC, the bisector AD of ∠ A is perpendicular to side BC (see Fig. 7.27). Show that AB = AC and Δ ABC is isosceles.



Sol: In ΔABD and ΔACD,

∠ BAD = ∠ CAD (Given)

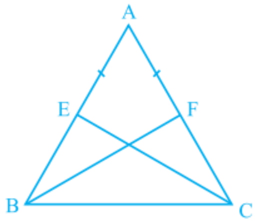
AD = AD (Common)

∠ ADB = ∠ ADC = 90° (Given)

So, Δ ABD ≅ Δ ACD (ASA rule)

So, AB = AC (CPCT) or, Δ ABC is an isosceles triangle.

Example 5 : E and F are respectively the mid-points of equal sides AB and AC of Δ ABC (see Fig. 7.28). Show that BF = CE.



Sol: In Δ ABF and Δ ACE,

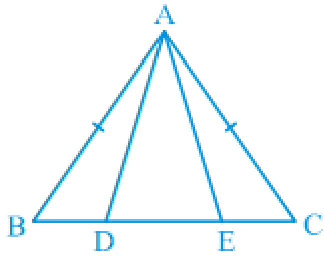
AB = AC (Given)

∠ A = ∠ A (Common)

AF = AE (Halves of equal sides)

So, Δ ABF ≅ Δ ACE (SAS rule) Therefore, BF = CE (CPCT)

Example 6 : In an isosceles triangle ABC with AB = AC, D and E are points on BC such that BE = CD (see Fig. 7.29). Show that AD = AE.



Sol: In Δ ABE and Δ ACD,

AB = AC (Given)

∠ B = ∠ C (Angles opposite to equal sides)

BE = CD (Given)

So, Δ ABE ≅ Δ ACD (SAS congruence rule)

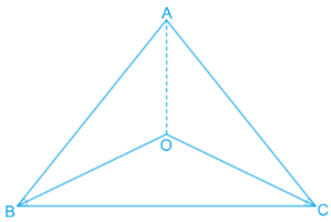
⇒AE = AD (CPCT)

EXERCISE 7.2

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠ B and ∠ C intersect each other at O. Join A to O. Show that : (i) OB = OC (ii) AO bisects ∠A

Sol: (i) In ΔABC, The bisectors of ∠ B and ∠ C intersect each other at O

$\angle OBA = \angle OBC = \frac{1}{2}\angle ABC$ and $\angle OCA = \angle OCB = \frac{1}{2}\angle ACB$



Given AB = AC

$\Rightarrow \angle ABC = \angle ACB$ (Angles opposite to equal sides)

$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB \Rightarrow \angle OBC = \angle OCB$

$In\ \Delta OBC, \angle OBC = \angle OCB$

$\Rightarrow OB = OC$ (Sides opposite to equal angles) $\rightarrow (i)$

$(ii) \Delta OAB$ and ΔOAC

OA=OA(Common)

AB=AC (Given)

OB = OC $(From\ (i))$

ΔOAB ≅ ΔOAC (SSS $congruence$ $rule$)

⇒ ∠OAB = ∠OAC ($CPCT$)

$\Rightarrow$ AO bisects $\angle A$

2. In Δ ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that Δ ABC is an isosceles triangle in which AB = AC.

Sol: In ΔADB and ΔADC

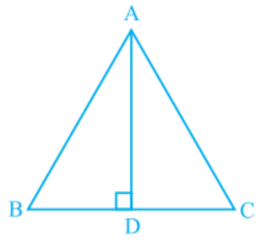
$AD = AD$ (Common)

$\angle ADB = \angle ADC = 90^0 (AD \perp BC)$

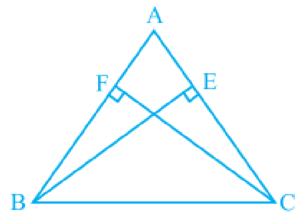
$BD = DC$(AD is bisector of BC)

$\Delta ADB \cong \Delta ADC$ (SAS congruence rule)

$\Rightarrow AB = AC$ (CPCT)



3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.



Sol: Δ ABC is an isosceles triangle. AB=AC

In ΔAEB and ΔAFC

$\angle A = \angle A$ (Common angle)

$\angle AEB = \angle AFC = 90^0$ ($BE \perp AC$ and $CF \perp AB$)

$AB = AC$(Given)

$\Delta AEB \cong \Delta AFC$ (AAS congruence rule)

$\Rightarrow BE = CF$ (CPCT)

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) Δ ABE ≅ Δ ACF (ii) AB = AC, i.e., ABC is an isosceles triangle.

Sol: In Δ ABE and Δ ACF

$\angle A = \angle A$ (Common angle)

$\angle AEB = \angle AFC = 90^0$ ($BE \perp AC$ and $CF \perp AB$)

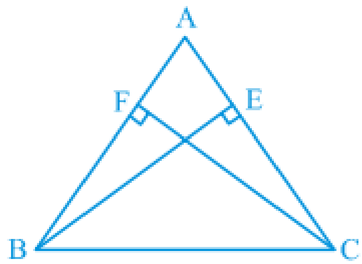
$BE = CF$(Given)

$\Delta ABE \cong \Delta ACF$ (AAS Congruence rule)

(ii) $\Delta ABE \cong \Delta ACF$

$\Rightarrow AB = AC$ (CPCT)

ΔABC is an isosceles triangle.



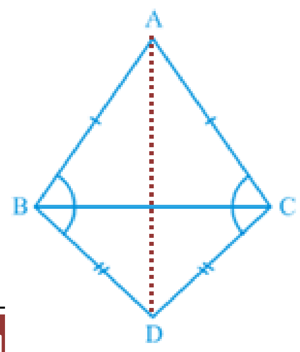
5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ ABD = ∠ACD

Sol: In Δ ABD and Δ ACD

$AB = AC$ (Given)

$BD = CD$(Given)

$AD = AD$ (Common side)



$\Delta ABD \cong \Delta ACD$ (SSS Congruence rule)

$\Rightarrow \angle ABD = \angle ACD$(CPCT)

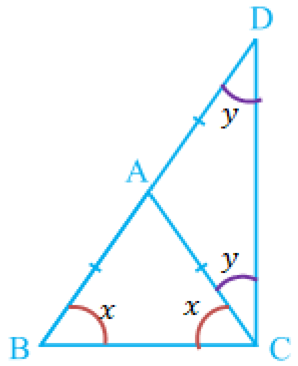
6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠ BCD is a right angle.

Sol: $In\ \Delta ABC, AB = AC$

$\Rightarrow \angle ABC = \angle ACB = x$ (Angles opposite to equal sides are equal)

In $\Delta ADC, AD = AC$

$\Rightarrow \angle ACD = \angle ADC = y$ (Angles opposite to equal sides are equal)



$\angle BCD = \angle ACB + \angle ACD = x + y$

In ΔBDC,

$\angle ABC + \angle ADC + \angle BCD = 180^0$ (Angle sum property of a triangle)

$x + y + (x + y) = 180^0$

$2(x + y) = 180^0$

$(x + y) = \frac{180^0}{2} = 90^0$

$\angle BCD = 90^0$

7. ABC is a right angled triangle in which ∠ A = 90° and AB = AC. Find ∠ B and ∠ C.

Sol: $In\ \Delta ABC$, AB = AC

$\Rightarrow \angle C = \angle B = x$ (*Equal sides opposite angles are equal*)

$\angle A + \angle B + \angle C = 180^0$ (*Angle sum property of a triangle*)

$90^0 + x + x = 180^0$

$2x = 90^0$

$x = \frac{90^0}{2} = 45^0$

$\angle B = \angle C = 45^0$

8. Show that the angles of an equilateral triangle are 60° each.

Sol: *Let* ΔABC *is an equlateral triangle*

$\Rightarrow AB = BC = AC$

$\Rightarrow \angle A = \angle B = \angle C = x$ (*Equal sides opposite angles are equal*)

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow x + x + x = 180^0$

$\Rightarrow 3x = 180^0$

$\Rightarrow x = \frac{180^0}{3} = 60^0$

So, each angle of an equilateral triangle is 60^0

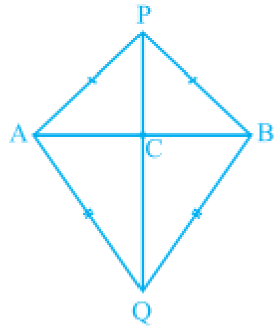
SSS congruence rule:

If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent

RHS congruence rule:

If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Example 7 : AB is a line-segment. P and Q are points on opposite sides of AB such that each of them is equidistant from the points A and B (see Fig. 7.37). Show that the line PQ is the perpendicular bisector of AB.



Sol: In Δ PAQ and Δ PBQ.

PA=PB(Given)

AQ=BQ(Given)

PQ=PQ(Common)

Δ PAQ ≅ Δ PBQ (SSS rule)

$\angle APQ = \angle BPQ\ (CPCT) \Rightarrow \angle APC = \angle BPC \rightarrow (1)$

In Δ PAC and Δ PBC.

AP = BP (Given)

∠ APC = ∠ BPC (From (1))

PC = PC (Common)

Δ PAC ≅ Δ PBC (SAS rule)

$AC = BC\ (CPCT) \rightarrow (2)$

∠ ACP = ∠ BCP (CPCT)

∠ ACP + ∠ BCP = 180° (Linear pair)

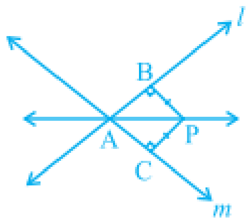
2∠ ACP = 180°

$\angle ACP = 90° \rightarrow (3)$

From (2) and (3)

PQ is perpendicular bisector of AB

Example 8 : P is a point equidistant from two lines l and m intersecting at point A (see Fig. 7.38). Show that the line AP bisects the angle between them.



Sol: Let PB ⊥ l, PC ⊥ m. It is given that PB = PC

In Δ PAB and Δ PAC

∠ PBA = ∠ PCA = 90° (Given)

PA = PA (Common)

PB = PC (Given)

Δ PAB ≅ Δ PAC (RHS rule)

∠ PAB = ∠ PAC (CPCT)

AP bisects the angle between l and m

EXERCISE 7.3

1. Δ ABC and Δ DBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that (i) Δ ABD ≅ Δ ACD (ii) Δ ABP≅ Δ ACP (iii) AP bisects ∠ A as well as ∠ D. (iv) AP is the perpendicular bisector of BC.

Sol: $(i) In\ \Delta ABD\ and\ \Delta ACD$

$AB = AC\ (Given)$

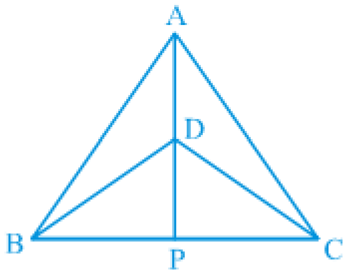
BD = CD(Given)

AD = AD (Common)

ΔABD ≅ ΔACD (SSS rule)

∠BAD = ∠CAD (CPCT)

i.e ∠BAP = ∠CAP → (1)



(ii) Δ ABP≅ Δ ACP

AB = AC (Given)

∠BAP = ∠CAP (From (1))

AP = AP (Common)

Δ ABP ≅ Δ ACP (SAS rule)

∠APB = ∠APC(CPCT) → (2)

i.e ∠DPB = ∠DPC → (3)

Also BP = PC (CPCT) → (4)

(iii) From (1) and (2)

AP bisects ∠ A as well as ∠ D

(iv) $\angle APB + \angle APC = 180^0$(Linear pair)

From (2) ;∠APB = ∠APC

$\therefore \angle APB = \angle APC = 90^0 \rightarrow (5)$

(iv) From (4) and (5)

AP is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that (i) AD bisects BC (ii) AD bisects ∠ A.

Sol: In ΔADB and ΔADC

AB = AC(Given)

$\angle ADB = \angle ADC = 90^0$ (AD ⊥ BC)

AD = AD(Common)

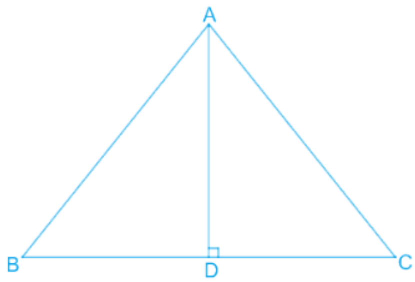
ΔADB ≅ ΔADC (RHS Congruence rule)

BD=CD (CPCT)

Hence , AD bisects BC

∠BAD = ∠CAD (CPCT)

AD bisects ∠A

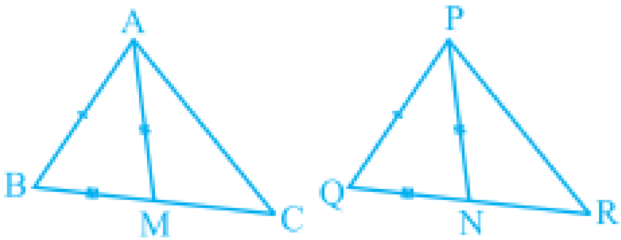


3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig. 7.40). Show that (i) Δ ABM ≅ Δ PQN (ii) Δ ABC ≅ Δ PQR

Sol: In ΔABC , AM is median

$BM = CM = \frac{1}{2}BC$

In ΔPQR , PN is median



$QN = NR = \frac{1}{2}QR$

Given BC=QR

$\frac{1}{2}BC = \frac{1}{2}QR$

$BM = QN \rightarrow (1)$

(i) Δ ABM ≅ Δ PQN

$AB = PQ(Given)$

$AM = PN(Given)$

$BM = QN\ \ (From(1))$

$\Delta\, ABM\ \cong\ \Delta\, PQN\ (SSS\ rule)$

$BM = QN \rightarrow (1)$

$\angle ABM = \angle \text{PQN (CPCT)}$

$\angle ABC = \angle \text{PQR} \rightarrow (2)$

(ii) In Δ ABC and Δ PQR

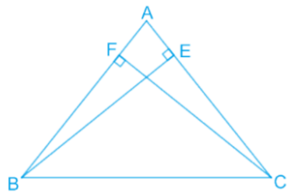
$AB = PQ(Given)$

$\angle ABC = \angle \text{PQR(From(2))}$

$BC = QR\ (Given)$

$\Delta\, ABC\ \cong\ \Delta\, PQR\ (ASA\ rule)$

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Sol: *In* Δ BEC and Δ CFB

∠ BEC = ∠ CFB = 90^0 (BE and CF are two altitudes)

$BC = BC\ (Common)$

$BE = CF\ (Given)$

Δ BEC ≅ Δ CFB (RHS rule)

∠BCE = ∠CBF (CPCT)

i.e, ∠BCA = ∠CBA

AB=AC (Sides opposite to equal angles are equal)

Hence, ΔABC is isosceles triangle.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠ B = ∠ C.

Sol: *In* ΔAPB *and* ΔAPC

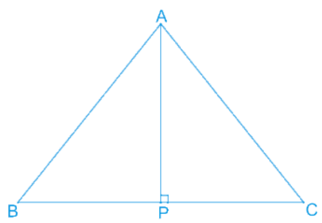
∠APB = ∠APC = 90^0

AB = AC

AP = AP (Common)

ΔAPB ≅ ΔAPC (RHS rule)

∠B = ∠C (CPCT)



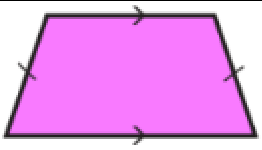
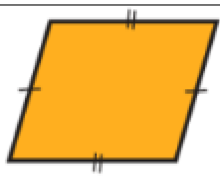
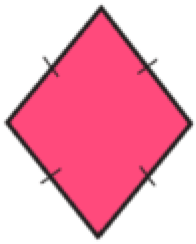
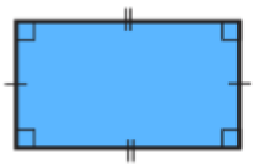
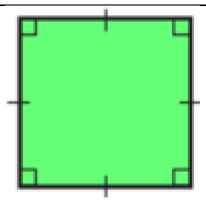
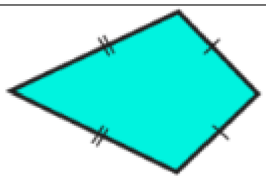
CHAPTER 8

IX-MATHEMATICS-NCERT-2023-24

8. QUADRILATERALS (NOTES)

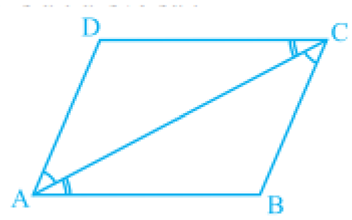
PREPARED BY: BALABHADRA SURESH

1. A quadrilateral has four sides, four angles and four vertices.

Quadrilateral	Figure	Properties
Trapezium A quadrilateral with a pair of parallel sides.		1. One pair of parallel lines
Parallelogram: A quadrilateral with each pair of opposite sides parallel		1. Opposite sides are equal. 2. Opposite angles are equal. 3. Diagonals not equal and bisect one another. 4. Adjacent angles are supplementary
Rhombus: A parallelogram with sides of equal length.		1. All sides are equal. 2. Opposite angles are equal 3. Diagonals are not equal and perpendicularly bisect one another. 4. Adjacent angles are supplementary
Rectangle: A parallelogram with a right angle		1. Opposite sides are equal 2. All angles are right angles. 3. Diagonals are equal and bisect one another.
Square: A rectangle with sides of equal length.		1. All sides are equal. 2. All angles are right angles. 3. Diagonals are equal and perpendicularly bisect one another.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides		1. The diagonals are perpendicular to one another. 2. Diagonals bisect each other.

Theorem 8.1 : A diagonal of a parallelogram divides it into two congruent triangles.

Proof : Let ABCD be a parallelogram and AC be a diagonal



In Δ ABC and Δ CDA,

BC || AD and AC is a transversal.

$\angle$ BCA = $\angle$ DAC (Pair of alternate angles)

AB || DC and AC is a transversal.

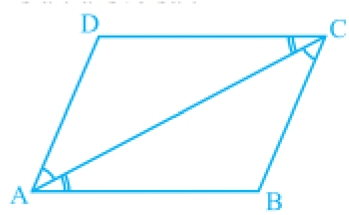
$\angle$ BAC = $\angle$ DCA (Pair of alternate angles)

AC = CA (Common)

Δ ABC $\cong$ Δ CDA (ASA rule)

Diagonal AC divides parallelogram ABCD into two congruent triangles ABC and CDA.

Theorem 8.2 : In a parallelogram, opposite sides are equal.

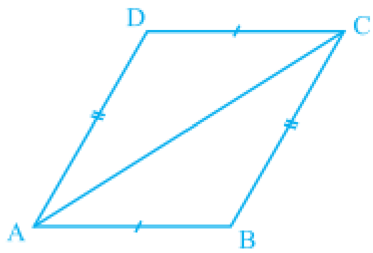


Proof: Let ABCD be a parallelogram and AC be a diagonal.

Δ ABC $\cong$ Δ CDA (ASA rule)

So, AB = DC and BC = AD (CPCT)

Theorem 8.3 : If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.



Proof: ABCD be a quadrilateral and AB = DC and BC = AD

In Δ ABC and Δ CDA

AB = DC (given)

BC = AD(given)

AC=AC (common)

Δ ABC $\cong$ Δ CDA (SSS congruence rule)

$\angle$ BAC = $\angle$ DCA(CPCT)

Alternate interior angles are equal$\Rightarrow$ AB || CD

Similarly BC || DA

Each pair of opposite sides are parallel .

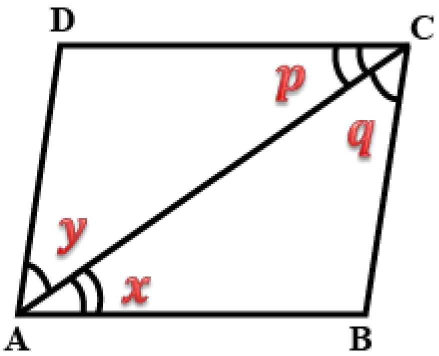
ABCD is a parallelogram.

Theorem 8.4 : In a parallelogram, opposite angles are equal.

Proof: ABCD is a parallelogram.

$AB \parallel CD$ *and* AC *is transversal*

$x = p($ *Alternate interior angles*$)$



$BC \parallel AD$ *and* AC *is transversal*

$y = q$ (*Alternate interior angles*)

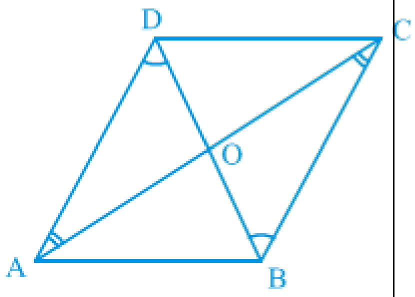
$x + y = p + q$

$\angle BAD = \angle BCD \Rightarrow \angle A = \angle C$

Similarly $\angle B = \angle D$

Theorem 8.5 : If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

Proof: In quadrilateral ABCD , $\angle A = \angle C$ *and* $\angle B = \angle D$



$\angle A + \angle B + \angle C + \angle D = 360^0$ (*Sum of angles in quadrilateral*)

$\angle A + \angle D + \angle A + \angle D = 360^0$

$2(\angle A + \angle D) = 360^0$

$\angle A + \angle D = 180^0$

Co interior angles are supplementary

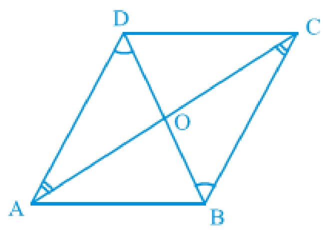
$\Rightarrow AB \parallel DC$

Similarly $BC \parallel AD$

$\therefore$ ABCD is a parallelogram.

Theorem 8.6 : The diagonals of a parallelogram bisect each other.

Proof: In parallelogram ABCD diagonals AC,BD intersect at O



$\Delta\, AOD = \Delta\, COB$ (*ASA rule*)

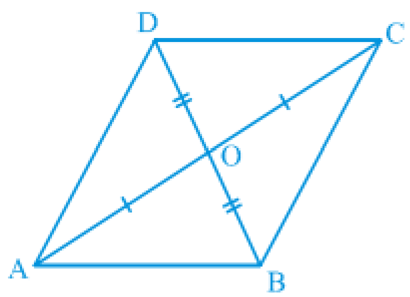
$AO = CO$ *and* $OD = OB$ (*CPCT*)

$\Rightarrow O$ *is mid point of* AC *and* BD

$\Rightarrow AC$ *and* BD *are bisect each other*

Theorem 8.7 : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram

Proof: ABCD is a quadrilateral. The diagonals AC and BD bisect at O



In Δ AOB , Δ COD

OA = OC and OB = OD (given)

∠AOB=∠COD (vertically opposite angles)

Δ AOB $\cong \Delta$ COD (SAS congruence rule)

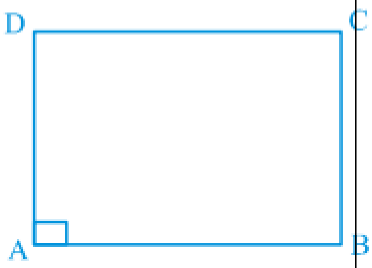
$\therefore \angle$ ABO = $\angle$ CDO (By CPCT)

Alternate interior angles are equal

$\therefore$ AB || CD

Similarly BC || AD

Therefore ABCD is a parallelogram.



Example 1 : Show that each angle of a rectangle is a right angle.

Sol: Rectangle is a parallelogram in which one angle is a right angle.

ABCD is a rectangle. Let one angle is $\angle A = 90^0$

We have, AD || BC and AB is a transversal.

$\angle A + \angle B = 180°$ (Interior angles on the same side of the transversal)

$90^0 + \angle B = 180°$

$\angle B = 180° - 90^0 = 90^0$

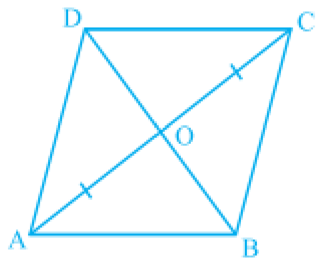
$\angle C = \angle A$ and $\angle D = \angle B$(Opposite angles of the parallelogram)

$\angle C = 90^0$ and $\angle D = 90^0$

Therefore, each of the angles of a rectangle is a right angle.

Example 2 : Show that the diagonals of a rhombus are perpendicular to each other.

Sol: Let ABCD is a rhombus.



AB = BC = CD = DA (All sides are equal in rhombus)

In Δ AOD and Δ COD

OA = OC (Diagonals of a parallelogram bisect each other)

OD = OD (Common)

AD = CD (given)

Δ AOD $\cong \Delta$ COD (SSS congruence rule)

$\angle AOD = \angle COD$ (CPCT)

But, $\angle AOD + \angle COD = 180°$ (Linear pair)

$2\angle AOD = 180°$

$\angle AOD = 90°$

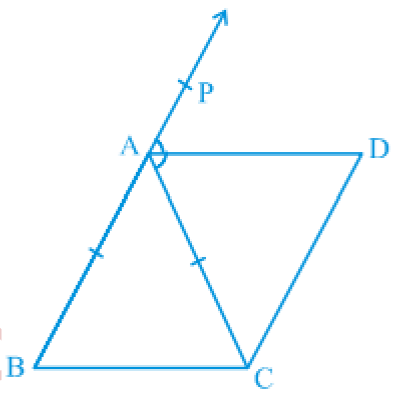
So, the diagonals of a rhombus are perpendicular to each other.

Example 3 : ABC is an isosceles triangle in which AB = AC. AD bisects exterior angle PAC and CD || AB (see Fig. 8.8). Show that (i) ∠ DAC = ∠ BCA and (ii) ABCD is a parallelogram.

Sol: (i) Δ ABC is isosceles in which AB = AC (Given)

So, ∠ ABC = ∠ ACB (Angles opposite to equal sides)

Also, ∠ PAC = ∠ ABC + ∠ ACB (Exterior angle of a triangle)



or, ∠ PAC = 2∠ ACB →(1)

Now, AD bisects ∠ PAC.

So, ∠ PAC = 2∠ DAC →(2)

2∠ DAC = 2∠ ACB [From (1) and (2)]

∠ DAC = ∠ BCA

(ii) ∠ DAC = ∠ ACB i.e alternate interior angles are equal.

⇒ BC ||AD

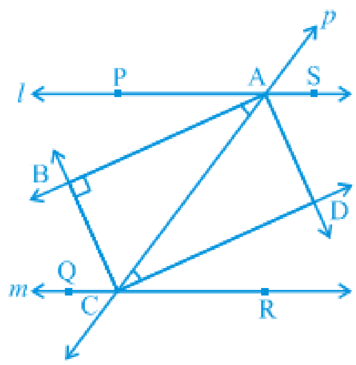
Also, BA || CD (*Given*)

Now, both pairs of opposite sides of quadrilateral ABCD are parallel.

So, ABCD is a parallelogram.

Example 4 : Two parallel lines l and m are intersected by a transversal p (see Fig. 8.9). Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Sol: PS || QR and transversal p intersects them at points A and C .



The bisectors of∠PAC and ∠ACQ intersect at B and bisectors of ∠ ACR and ∠SAC intersect at D.

Now, ∠ PAC = ∠ ACR (Alternate angles as l || m and p is a transversal)

$$\frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR$$

i.e., ∠ BAC = ∠ ACD ⇒ alternate interior angles are equal.

$AB \,||\, DC$

Similarly, BC || AD (Considering ∠ ACB and ∠ CAD)

Therefore, quadrilateral ABCD is a parallelogram.

∠ PAC + ∠ CAS = 180° (Linear pair)

$\frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = \frac{1}{2} \times 180° = 90°$

$\angle BAC + \angle CAD = 90°$

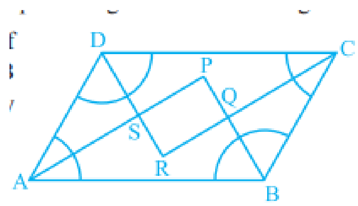
$\angle BAD = 90°$

So, ABCD is a parallelogram in which one angle is 90°.

Therefore, ABCD is a rectangle.

Example 5 : Show that the bisectors of angles of a parallelogram form a rectangle.

Sol : Let P, Q, R and S be the points of intersection of the bisectors of



$\angle A$ and $\angle B$, $\angle B$ and $\angle C$, $\angle C$ and $\angle D$, and $\angle D$ and $\angle A$ of parallelogram ABCD In Δ ASD , DS bisects $\angle D$ and AS bisects $\angle A$

$\therefore \angle DAS + \angle ADS = \frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{1}{2}(\angle A + \angle D)$

$= \frac{1}{2} \times 180^0$ ($\angle A$ and $\angle D$ are adjacent angles of parallelogram ABCD)

$= 90^0$

Also, $\angle DAS + \angle ADS + \angle DSA = 180°$ (Angle sum property of a triangle)

$90° + \angle DSA = 180°$

$\angle DSA = 90°$

$\angle PSR = 90°$ (Being vertically opposite to $\angle DSA$)

Similarly $\angle SPQ = 90°$,$\angle PQR = 90°$ and $\angle SRQ = 90°$

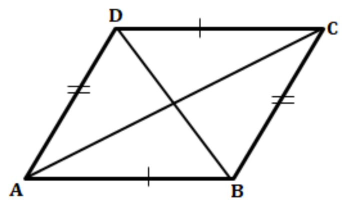
So, PQRS is a quadrilateral in which all angles are right angles.

So, PQRS is a rectangle.

EXERCISE 8.1

1. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Sol: ABCD is a parallelogram and diagonals AC=BD



$\Delta ABC \cong \Delta DCB$ (*SSS congruence rule*)

$\angle ABC = \angle DCB$ (*CPCT*)

$\angle ABC + \angle DCB = 180^0$ (*co − interior angles are supplementary*)

$\angle ABC + \angle ABC = 180^0$

$2\angle ABC = 180^0$

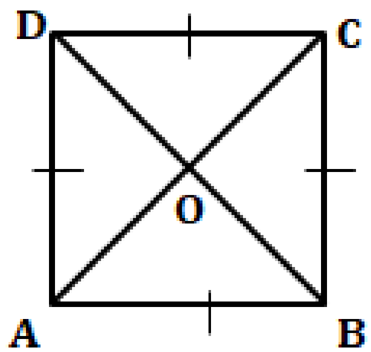
$\angle ABC = 90^0$

ABCD is a parallelogram and one of the angles is 90^0

ABCD is a rectangle.

2. How that the diagonals of a square are equal and bisect each other at right angles.

Sol: Let ABCD is square.



$\Delta\, ABC \cong \Delta\, DCB(\, SAS\ rule)$

$AC = BD\ (By\ CPCT) \Rightarrow Diagonals\ are\ equal$

$\Delta\, AOB \cong \Delta COD\ (ASA\ congruence\ rule)$

$\therefore\ AO = CO\ and\ OB = OD\ (by\ CPCT)$

$\Rightarrow Diogonals\ are\ bisect\ each\ other$

$\Delta\, AOB \cong \Delta COB\ (SSS\ congruence\ rule)$

$\angle AOB = \angle COB\ (by\ CPCT)$

$But\ \angle AOB + \angle COB = 180^0\ (Linear\ pair)$

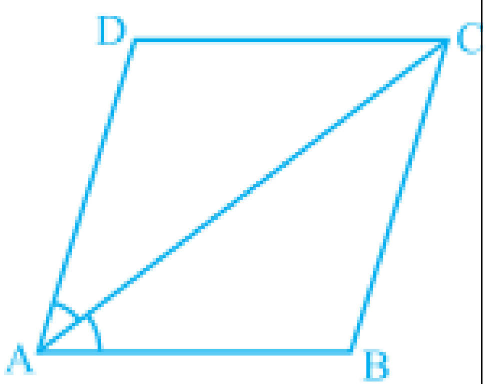
$\angle AOB + \angle AOB = 180^0$

$2\angle AOB = 180^0$

$\angle AOB = 90^0$

Diagonals are bisect each other at right angles.

3. Diagonal AC of a parallelogram ABCD bisects ∠ A Show that (i) it bisects ∠ C also, (ii) ABCD is a rhombus.



Sol: (i) ∠BAC=∠DAC (AC bisects ∠A)→(1)

∠BAC=∠DCA (Alternate interior angles)→(2)

∠DAC =∠BCA (Alternate interior angles)→(3)

From (1),(2),(3)

∠DCA=∠BCA

Hence AC bisects ∠ C also.

(ii) In ΔBAC

∠BAC=∠BCA (From (1),(2),(3))

AB=BC (opposite sides of equal angles are equal) →(4)

But AB=DC and BC=AD (Opposite sides of parallelogram)→(5)

From (4) ,(5)

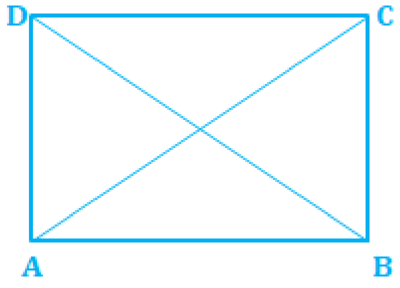
AB =BC= CD= DA

Hence, ABCD is a rhombus.

4. ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that: (i) ABCD is a square (ii) diagonal BD bisects ∠ B as well as ∠ D.

Sol: ABCD is a rectangle

Diagonal AC bisects ∠ A as well as ∠ C



$\angle BAC = \angle DAC = \frac{1}{2}\angle A \text{ and } \angle BCA = \angle DCA = \frac{1}{2}\angle C$

$But\ \angle A = \angle C \Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle BAC = \angle BCA$

$\Rightarrow AB = BC(\text{ Sides opposite to equal angles are equal})$

$But\ AB = DC \text{ and } BC = DA \text{ (opposite sides of a rectangle are equal)}$

$\therefore AB = BC = CD = DA$

∴ ABCD is a square.

(ii) In ΔBCD , BC=CD (ABCD is a square)

$\angle CBD = \angle CDB \text{ (Angles opposite to equal sides are equal)}$

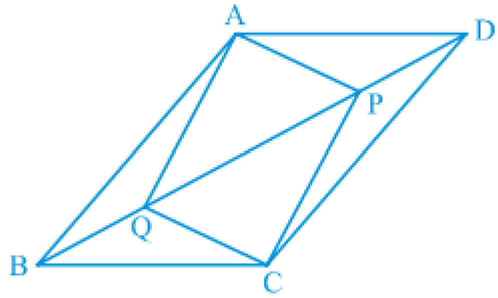
$But\ \angle CBD = \angle ADB \text{ and } \angle CDB = \angle ABD \text{ (alternate interior angles)}$

$\therefore \angle CBD = \angle ABD \text{ and } \angle CDB = \angle ADB$

BD bisects ∠D and ∠B.

5. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.12). Show that:
(i) Δ APD ≅ Δ CQB (ii) AP = CQ (iii) Δ AQB ≅Δ CPD (iv) AQ = CP (v) APCQ is a parallelogram.

Sol: (i) In ΔAPD and ΔCQB



∠ADP=∠CBQ (Alternate interior angles)

AD=BC (Opposite sides of a parallelogram are equal)

DP=BQ (Given)

Δ APD ≅ Δ CQB (SAS congruence rule)

(ii) Δ APD ≅ Δ CQB (From (i))

∴AP=CQ (CPCT)

(iii) In ΔAQB and ΔCPD

∠ABQ=∠CDP (Alternate interior angles)

AB=CD (Opposites of a parallelogram are equal)

BQ=DP (Given)

ΔAQB ≅ ΔCPD (SAS congruence rule)

(iv) ΔAQB ≅ ΔCPD (From (iii))

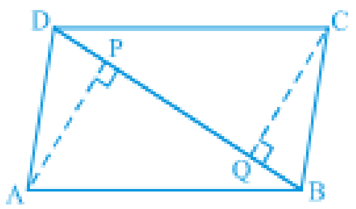
∴AQ=CP (CPCT)

(v) In quadrilateral APCQ

AP=CQ and AQ=CP

Hence APCQ is a parallelogram.

6. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.13). Show that (i) Δ APB ≅ Δ CQD (ii) AP = CQ



Sol: (i) In ΔAPB and ΔCQD

∠APB=∠CQD=90^0

AB=CD (opposite sides of parallelogram are equal)

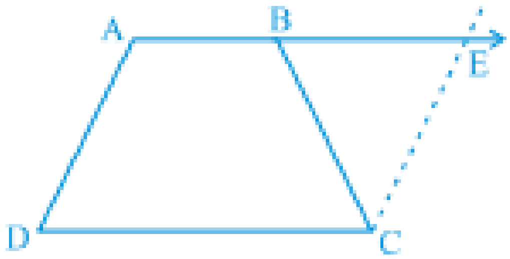
∠ABD=∠CDQ (AB||CD , alternate interior angles)

∴ΔAPB ≅ ΔCQD (AAS congruence rule)

(ii) Δ APB ≅ Δ CQD (from (i))

∴AP=CQ (CPCT)

7. ABCD is a trapezium in which AB || CD and AD = BC (see Fig. 8.14). Show that (i) ∠ A = ∠ B (ii) ∠ C = ∠ D (iii) Δ ABC ≅ Δ BAD (iv) diagonal AC = diagonal BD
[Hint: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]



Sol: Draw AE ||DC and CE||DA

ADCE is a parallelogram

AD=CE (opposite sides of ADCE)

AD = BC (given)

∴BC=CE

∠CEB=∠CBE (equal sides opposite angles are equal)

$\angle A + \angle CEB = 180^0$(co-interior angles are supplementary)

$\angle A + \angle CBE = 180^0 (\angle CEB = \angle CBE) \rightarrow (1)$

$\angle B + \angle CBE = 180^0$ (Linear pair)$\rightarrow$(2)

From (1) and (2)

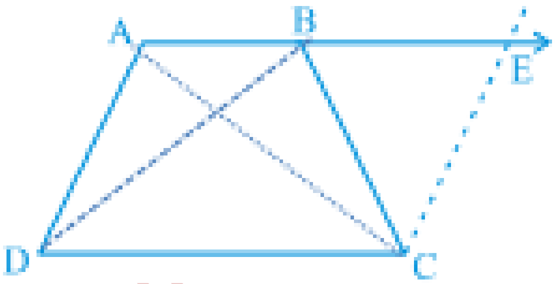
$\angle A = \angle B$

(ii) $\angle A + \angle D = 180^0$(co-interior angles)$\rightarrow$(3)

$\angle B + \angle C = 180^0$(co-interior angles)$\rightarrow$(4)

From (3) and (4)

$\angle B + \angle C = \angle A + \angle D$



But $\angle A = \angle B$

$\therefore \angle C = \angle D$

(iii) InΔABC and ΔBAD,

BC=AD (given)

AB=BA (common side)

$\angle B = \angle A$ (From (i))

$\Delta ABC \cong \Delta BAD$(SAS congruence rule)

(iv) $\Delta ABC \cong \Delta BAD$ (from (iii))

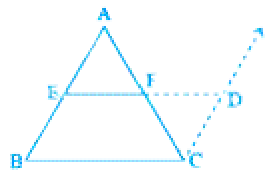
$\therefore$ AC=BD (by CPCT)

The Mid-point Theorem

Theorem 8.8 : The line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Proof: In ΔABC, E and F are mid-points of AB and AC respectively and draw CD || BA.

In ΔAEF , ΔCDF



$\angle AEF = \angle CDF$ (*Alternate interior angles*)

$\angle EAF = \angle FCD$ (*Alternate interior angles*)

$AF = FC$ (*F is mid point of AC*)

$\Delta AEF \cong \Delta CDF$ (*ASA rule*)

$EF = DF$ *and* $BE = AE = DC$ (*CPCT*)

∴ BCDE is a parallelogram. So, EF || BC

Theorem 8.9 : The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

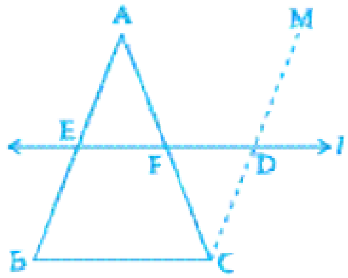
Proof: In ΔABC , E is midpoint of AB.

Draw a line l passing through E and parallel to BC . The line intersects AC at F.

Construct CD || BA

EB || DC and ED || BC

⇒EBCD is a parallelogram.



BE=DC (opposite sides of parallelogram)

But BE=AE (E is midpoint of AB)

∴ AE=CD →(1)

In ΔAFE and ΔCFD

∠EAF = ∠DCF (BA || CD and AC is transversal, alternate interior angles)

∠AEF = ∠CDF (BA || CD and ED is transversal, alternate interior angles)

AE=CD (from (1))

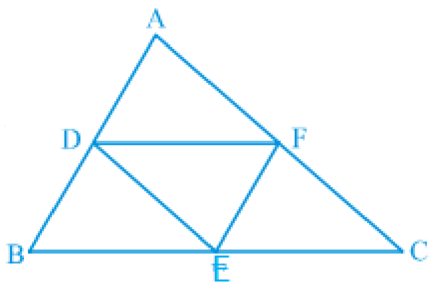
ΔAFE ≅ ΔCFD (ASA congruence rule)

∴ AF = CF (CPCT)

⇒ l bisects AC.

Example 6 : In ΔABC, D, E and F are respectively the mid-points of sides AB, BC and CA (see Fig. 8.18). Show that Δ ABC is divided into four congruent triangles by joining D, E and F.

Solution: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.



DE || AC , DF || BC and EF || AB

Therefore ADEF, BDFE and DFCE are all parallelograms.

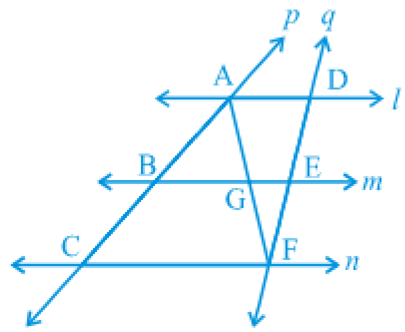
Now DE is a diagonal of the parallelogram BDFE,

Therefore, ΔBDE ≅ Δ FED

Similarly ΔDAF ≅ ΔFED and ΔEFC ≅ ΔFED

So, all the four triangles are congruent

Example 7 : l, m and n are three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts AB and BC on p (see Fig. 8.19). Show that l, m and n cut off equal intercepts DE and EF on q also.



Sol: Let us join A to F intersecting m at G.

The trapezium ACFD is divided into two triangles; namely ΔACF and ΔAFD.

In ΔACF, it is given that B is the mid-point of AC (AB = BC)

and BG || CF (since m || n).

So, G is the mid-point of AF .

Now, in ΔAFD, we can apply the same argument as G is the mid-point of AF, GE || AD, so E is the mid-point of DF,

i.e., DE = EF

$\Rightarrow l, m\ and\ n$ cut off equal intercepts on q also.

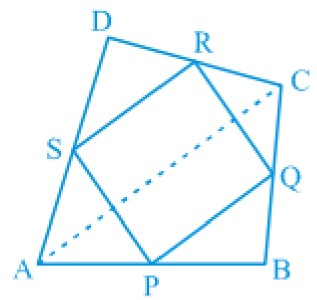
EXERCISE 8.2

1. **ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that :**

(i) SR || AC and SR $= \frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.



Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.

(i) In ΔABC , P and Q are midpoints of AB and BC.

PQ || AC and PQ $= \frac{1}{2}$ AC$\rightarrow$(1)

(ii) In ΔADC , S and R are midpoints of DA and DC.

PQ || AC and PQ $= \frac{1}{2}$ AC$\rightarrow$(2)

From (1) and (2)

PQ = SR

(iii) From (1) and (2)

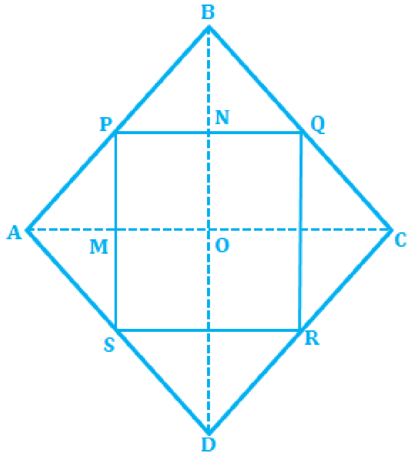
SR || AC and PQ || AC

$\Rightarrow$ PQ ||SR also PQ = SR

$\therefore$ PQRS is a parallelogram.

2. **ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.**

Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.



In ΔABC , P and Q are midpoints of AB and BC.

$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \rightarrow (1)$

In ΔADC , S and R are midpoints of AD and DC.

$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \rightarrow (2)$

$From\ (1) and\ (2) : PQ \parallel SR \text{ and } PQ = SR$

Similarly : $PS \parallel QR \text{ and } PS = QR$

$\therefore$ PQRS is a parallelogram.

$MO \parallel PN \text{ and } PM \parallel NO$

PMON also a parallelogram.

$\angle MPN = \angle MON$ (opposite angles in a parallelogram)

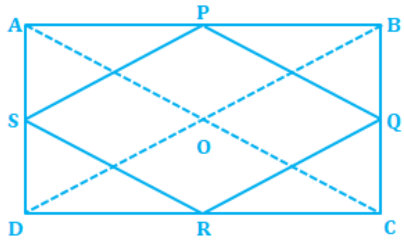
But $\angle MON = 90^0$ (Diagonals of a rhombus perpendicular to each other)

$\therefore \angle MPN = 90^0$

In parallelogram PQRS one angle is 90^0

So, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.



Sol: We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and also half of it.

In ΔABC , P and Q are midpoints of AB and BC.

$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC \rightarrow (1)$

In ΔADC , S and R are midpoints of AD and DC.

$SR \parallel AC \text{ and } SR = \frac{1}{2} AC \rightarrow (2)$

$From\ (1) and\ (2) : PQ \parallel SR \text{ and } PQ = SR = \frac{1}{2} AC$

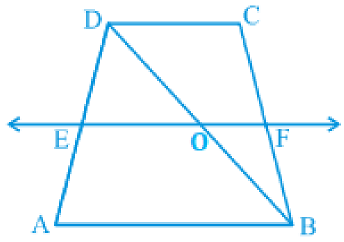
Similarly : $PS \parallel QR \text{ and } PS = QR = \frac{1}{2} BD$

$Also, AC = BD\ (Diagonals\ of\ a\ rectangle\ AC, BD\ are\ equal)$

$\therefore PQ=QR=RS=SP$

So, PQRS is a rhombus.

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.21). Show that F is the mid-point of BC.



Sol: We know that the line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

$In\ \Delta ABD, EO \parallel AB\ and\ E\ is\ mid\ point\ of\ AD$

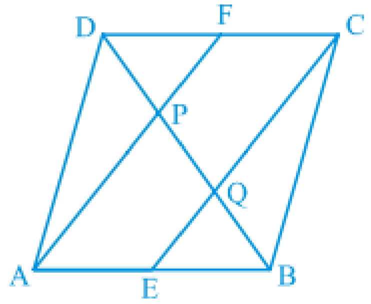
$\Rightarrow O\ is\ mid\ point\ of\ BD$

$In\ \Delta CBD, OF \parallel CD\ and\ O\ is\ mid\ point\ of\ BD$

$\Rightarrow F\ is\ mid\ point\ of\ BC$

5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.22). Show that the line segments AF and EC trisect the diagonal BD.

Sol: E and F are the mid-points of sides AB and CD



$DF = FC = \frac{1}{2}DC\ and\ AE = EB = \frac{1}{2}AB$

$AB \parallel DC\ \ and\ AB = CD\ (ABCD\ is\ a\ parallelogram)$

$\Rightarrow AE \parallel FC\ \ and\ \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow AE \parallel FC\ \ and\ AE = FC$

$\therefore AEFC\ is\ a\ parallelogram.$

$\Rightarrow AF \parallel EC$

$In\ \Delta ABP, EQ \parallel AP\ and\ E\ is\ midpoint\ of\ AB.$

$\Rightarrow Q\ is\ midpoint\ of\ BP$

$\Rightarrow BQ = QP \rightarrow (1)$

$In\ \Delta DQC, FP \parallel CQ\ and\ F\ is\ midpoint\ of\ DC.$

$\Rightarrow P\ is\ midpoint\ of\ DQ$

$\Rightarrow QP = PD \rightarrow (2)$

From (1) and (2)

$BQ = QP = PD$

$\therefore$ The line segments AF and EC trisect the diagonal BD.

6. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that (i) D is the mid-point of AC (ii) MD ⊥ AC

(iii) $CM = MA = \frac{1}{2}AB$

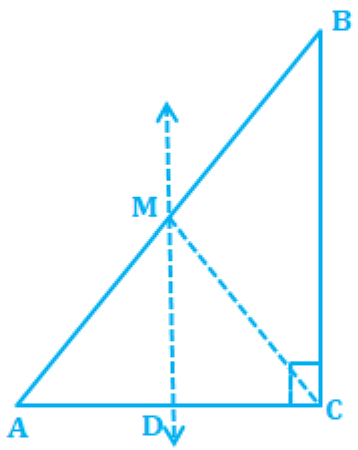
Sol: We know that the line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

(i) $In\ \Delta ABC, MD \parallel BC\ and\ M\ is\ midpoint\ of\ AB$

$\Rightarrow D\ is\ midpoint\ of\ AC.$

(ii) $MD \parallel BC\ and\ AC\ is\ transversal$

$\angle MDC + \angle BCD = 180^0 (Co-interior\ angles\ are\ supplementary)$



$\angle MDC + 90^0 = 180^0$

$\angle MDC = 180^0 - 90^0 = 90^0$

$\therefore MD \perp AC$

(iii) $In\ \Delta AMD\ and\ \Delta CMD$

AD=DC (D is midpoint of AC)

$\angle ADM = \angle CDM (= 90^0)$

$MD = MD\ (Common)$

$\Delta AMD \cong \Delta CMD\ (SAS\ congruence\ rule)$

$AM = CM\ (By\ CPCT)$

$But\ AM = \frac{1}{2} AB\ (\ M\ is\ mid\ point\ of\ AB)$

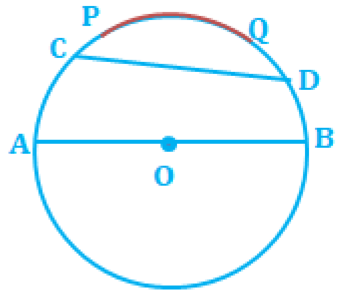
$\therefore CM = MA = \frac{1}{2} AB$

9.CIRCLES (Notes)

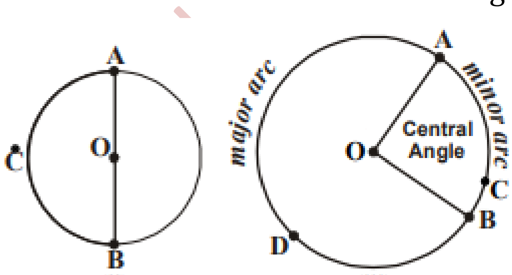
PREPARED BY: BALABHADRA SURESH

https://sureshmathsmaterial.com

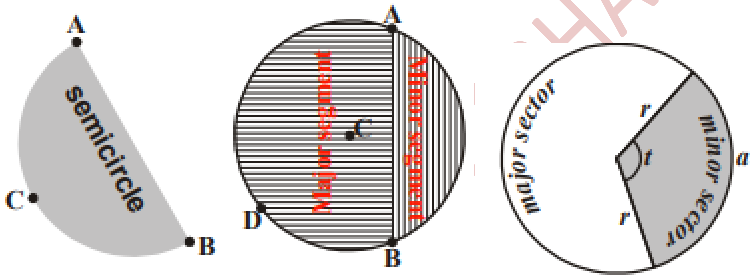
1. **Circle**: A circle is a collection of all the points in a plane which are at a fixed distance from a fixed point on the plane. The fixed point 'O' is called the centre of the circle and the fixed distance OA, is called the radius of the circle.



2. A line segment joining any two points on the circle that passes through the centre is called the diameter (AB)
3. A line segment joining any two points on the circle is called a chord (CD)
4. The part of the circle between any two points on it is called an arc.
5. If the end points of an arc become the end points of a diameter then such an arc is called a semi-circular arc or a semicircle.
6. If the arc is smaller than a semicircle, then the arc is called a minor arc and if the arc is longer than a semicircle, then the arc is called a major arc
7. The region between the chord and the minor arc is called the minor segment and the region between the chord and the major arc is called the major segment.
8. The area enclosed by an arc and the two radii joining the centre to the end points of an arc is called a sector. One is minor sector and another is major sector.

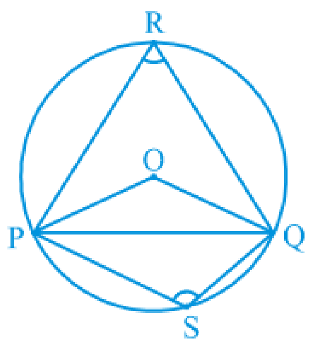


(ii) (iii)

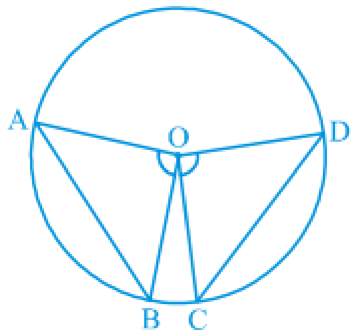


Angle Subtended by a Chord at a Point

1. In adjacent figure ∠ PRQ is called the angle subtended by the line segment PQ at the point R and ∠ POQ is the angle subtended by the chord PQ at the centre O
2. ∠ PRQ and ∠ PSQ are respectively the angles subtended by PQ at points R and S on the major and minor arcs PQ.



Theorem 9.1 : Equal chords of a circle subtend equal angles at the centre.



Proof: AB and CD are two equal chords of a circle with centre O.

In ΔAOB and ΔCOD

OA = OC (Radii of a circle)

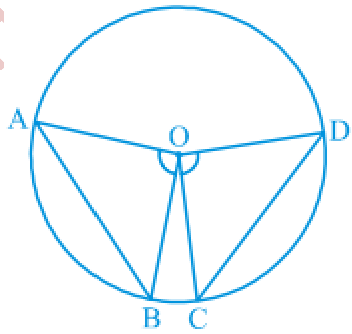
OB = OD (Radii of a circle)

AB = CD (Given)

Δ AOB $\cong$ Δ COD (By SSS congruence rule)

$\angle AOB = \angle COD$ (By CPCT)(Corresponding parts of congruent triangles)

Theorem 9.2 : If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



Proof: The angles subtended by the chords AB and CD of a circle at the centre O are $\angle AOB$ and $\angle COD$ respectively.

Given $\angle AOB = \angle COD$

In ΔAOB and ΔCOD

OA = OC (Radii of a circle)

OB = OD (Radii of a circle)

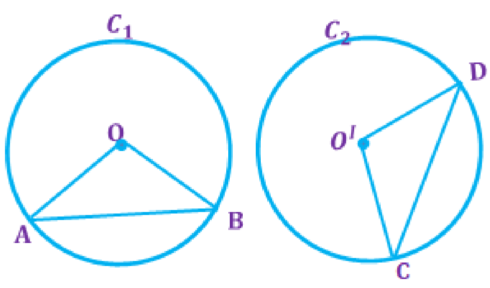
$\angle AOB = \angle COD$ (Given)

Δ AOB $\cong$ Δ COD (By SAS congruence rule)

AB=CD (By CPCT)

EXERCISE 9.1

1. **Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.**



Sol: C_1, C_2 are two congruent circles.

In ΔAOB and ΔCO^ID

$OA = O^IC$ (Radii ofcongruent circles)

$OB = O^ID$ (Radii ofcongruent circles)

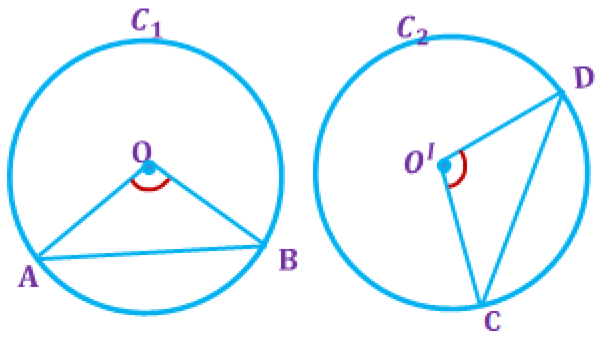
AB = CD (Given)

$\Delta AOB \cong \Delta CO^ID$(By SSS congruence rule)

$\angle AOB = \angle CO^{I}D$ (By CPCT)

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Sol: C_1, C_2 are two congruent circles



$OA = O^{I}C$ (Radii ofcongruent circles)

$OB = O^{I}D$ (Radii ofcongruent circles)

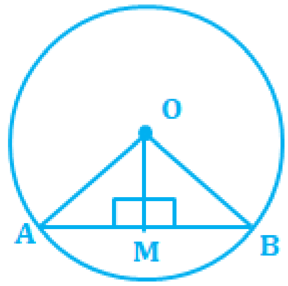
$\angle AOB = \angle CO^{I}D$ (Given)

$\Delta AOB \cong \Delta CO^{I}D$(By SAS congruence rule)

$AB = CD$ (By CPCT)

Perpendicular from the Centre to a Chord

Theorem 9.3 : The perpendicular from the centre of a circle to a chord bisects the chord.



Proof: AB is a chord for the circle with centre O and $OM \perp AB$.

Joining O to A and B

In ΔAMO and ΔBMO

$OA = OB$ (Radii of a circle)

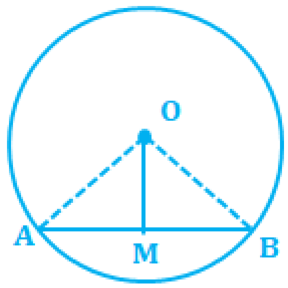
$OM = OM$ (Common))

$\angle AMO = \angle BMO = 90^0$ ($OP \perp AB$)

$\Delta\ AMO \cong \Delta\ BMO$ (By RHS congruence rule)

AM=BM (By CPCT)

Theorem 9.4 : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.



Proof: Let AB be a chord of a circle with centre O and O is joined to the mid-point M of AB. Join OA and OB.

In ΔAMO and ΔBMO

$OA = OB$ (Radii of a circle)

$OM = OM$ (Common))

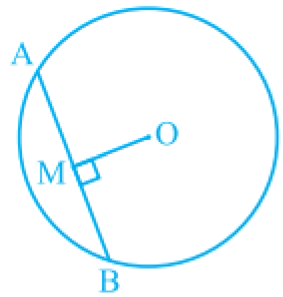
$AM = BM$ (M is midpoint of AB)

$\Delta\ AMO \cong \Delta\ BMO$ (By SSS congruence rule)

$\angle AMO = \angle BMO$ (By CPCT)

But ∠AMO and ∠BMO are linear pair angles.

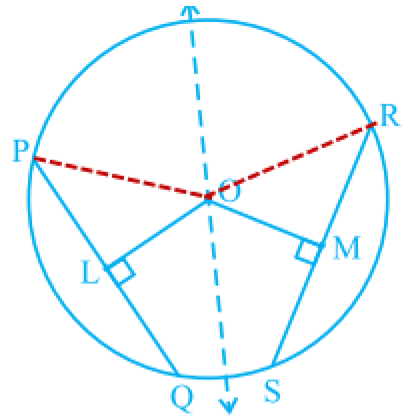
So, ∠AMO = ∠BMO=90^0



Equal Chords and their Distances from the Centre

The length of the perpendicular from a point to a line is the distance of the line from the point.

Theorem 9.5 : Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres).



Proof:PQ and RS are two equal chords of circle with center O.

OL and OM are perpendiculars to PQ and RS respectively.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$PL = QL = \frac{PQ}{2} \text{ and } SM = RM = \frac{RS}{2}$$

But PQ=RS (Given)

⇒PL=RM→(1)

In ΔPOL and ΔROM

∠OLP=∠OMR=90^0

OP=OR (Radii of same circle)

PL=RM (from(1))

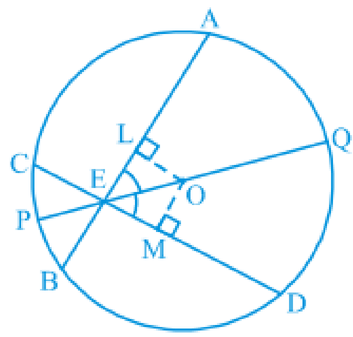
ΔPOL ≅ ΔROM (by RHS rule)

OL=OM(by CPCT)

Hence proved.

Theorem 9.6 : Chords equidistant from the centre of a circle are equal in length.

Example 1 : If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.



Sol: AB and CD are two chords of a circle, with centre O intersecting at a point E.

PQ is a diameter through E, such that ∠ AEQ = ∠ DEQ.

Draw OL ⊥ AB and OM ⊥ CD.

$\angle LEO = \angle MEO \rightarrow (1)$

In Δ OLE and ΔOME

$\angle LEO = \angle MEO$ (from(1))

$\angle OLE = \angle OME$ ($=90^0$)

EO = EO (Common)

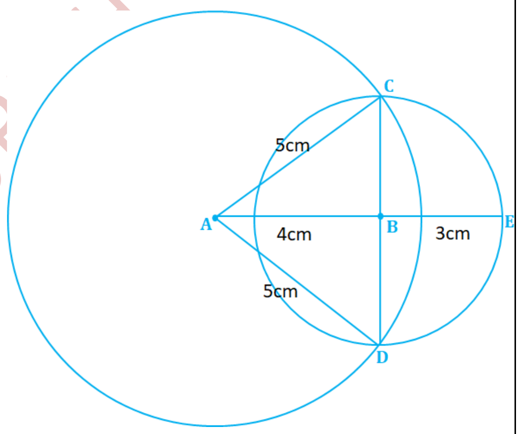
Δ OLE ≅ Δ OME (AAS congruency rule)

⇒ OL = OM (by CPCT)

⇒ AB = CD

EXERCISE 9.2

1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.



Sol: Let A,B are the centers of the circles.

CD is the common chord.

AC=AD=5 cm; BE=3 cm

AB⊥CD

From Pythagoras theorem

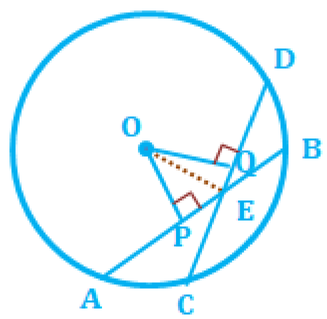
$BC^2 = AC^2 - AB^2 = 5^2 - 4^2 = 25 - 16 = 9$

$BC = 3\ cm$

CD=3+3=6 cm

Length of the chord=6 cm

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.



Sol: AB,CD are two chords and AB=CD.

Let AB,CD intersect at E.

Now we prove AE=DE and CE=BE

OP and OQ are perpendiculars to AB and CD from O.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

AP=PB =CQ=QD→(1)

$In\ \Delta OPE\ and\ \Delta OQE$

$\angle OPE = \angle OQE = 90^0$

$OP = OQ$ (Distance from centre to equal chords)

$OE = OE\ (common)$

$\Delta OPE \cong \Delta OQE(\ RHS\ congruence\ rule)$

$PE = QE\ (By\ CPCT) \rightarrow (2)$

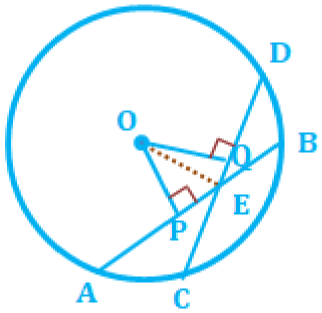
$Now\ AE = AP + PE = DQ + QE\ [\ From\ (1) and\ (2)]$

$\therefore$ AE = DE

$Given\ AB = CD$

$AB - AE = CD - DE\ (\because AE = DE)$
$\therefore$ BE=CE
Hence proved.

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.



Sol: AB,CD are two chords and AB=CD.

Let AB,CD intersect at E.

Now we show that ∠OEA=∠OED

Let OP and OQ are perpendiculars to AB and CD from O.

$In\ \Delta OPE\ and\ \Delta OQE$

$\angle OPE = \angle OQE = 90^0$

$OP = OQ$ (Distance from centre to equal chords)

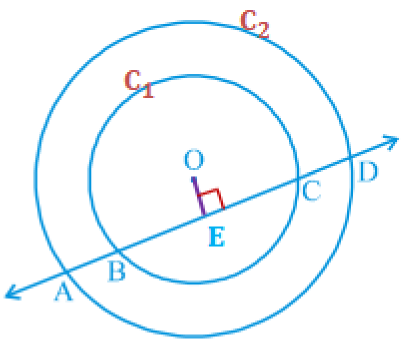
$OE = OE\ (common)$

$\Delta OPE \cong \Delta OQE(\ RHS\ congruence\ rule)$

$\angle OEP = \angle OEQ\ (By\ CPCT)$

$\Rightarrow \angle OEA = \angle OED$

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see Fig. 9.12).



Sol: $Let\ C_1$ and C_2 are two concentric circles with centre O.

$A\ line\ intersects\ C_1\ and\ C_2\ at\ B, C\ and\ A, D.$

Let $OE \perp AD$

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$In\ circle\ C_2 : AE = ED \longrightarrow (1)$

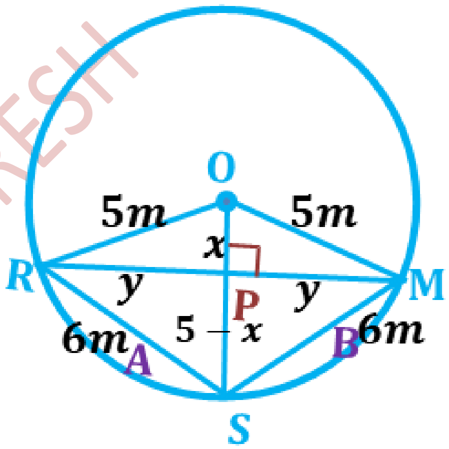
$In\ circle\ C_1 : BE = EC \longrightarrow (2)$

From (1)-(2)

$AE - BE = ED - EC$

AB=CD

5. **Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?**



Sol: Join OR,OS,OM,RS,MS. OS intersect RM at P

In ΔORS and ΔOMS

OR=OM (Radii)

RS=MS (Given)

OS=OS (Common)

ΔORS ≅ ΔOMS (By SSS congruence rule)

∠ROS=∠MOS (By CPCT)

∴∠ROP=∠MOP →(1)

In ΔROP and ΔMOP

RO=MO (Radii)

∠ROP=∠MOP (From (1))

OP=OP (Common)

ΔROP ≅ ΔMOP (By SAS congruence rule)

RP=PM and ∠RPO=∠MPO (By CPCT)→(2)

∠RPO=∠MPO=90^0

So, OP bisects the chord RM and OP⊥RM

Let RP = PM = y ; $OP = x \Rightarrow$ PS = $5 - x$

In ΔOPR

$x^2 + y^2 = 5^2$(From Pythagoras theorem)

$y^2 = 25 - x^2 \rightarrow (3)$

In ΔRPS

$(5 - x)^2 + y^2 = 6^2$

$y^2 = 36 - (5 - x)^2$

$y^2 = 36 - (25 - 10x + x^2)$

$y^2 = 36 - 25 + 10x - x^2$

$y^2 = 11 + 10x - x^2 \rightarrow (4)$

From (3)and (4)

$11 + 10x - x^2 = 25 - x^2$

$10x = 25 - 11 = 14$

$x = \frac{14}{10} = 1.4\ cm$

From (3)

$y^2 = 25 - x^2 = 25 - (1.4)^2 = 25 - 1.96 = 23.04$

$y = \sqrt{23.04} = 4.8\ cm$

The distance between Reshma and Mandip $= 2y = 2 \times 4.8\ cm = 9.6\ cm$

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Sol: Ankur (A), Syed(S) and David(D)

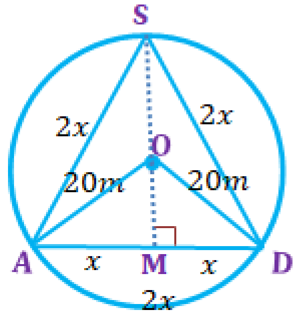
Let AS = SD = DA = $2x$

Let OM$\perp$AD

ΔAMO $\cong$ ΔDMO (By RHS congruence rule)

AM = MD (By CPCT)

$\therefore$ AM = MD = x



From ΔAMS

$SM^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$

$SM = \sqrt{3x^2} = \sqrt{3}x$

$OM = SM - OS = \sqrt{3}x - 20$

From ΔAMO

$OM^2 + AM^2 = OA^2$

$OM^2 = OA^2 - AM^2$

$OM^2 = 20^2 - x^2 = 400 - x^2$

$(\sqrt{3}x - 20)^2 = 400 - x^2$

$3x^2 - 40\sqrt{3}x + 400 = 400 - x^2$

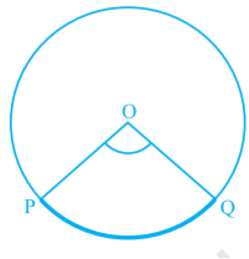
$3x^2 + x^2 = 40\sqrt{3}x$

$4x^2 = 40\sqrt{3}x$

$x = 10\sqrt{3}\, m$

The length of the string of each phone $= 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}\, m$

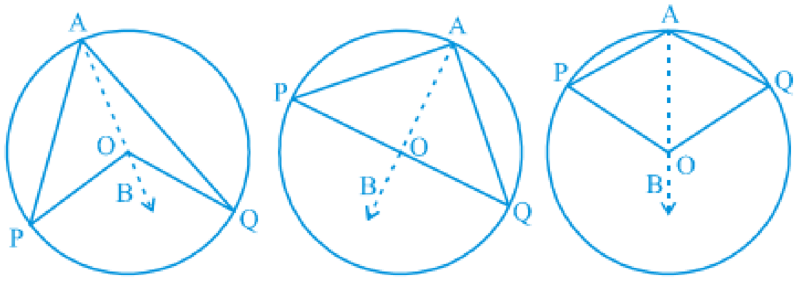
Angle Subtended by an Arc of a Circle



1. If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal.

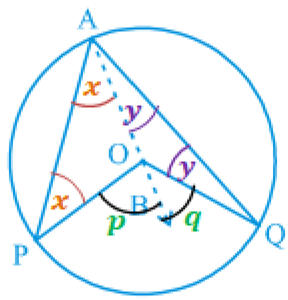
2. Congruent arcs (or equal arcs) of a circle subtend equal angles at the centre.

Theorem 9.7 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



(i) (ii) (iii)

Given: : Let O be the centre of the circle. PQ is an arc subtending $\angle POQ$ at the centre. Let A be a point on the remaining part of the circle.



Proof: Draw ray $\overrightarrow{OA}$

In ΔAOP, $OA = OP$ (radii of the same circle)

$\Rightarrow \angle OAP = \angle OPA = x$(say)(Angles opposite to equal sides are equal)

simlarly In ΔAOQ, $OA = OQ$

$\Rightarrow \angle OAQ = \angle OQA = y$(say)

$Let\ \angle POB = p$ and $\angle QOB = q$

$p = x + x$(exterior angle is equal to sum of the opposite interior angles)

$p = 2x$

Similarly $q = 2y$

$p + q = 2x + 2y$

$p + q = 2(x + y)$

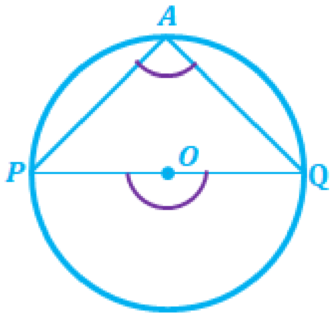
$\angle POQ = 2\angle PAQ$

For the case (iii), where PQ is the major arc

Reflex of $\angle POQ = 2\angle PAQ$

Theorem 9.8 : Angles in the same segment of a circle are equal.

Example: Angle in a semicircle is a right angle.



Sol: PQ is a diameter and 'O' is the centre of the circle.

$\therefore \angle POQ = 180o$ [Angle on a straight line]

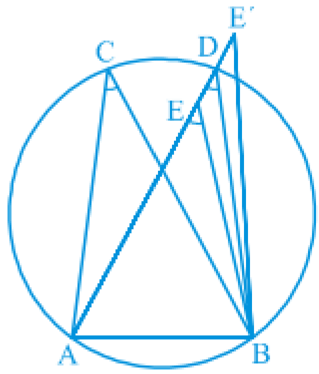
$\angle POQ = 2\angle PAQ$ [Angle subtended by an arc at the centre is twice the angle subtended by it at any other point on circle]

$$\therefore \angle PAQ = \frac{\angle POQ}{2} = \frac{180^0}{2} = 90^0$$

Theorem 9.9 : If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

Proof: AB is a line segment, which subtends equal angles at two points C and D. That is $\angle ACB = \angle ADB$.

R.T.P : A, B, C and D are concyclic.(they lie on the same circle)



let us draw a circle through the points A, C and B. Suppose it does not pass through the point D. Then it will intersect AD (or extended AD) at a point, say E (or E′).

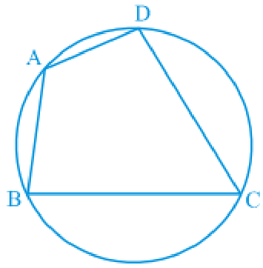
If points A, C, E and B lie on a circle,

$\angle ACB = \angle AEB$

But it is given that $\angle ACB = \angle ADB$.

$\therefore \angle AEB = \angle ADB$.

This is not possible unless E coincides with D.



Cyclic Quadrilateral: A quadrilateral ABCD is called cyclic if all the four vertices A, B, C, D of it lie on a circle.

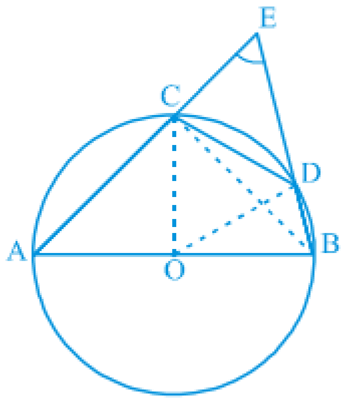
Theorem 9.10 : The sum of either pair of opposite angles of a cyclic quadrilateral is 180º.(OR)

If ABCD is a Cyclic quadrilateral then $\angle A+\angle C=180^0$ and $\angle B+\angle D=180^0$

Theorem 9.11 : If the sum of a pair of opposite angles of a quadrilateral is 180º, the quadrilateral is cyclic.(OR)

If in quadrilateral $\angle A+\angle C=180^0$ and $\angle B+\angle D=180^0$ then ABCD is a Cyclic quadrilateral.

Example 2 : In Fig. 9.19, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at a point E. Prove that $\angle AEB = 60°$.



Solution : Join OC, OD and BC.

In ΔODC , OC=OD=DC

$\therefore$ ΔODC is an equilateral.

$\Rightarrow \angle COD = 60°$

$\angle CBD = \frac{1}{2}\angle COD$ (By Angle subtended theorem)

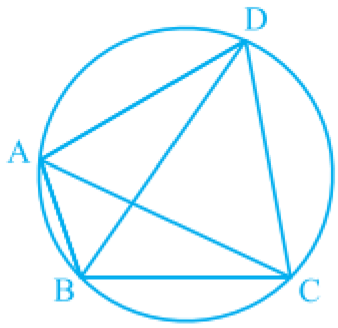
$\angle CBD = \frac{1}{2} \times 60^0 = 30^0 \Rightarrow \angle CBE = 30^0$

$\angle ACB = 90°$ (angle subtended by semi – circle is 90^0)

$\angle BCE = 180° – \angle ACB = 180° – 90° = 90°$

$\angle CEB = 90° – \angle CBE = 90° - 30° = 60°$, i.e., $\angle AEB = 60°$

Example 3 : In Fig 9.20, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.



Solution: $\angle CAD = \angle DBC = 55°$ (Angles in the same segment)

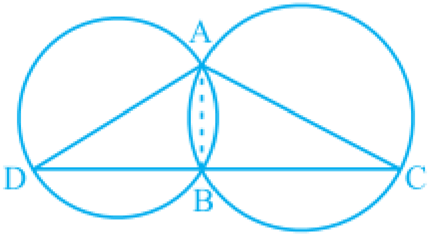
$\therefore$ $\angle DAB = \angle CAD + \angle BAC = 55° + 45° = 100°$

But $\angle DAB + \angle BCD = 180°$ (Opposite angles of a cyclic quadrilateral)

So, $\angle BCD = 180° – 100° = 80°$

Example 4 : Two circles intersect at two points A and B. AD and AC are diameters to the two circles (see Fig. 9.21). Prove that B lies on the line segment DC.

Solution : Join AB.



$\angle ABD = 90°$ (Angle in a semicircle)

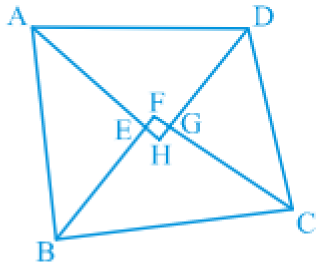
$\angle ABC = 90°$ (Angle in a semicircle)

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

Therefore, DBC is a line. That is B lies on the line segment DC.

Example 5: Prove that the quadrilateral formed (if possible) by the internal angle bisectors of any quadrilateral is cyclic.

Solution : In Fig. 9.22, ABCD is a quadrilateral in which the angle bisectors AH, BF, CF and DH of internal angles A, B, C and D respectively form a quadrilateral EFGH.



Now, $\angle FEH = \angle AEB = 180° - \angle EAB - \angle EBA$

$= 180° - \frac{1}{2}(\angle A + \angle B)$

$\angle FGH = \angle CGD = 180° - \angle GCD - \angle GDC$

$= 180° - \frac{1}{2}(\angle C + \angle D)$

Therefore, $\angle FEH + \angle FGH = 180° - \frac{1}{2}(\angle A + \angle B) + 180° - \frac{1}{2}(\angle C + \angle D)$

$= 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = 360° - \frac{1}{2}(360°) = 360° - 180° = 180°$

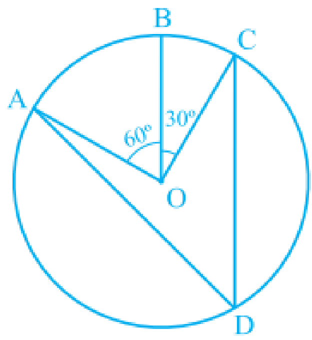
In EFGH the pair of opposite angles are supplementary.

So, the quadrilateral EFGH is cyclic.

EXERCISE 9.3

1. **In Fig. 9.23, A,B and C are three points on a circle with centre O such that $\angle BOC = 30°$ and $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.**

Sol: $\angle AOC = \angle AOB + \angle BOC = 60^0 + 30^0 = 90^0$.



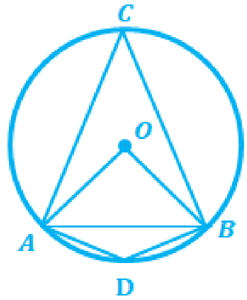
The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$2 \times \angle ADC = \angle AOC$

$2 \times \angle ADC = 90^0$

$\angle ADC = \frac{90^0}{2} = 45^0$

2. **A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.**



Sol: O is the center of the circle and AB is a chord with length of radius.

OA=OB=AB (Radius)

ΔABO becomes an equilateral triangle.

$\therefore \angle AOB = 60^0$

Let C be a point on the major arc and D be a point on the minor arc.

The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$2 \times \angle ACB = \angle AOB$

$2 \times \angle ACB = 60^0$

$\angle ACB = \frac{60^0}{2} = 30^0$

Since A, B. C and D lie on the same circle. ADBC is a cyclic quadrilateral.

$\angle ACB + \angle ADB = 180^0$ (*In a cyclic quadrilateral opposite angles are supplementary*)

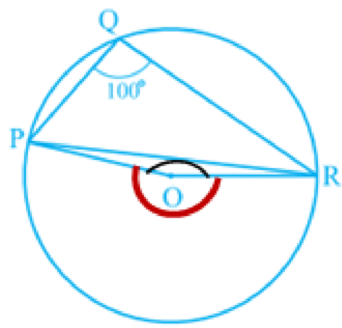
$30^0 + \angle ADB = 180^0$

$\angle ADB = 180^0 - 30^0 = 150^0$

Required angles are $150^0, 30^0$.

3. **In Fig. 9.24, ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.**

Sol: We know that, the angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.



Reflex $\angle POR = 2 \times \angle PQR$

Reflex$\angle POR = 2 \times 100^0 = 200^0$

$\angle POR = 360^0 - \text{Reflex}\angle POR = 360^0 - 200^0 = 160^0$

In ΔPOR, OP=OR (Radii of the circle)

$\angle OPR = \angle ORP = x$ (Angles of opposite to equal sides are equal)

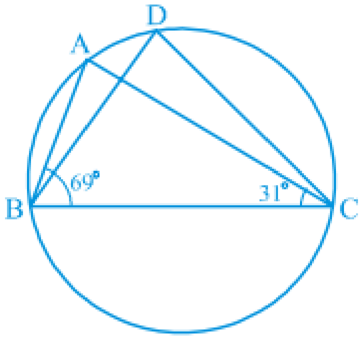
$\angle OPR + \angle ORP + \angle POR = 180^0$ (*Angle sum property of triangle*)

$x + x + 160^0 = 180^0$

$2x = 180^0 - 160^0 = 20^0$

$x = \frac{20^0}{2} = 10^0 \Rightarrow \angle OPR = 10^0$

4. **In Fig. 9.25, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.**



Sol: In ΔABC, the sum of all angles = 180º

$\angle BAC + \angle ABC + \angle ACB = 180^0$

$\angle BAC + 69^0 + 31^0 = 180^0$

$\angle BAC + 100^0 = 180^0$

$\angle BAC = 180^0 - 100^0$

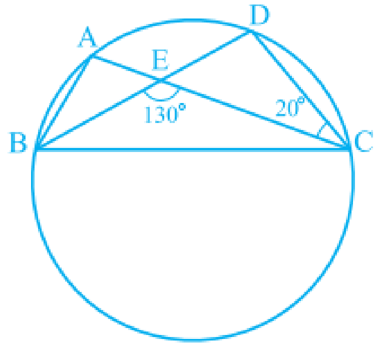
$\angle BAC = 80^0$

We know that angles in the same segment of a circle are equal.

$\angle BDC = \angle BAC$

$\angle BDC = 80^0$

5. **In Fig. 9.26, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find ∠ BAC.**



Sol: $\angle BEC + \angle CED = 180^0 (\textit{ Linear pair})$

$130^0 + \angle CED = 180^0$

$\angle CED = 180^0 - 130^0 = 50^0$

In ΔDEC, the sum of all angles = 180^0

$\angle CDE + \angle CED + \angle ECD = 180^0$

$\angle CDE + 50^0 + 20^0 = 180^0$

$\angle CDE + 70^0 = 180^0$

$\angle CDE = 180^0 - 70^0 = 110^0$

$\angle CDB = 110^0$

We know that angles in the same segment of a circle are equal.

$\angle BAC = \angle CDB$

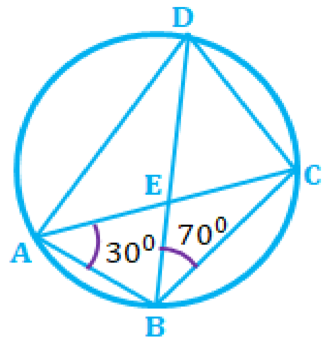
$\angle BAC = 110^0$

6. **ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.**

Sol: We know that angles in the same segment of a circle are equal.

$\angle CAD = \angle CBD = 70^0$

$\angle BAC = \angle BDC = 30^0$



In ΔABC, AB=BC

$\angle BAC = \angle BCA = 30^0$(*Angles opposite to equal sides are equal*)

$\angle BAD = \angle BAC + \angle CAD = 30^0 + 70^0 = 100^0$

ABCD is a cyclic quadrilateral

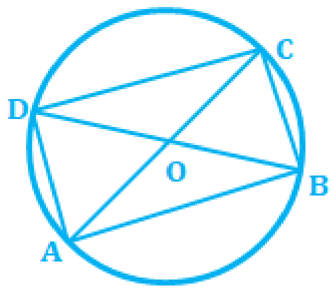
$\angle BAD + \angle BCD = 180^0$

$100^0 + \angle BCD = 180^0$

$\angle BCD = 180^0 - 100^0 = 80^0$

$\angle ECD = \angle BCD - \angle BCA = 80^0 - 30^0 = 50^0$.

7. **If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.**



Sol: ABCD is a cyclic quadrilateral.

AC and BD are diameters.

We know that, the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$2 \times \angle ADC = \angle AOC$

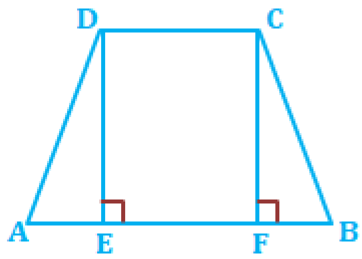
$2 \times \angle ADC = 180^0$

$\angle ADC = \frac{180^0}{2} = 90^0$

In ABCD one angle is 90^0 and diagonals are equal.

So, ABCD is a rectangle.

8. **If the non-parallel sides of a trapezium are equal, prove that it is cyclic.**



Sol: ABCD is a trapezium with AB || DC and AD=BC.

Draw DE⊥AB and CF⊥AB

In ΔADE and ΔBCF

AD=BC (Given)

$\angle AED = \angle BFC = 90^0$

DE=CF (Distance between parallel lines)

$\Delta ADE \cong \Delta BCF$ (By RHS congruence rule)

$\angle DAE = \angle CBF$ (By CPCT)

$\angle DAB = \angle CBA \rightarrow (1)$

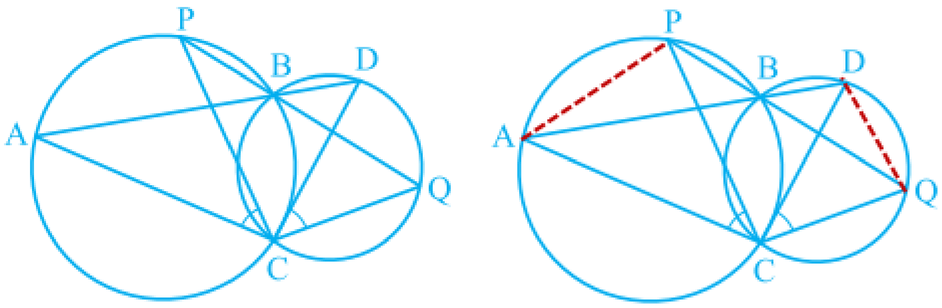
$\angle ADC + \angle DAB = 180^0$(co- interior angles are supplementary)

$\angle ADC + \angle CBA = 180^0$

Opposite angles are supplementary.

Hence, ABCD is a cyclic quadrilateral.

9. **Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 9.27). Prove that ∠ACP = ∠ QCD.**



Sol: Join AP and DQ

For chord AP

$\angle ABP = \angle ACP($ *Angles in the sqme segment are equal*$) \rightarrow (1)$

For chord DQ

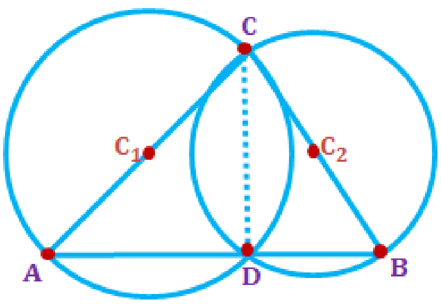
$\angle QBD = \angle QCD$ (*Angles in the sqme segment are equal*$) \rightarrow (2)$

$\angle ABP = \angle QBD$ (*vertically opposite angles*$) \rightarrow (3)$

From (1) ,(2) and (3)

$\angle ACP = \angle QCD$

10. **If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.**



Sol: Two circles are drawn on the sides AC and BC of the ΔABC

The circles intersected at D.

Join CD.

$\angle ADC = \angle BDC = 90^0($ *Angle in semi circle*$)$

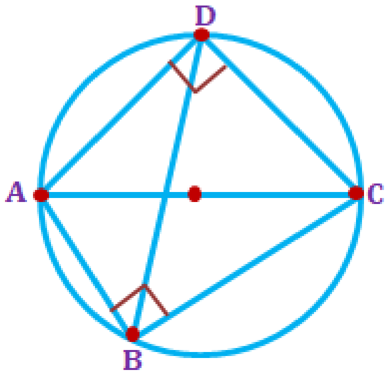
$\angle ADC + \angle BDC = 90^0 + 90^0$

$\angle ADC + \angle BDC = 180^0$

ΔADB is a straight angle

So, D lies on AB.

11. **ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠ CAD = ∠ CBD.**



Sol: Draw a circle with diameter AC.

ABCD is a quadrilateral.

$\angle B + \angle D = 90^0 + 90^0 = 180^0$

In quadrilateral ABCD , opposite angles are supplementary

ABCD is a cyclic quadrilateral.

CD is a chord.

∠ CAD = ∠ CBD(Angles in the same segment in a circle)

12. **Prove that a cyclic parallelogram is a rectangle.**

Sol: Let ABCD is a cyclic parallelogram.

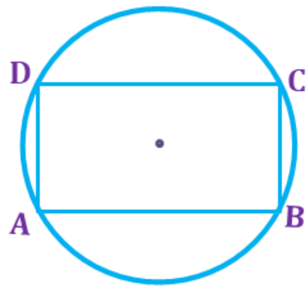
$\angle A + \angle C = 180^0$ (*opposite angles are supplementary in cyclic quadrilateral*)

But $\angle A = \angle C$ (*In a parallelogramopposite angle are equal*)

$\therefore \angle A = \angle C = 90^0$

ABCD is a parallelogram and one interior angle is 90^0

So, ABCD is a rectangle.



CHAPTER 10

IX-MATHEMATICS-NCERT(2023-24)

10. HERON'S FORMULA

PREPARED BY: BALABHADRA SURESH

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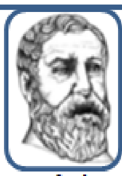
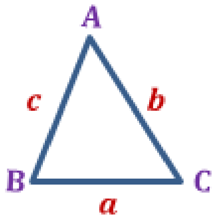

Heron of Alexandria

1. **Heron's formula**:

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

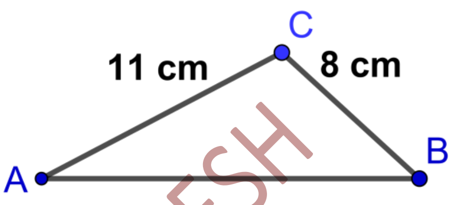
Where a, b, c are sides of the triangle and $s = \frac{a+b+c}{2}$

s=semi-perimeter i.e., half the perimeter of the triangle



Example 1 : Find the area of a triangle, two sides of which are 8 cm and 11 cm and the perimeter is 32 cm

Sol: a = 8 cm and b = 11 cm.



Perimeter of the triangle = 32 cm

$a + b + c = 32$

$8 + 11 + c = 32$

$c = 32 - 19 = 13\ cm$

2s =32 cm, i.e., s = 16 cm,

s – a =(16 – 8) cm = 8 cm,

s – b =(16 – 11) cm = 5 cm,

s – c =(16 – 13) cm = 3 cm.

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

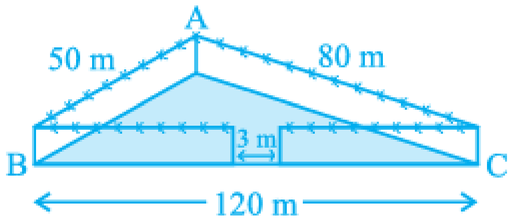
$= \sqrt{16 \times 8 \times 5 \times 3}$

$= \sqrt{2 \times 8 \times 8 \times 5 \times 3}$

$= 8\sqrt{2 \times 5 \times 3} = 8\sqrt{30}\ cm^2$

Example 2 : A triangular park ABC has sides 120m, 80m and 50m (see Fig. 10.4). A gardener Dhania has to put a fence all around it and also plant grass inside. How much area does she need to plant? Find the cost of fencing it with barbed wire at the rate of `20 per metre leaving a space 3m wide for a gate on one side.

Solution: $a = 120\ m, b = 80\ m, c = 50\ m$



$s = \frac{a+b+c}{2} = \frac{50 + 80 + 120}{2} = \frac{250}{2} = 125\ m$

$s = 125\ m$

$s - a = (125 - 120)\ m = 5\ m,$

$s - b = (125 - 80)\ m = 45\ m,$

$s - c = (125 - 50)\ m = 75\ m$

Area of the park $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{125 \times 5 \times 45 \times 75}$

$= \sqrt{25 \times 5 \times 5 \times 3 \times 15 \times 25 \times 3}$

$= 25 \times 5 \times 3 \times \sqrt{15} = 375\sqrt{15} m^2$

Perimeter of the park = AB + BC + CA = 250 m

Length of the wire needed for fencing = 250 m – 3 m=247 m

The cost of fencing = ₹20 × 247 = ₹4940.

Example 3 : The sides of a triangular plot are in the ratio of 3 : 5 : 7 and its perimeter is 300 m. Find its area.

Sol: The ratio of sides of triangle=3:5:7

Let the sides $a = 3x, b = 5x, c = 7x$

Perimeter of the triangle=300m

$3x + 5x + 7x = 300$

$15x = 300$

$x = \frac{300}{15} = 20$

$a = 3 \times 20 = 60m, b = 5 \times 20 = 100m, c = 7 \times 20 = 140m$

$s = \frac{60 + 100 + 140}{2} = \frac{300}{2} = 150m$

Area of the plot $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150 \times (150-60) \times (150-100) \times (150-140)}\ m^2$

$= \sqrt{150 \times 90 \times 50 \times 10}\ m^2$

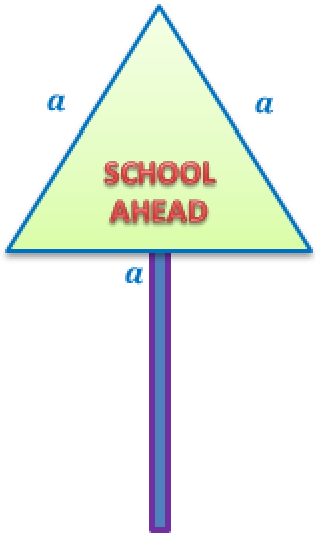
$= \sqrt{3 \times 50 \times 3 \times 3 \times 10 \times 50 \times 10}\ m^2$

$= 3 \times 10 \times 50 \times \sqrt{3}\ m^2$

$= 1500\sqrt{3}\ m^2$

EXERCISE 10.1

1. A traffic signal board, indicating 'SCHOOL AHEAD', is an equilateral triangle with side 'a'. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?



Sol: $2s = 3a$

$$s = \frac{3a}{2}$$

$$s - a = \frac{3a}{2} - a = \frac{3a - 2a}{2} = \frac{a}{2}$$

Area of the signal board $= \sqrt{s(s-a)(s-a)(s-a)}$

$$= \sqrt{\frac{3a}{2} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \frac{\sqrt{3} \times a \times a}{2 \times 2} = \frac{\sqrt{3}}{4}a^2$$

If perimeter of the board=180 cm

$$3a = 180\ cm$$

$$a = \frac{180}{3} = 60\ cm$$

Area of the signal board $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 60 \times 60 = 900\sqrt{3}\ cm^2$

Method 2:

Let a=2x

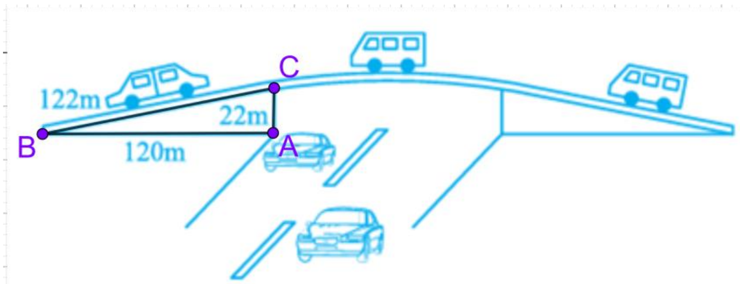
$$2s = 6x \Longrightarrow s = \frac{6x}{2} = 3x$$

$$s - a = 3x - 2x = x$$

Area of the signal board $= \sqrt{s(s-a)(s-a)(s-a)}$

$$= \sqrt{3x \times x \times x \times x} = \sqrt{3x^4} = \sqrt{3}x^2 = \sqrt{3} \times \left(\frac{a}{2}\right)^2 = \frac{\sqrt{3}}{4}a^2$$

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 122 m, 22 m and 120 m (see Fig. 10.6). The advertisements yield earnings of ₹ 5000 per m² per year. A company hired one of its walls for 3 months. How much rent did it pay?



Sol: $a = 122$ m, b $= 22$ m , c $= 120$ m

$$s = \frac{a+b+c}{2} = \frac{122+22+120}{2} = \frac{264}{2} = 132\ m$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{132(132-122)(132-22)(132-120)}$

$= \sqrt{132 \times 10 \times 110 \times 12}$

$= \sqrt{11 \times 12 \times 10 \times 11 \times 10 \times 12}$

$= 10 \times 11 \times 12 = 1320\ m^2$

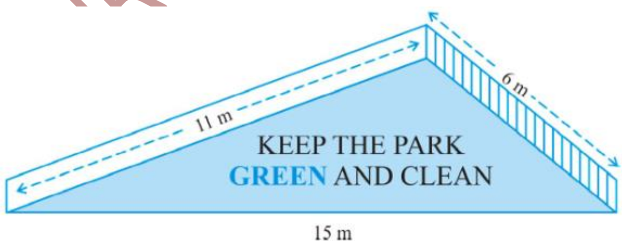
Rent for 1 m^2 area per year=₹ 5000

Rent for 1 m^2 area per 3 months $= \frac{₹\ 5000}{12} \times 3 = ₹1250$

Rent for 1320 m^2 area per 3 months $= ₹1250 \times 1320 = ₹16{,}50{,}000$

3. There is a slide in a park. One of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN" (see Fig. 10.7). If the sides of the wall are 15 m, 11 m and 6 m, find the area painted in colour.

Sol: $a = 15$ m, $b = 11$ m, $c = 6$ m



$$s = \frac{a+b+c}{2} = \frac{15+11+6}{2} = \frac{32}{2} = 16\ m$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-15)(16-11)(16-6)}$

$= \sqrt{16 \times 1 \times 5 \times 10}$

$= \sqrt{4 \times 4 \times 5 \times 5 \times 2}$

$= 20\sqrt{2}\ m^2$

$\therefore$ The area painted in colour= $20\sqrt{2}\ m^2$

4. Find the area of a triangle two sides of which are 18cm and 10cm and the perimeter is 42cm.

Sol: a = 18 cm and b = 10 cm.

Perimeter of the triangle = 42 cm

$a + b + c = 42$

$18 + 10 + c = 42$

$c = 42 - 28 = 14\ cm$

2s =42 cm, i.e., s = 21 cm,

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-18)(21-10)(21-14)}$

$= \sqrt{21 \times 3 \times 11 \times 7}$

$= \sqrt{3 \times 7 \times 3 \times 11 \times 7} = 3 \times 7 \times \sqrt{11}\ cm^2 = 21\sqrt{11}\ cm^2$

5. **Sides of a triangle are in the ratio of 12 : 17 : 25 and its perimeter is 540cm. Find its area.**

Sol: The ratio of sides=12 : 17 : 25

Let the sides are $a = 12x, b = 17x, c = 25x$

Perimeter = 540cm

$12x + 17x + 25x = 540$

$54x = 540$

$x = 10$

$a = 12 \times 10 = 120cm\ , b = 17 \times 10 = 170\ cm, c = 25 \times 10 = 250\ cm$

2s=540cm

s=270 cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-120)(270-170)(270-250)}$

$= \sqrt{270 \times 150 \times 100 \times 20}$

$= \sqrt{3 \times 3 \times 3 \times 10 \times 3 \times 5 \times 5 \times 2 \times 10 \times 10 \times 2 \times 10}$

$= 3 \times 3 \times 2 \times 5 \times 10 \times 10 = 9000\ cm^2$

6. **An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.**

Sol: $a = 12\ cm\ , b = 12\ cm$

Perimeter= 30 cm

$a + b + c = 30$

$12 + 12 + c = 30$

$24 + c = 30$

$c = 30 - 24 = 6\ cm$

$2s = 30\ cm \Rightarrow s = 15\ cm$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-12)(15-12)(15-6)}$

$= \sqrt{15 \times 3 \times 3 \times 9} = 9\sqrt{15}\ cm^2$

Proof of Heron's formula:

$2s = P = a + b + c$

From ΔABD

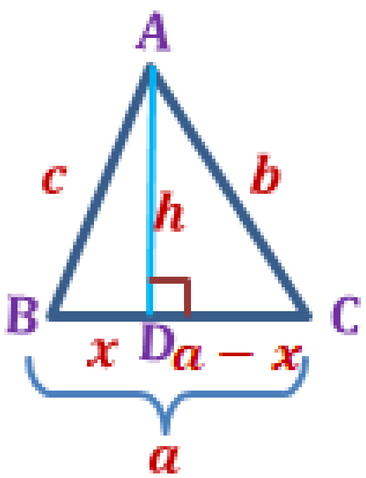
$x^2 + h^2 = c^2$

$h^2 = c^2 - x^2 \rightarrow (1)$

From ΔADC

$(a-x)^2 + h^2 = b^2$

$h^2 = b^2 - (a-x)^2 \rightarrow (2)$



From (1) and (2)

$c^2 - x^2 = b^2 - (a-x)^2$

$c^2 - x^2 = b^2 - a^2 + 2ax - x^2$

$c^2 = b^2 - a^2 + 2ax$

$x = \dfrac{a^2 + c^2 - b^2}{2a}$

Substitute x value in (1)

$h^2 = c^2 - x^2 = (c+x)(c-x)$

$= \left(c + \dfrac{a^2+c^2-b^2}{2a}\right)\left(c - \dfrac{a^2+c^2-b^2}{2a}\right)$

$= \left(\dfrac{2ac + a^2 + c^2 - b^2}{2a}\right)\left(\dfrac{2ac - a^2 - c^2 + b^2}{2a}\right)$

$= \left(\dfrac{(a+c)^2 - b^2}{2a}\right)\left(\dfrac{b^2 - (a-c)^2}{2a}\right)$

$= \left[\dfrac{(a+c+b)(a+c-b)}{2a}\right]\left[\dfrac{(b+a-c)(b-a+c)}{2a}\right]$

$= \left[\dfrac{2s(2s-2b)}{2a}\right]\left[\dfrac{(2s-2c)(2s-2a)}{2a}\right]$

$= \left[\dfrac{16s(s-a)(s-b)(s-c)}{4a^2}\right]$

$= \dfrac{4s(s-a)(s-b)(s-c)}{a^2}$

$h = \sqrt{\dfrac{4s(s-a)(s-b)(s-c)}{a^2}}$

$$h = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$$

Area of the triangle $= \frac{1}{2} \times base \times height$

$$= \frac{1}{2} \times a \times \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Heron (10 C.E. – 75 C.E.)

For VI to X all chapters notes visit my website

https://sureshmathsmaterial.com/

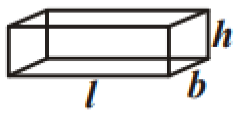
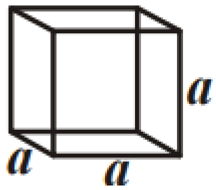
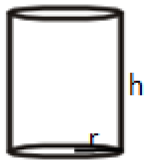
CHAPTER 11

IX-MATHEMATICS-NCERT-2023-24

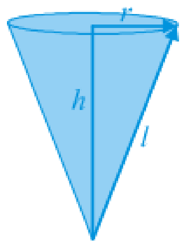
11. SURFACE AREAS AND VOLUMES (Notes)

PREPARED BY: BALABHADRA SURESH

1.

S.N	Figure	Name of the solid and Nomenclature	Lateral/Curved surface area L.S.A/C.S.A($unit^2$)	Total surface are T.S.A($unit^2$)	Volume ($unit^3$)
1		**Cuboid** $l = length$ b = breadth h = height	$2h(l+b)$	$2(lb+bh+lh)$	lbh
2		**Cube** $a = side$	$4a^2$	$6a^2$	a^3
3		**Regular circular Cylinder** $r = radius$ h = height	$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$

Surface Area of a Right Circular Cone



Base radius $= r$, height $= h$, slant height $= l$

$l^2 = h^2 + r^2 \Rightarrow l = \sqrt{h^2 + r^2}$

Curved Surface Area of a Cone $= \pi rl$

Total Surface Area of a Cone $= \pi rl + \pi r^2 = \pi r(l + r)$

Example1: Find the curved surface area of a right circular cone whose slant height is 10 cm and base radius is 7 cm

Sol: $l = 10\ cm, \quad r = 7\ cm$

Curved Surface Area of a Cone $= \pi rl = \frac{22}{7} \times 7 \times 10 = 220\ cm^2$

Example 2 : The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone (Use p = 3.14).

Solution : Here, h = 16 cm and r = 12 cm.

$l = \sqrt{h^2 + r^2} = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20\ cm$

Curved Surface Area $= \pi rl = 3.14 \times 12 \times 20 = 753.6\ cm^2$

Total Surface Area of a Cone $= \pi r(l + r) = 3.14 \times 12 \times (20 + 12)$

$= 3.14 \times 12 \times 32 = 1205.76\ cm^2$

Example 3 : A corn cob (see Fig. 11.5), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Sol: $r = 2.1\ cm, h = 20\ cm$

$l = \sqrt{h^2 + r^2} = \sqrt{2.1^2 + 20^2} = \sqrt{4.41 + 400} = \sqrt{404.41} = 20.11\ cm$

The curved surface area of the corn cob $= \pi rl = \frac{22}{7} \times 2.1 \times 20.11$

$= 22 \times 0.3 \times 20.11 = 132.726\ cm^2$

Number of grains of corn on 1 cm^2= 4

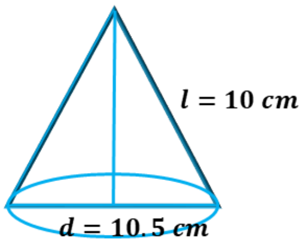
Number of grains on the entire curved surface of the cob $= 132.726 \times 4 = 530.904 \approx 531$

So, there would be approximately 531 grains of corn on the cob.

EXERCISE 11.1

1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol: Diameter$(d) = 10.5\ cm$



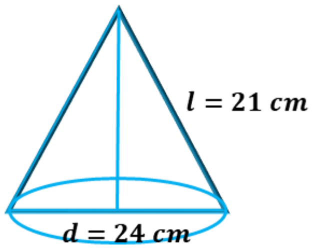
Radius$(r) = \frac{10.5}{2} cm$

Slant height$(l) = 10\ cm$

Curved Surface Area of Cone $= \pi rl = \frac{22}{7} \times \frac{10.5}{2} \times 10 = 165\ cm^2$.

2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Sol: Slant height$(l) = 21\ cm$



Diameter$(d) = 24\ cm$

Radius$(r) = \frac{24}{2} = 12cm$

Total Surface Area of a Cone $= \pi r(l + r) = \frac{22}{7} \times 12 \times (21 + 12)$

$= \frac{22}{7} \times 12 \times 33$

$= \frac{8712}{7} = 1244.57\ m^2$

3. Curved surface area of a cone is 308 cm² and its slant height is 14 cm. Find (i) radius of the base and (ii) total surface area of the cone.

Sol: Slant height$(l) = 14$ cm

Curved surface area of a cone $= 308$ cm^2

$\pi r l = 308$

$\frac{22}{7} \times r \times 14^2 = 308$

$r = \frac{308}{22 \times 2} = 7\ cm$

Total Surface Area of Cone $= \pi r(l + r) = \frac{22}{7} \times 7 \times (14 + 7) = 22 \times 21 = 462\ cm^2$

4. A conical tent is 10 m high and the radius of its base is 24 m. Find (i) slant height of the tent. (ii) cost of the canvas required to make the tent, if the cost of 1 m² canvas is ₹ 70.

Sol: Height$(h) = 10\ m$, radius$(r) = 24\ m$

(i)Slant height of the tent $= l = \sqrt{h^2 + r^2} = \sqrt{10^2 + 24^2} = \sqrt{100 + 576} = \sqrt{676} = 26\ m$

(ii) Curved surface area of the cone $= \pi r l = \frac{22}{7} \times 24 \times 26 = \frac{13728}{7}\ m^2$

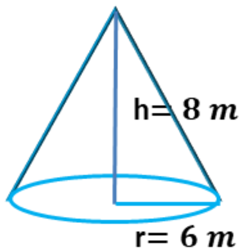
The cost of 1 m² canvas = ₹ 70

The cost of the required canvas = ₹ 70 × $\frac{13728}{7}$ = ₹137280

5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use p = 3.14).

Sol: Radius of cone$(r) = 6\ m$

Height$(h) = 8\ m$



Slant height of the tent $= l = \sqrt{h^2 + r^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$
$= \sqrt{100} = 10m$

Curved surface area $= \pi r l = 3.14 \times 6m \times 10m = 188.4\ m^2$

Width of the tarpaulin is 3m and area of the tarpaulin=188.4 m^2

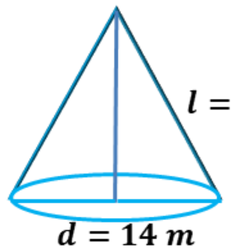
Length of tarpaulin $= \frac{188.4}{3} = 62.8\ m$

The extra length of material=20 cm=0.2 m

Required length of tarpaulin=62.8 m+0.2 m=63 m

6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹ 210 per 100 m² .

Sol: Slant height of cone$(l) = 25\ m$



Diameter$(d) = 14\ m$

Radius$(r) = 7\ m$

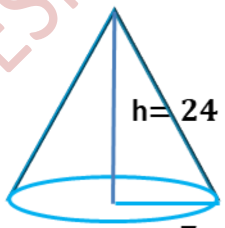
Curved surface area $= \pi r l = \frac{22}{7} \times 7m \times 25m = 550\ m^2$

Cost of white washing per 100 m^2=₹210

Total cost for white washing the tomb $= ₹\ \frac{210}{100} \times 550 = ₹21 \times 55 = ₹1155$

7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol: Radius of the cap (cone) =r=7 cm



Height of the cap (cone) =h=24 cm

Slant height of the cap $= l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\ \text{cm}$

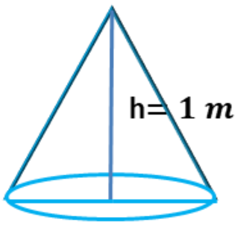
L. S. A of cap $= \pi r l$

$= \frac{22}{7} \times 7\text{cm} \times 25\text{cm} = 22 \times 25\text{cm}^2 = 550\text{cm}^2$

Area of the sheet required to make 1cap $= 550\ \text{cm}^2$.

Area of the sheet required to make 10 such caps $= 10 \times 550\ \text{cm}^2 = 5500\ \text{cm}^2$

8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is ₹ 12 per m² , what will be the cost of painting all these cones? (Use p = 3.14 and take $\sqrt{1.04}$= 1.02)



Sol: Diameter$(d) = 40\ cm = 0.4\ m$

Radius$(r) = 0.2\ m$

Height$(h) = 1$ m

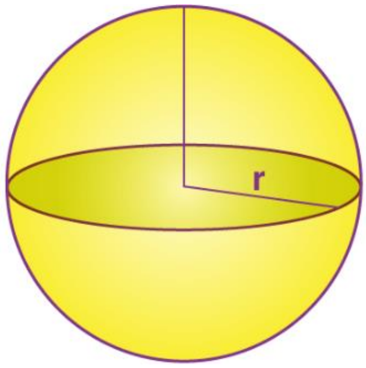
Slant height $= l = \sqrt{r^2 + h^2} = \sqrt{(0.2)^2 + 1^2} = \sqrt{0.04 + 1} = \sqrt{1.04} = 1.02\ m$

Curved surface area $= \pi rl = 3.14 \times 0.2 \times 1.02 = 0.64056\ m^2$

Curved surface area of 50 cones $= 50 \times 0.64056 = 32.028\ m^2$

Cost of painting per 1 m^2 = ₹ 12

Cost of painting for all cones = ₹ 12 × 32.028 = ₹384.336



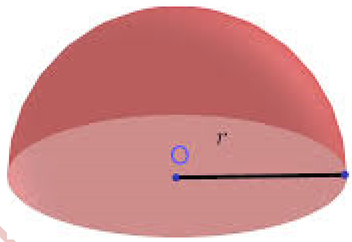
Surface Area of a Sphere

Surface Area of a Sphere $= 4\pi r^2$

Hemi sphere

Curved Surface Area of a Hemisphere $= 2\pi r^2$

Total Surface Area of a Hemisphere $= 3\pi r^2$



Example 4 : Find the surface area of a sphere of radius 7 cm.

Sol: $r = 7\ cm$

Surface Area of a Sphere $= 4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 = 616\ cm^2$

Example 5 : Find (i) the curved surface area and (ii) the total surface area of a hemisphere of radius 21 cm.

Sol: $r = 21\ cm$

(i) Curved Surface Area of a Hemisphere $= 2\pi r^2 = 2 \times \frac{22}{7} \times 21 \times 21 = 2772\ cm^2$

(ii) Total Surface Area of a Hemisphere $= 3\pi r^2 = 3 \times \frac{22}{7} \times 21 \times 21 = 4158\ cm^2$

Example 6 : The hollow sphere, in which the circus motorcyclist performs his stunts, has a diameter of 7 m. Find the area available to the motorcyclist for riding.

Solution : Diameter of the sphere(d) = 7 m

Radius$(r) = \frac{7}{2}\ m$

Surface Area of a Sphere $= 4\pi r^2 = \cancel{4}^{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} = 22 \times 7 = 154\ m^2$

Example 7 : A hemispherical dome of a building needs to be painted (see Fig. 11.9). If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹ 5 per 100 cm² .

Sol: circumference of the dome = 17.6 m

$2\pi r = 17.6$

$2 \times \frac{22}{7} \times r = 17.6$

$$r = \frac{17.6 \times 7}{2 \times 22} = 0.4 \times 7 = 2.8\ m$$

The curved surface area of the dome $= 2\pi r^2 = 2 \times \frac{22}{7} \times 2.8 \times 2.8 = 49.28\ m^2$

Cost of painting per 100 cm² = ₹ 5.

Cost of painting per 1m² = ₹ 5×100=₹500

Cost of painting the whole dome=₹500×49.28=₹24640

EXERCISE 11.2

1. Find the surface area of a sphere of radius:
(i) 10.5 cm

Sol: Radius$(r) = 10.5\ m$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{10.5}^{1.5} \times 10.5$

$= 88 \times 1.5 \times 10.5 = 1386\ m^2$

(ii) 5.6 cm

Sol: Radius$(r) = 5.6\ cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{5.6}^{0.8} \times 5.6$

$= 88 \times 0.8 \times 5.6 = 394.24\ cm^2$

(iii) 14 cm

Sol: Radius$(r) = 14\ cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{14}^{2} \times 14$

$= 88 \times 2 \times 14 = 2464\ cm^2$

2. Find the surface area of a sphere of diameter:
(i) 14 cm

Sol: Diameter$(d) = 14$ cm, Radius$(r) = 7cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7$

$= 88 \times 7 = 616\ cm^2$

(ii) 21 cm

Sol: Diameter$(d) = 21$ cm, Radius$(r) = \frac{21}{2} cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{22}{\cancel{7}} \times \frac{\cancel{21}^3}{\cancel{2}} \times \frac{21}{\cancel{2}}$

$= 22 \times 63 = 1386\ cm^2$

(iii) 3.5m

Sol: Diameter$(d) = 3.5 = \frac{35}{10} = \frac{7}{2}$cm, Radius$(r) = \frac{7}{4} cm$

Surface Area of the Sphere $= 4\pi r^2 = 4 \times \frac{\cancel{22}^{11}}{\cancel{7}} \times \frac{\cancel{7}}{4} \times \frac{7}{4_2}$

$= \frac{11 \times 7}{2} = \frac{77}{2} = 38.5\ cm^2$

3. Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)

Sol: Radius$(r) = 10\ cm$

Total surface area of a hemisphere $= 3\pi r^2 = 3 \times 3.14 \times 10 \times 10 = 942\ cm^2$

4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Sol: Radius of balloon before pumping $(r) = 7\ cm$

Radius of balloon after pumping (R) $= 14\ cm$

Ratio of the surface areas of the balloon $= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2} = \frac{7 \times 7}{14 \times 14} = \frac{1}{4} = 1:4$

5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of tin-plating it on the inside at the rate of ₹ 16 per 100 cm² .

Sol: Inner diameter(d) $= 10.5\ cm$

Inner radius$(r) = \frac{10.5}{2} = 5.25\ cm$

CSA of hemispherical bowl $= 2\pi r^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{7.5} \times 5.25$

$= 44 \times 0.75 \times 5.25 = 173.25\ cm^2$

The cost of tinplating the bowl per 100 cm²= ₹16 per

Total cost of tinplating to the boul $= ₹16 \times \frac{173.25}{100} = ₹\frac{2772}{100} = ₹27.72$

6. Find the radius of a sphere whose surface area is 154 cm² .

Sol: The surface area of the sphere $= 154\ \text{cm}^2$

$4\pi r^2 = 154\text{cm}^2$

$4 \times \frac{22}{7} \times r^2 = 154\text{cm}^2$

$r^2 = \frac{154 \times 7\text{cm}^2}{4 \times 22}$

$r^2 = \frac{49}{4}\text{cm}^2$

$r = \frac{7}{2} = 3.5\ cm$

Radius of the sphere=3.5 cm

7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol: Radius of moon $= r$ and radius of earth $= R$

The diameter of the moon = one fourth of the diameter of the earth'

$2r = \frac{1}{4} \times 2R$

$\frac{r}{R} = \frac{1}{4}$

The ratio of their surface areas $= \frac{4\pi r^2}{4\pi R^2} = \left(\frac{r}{R}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} = 1:16$

8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol: Inner radius of the bowl$(r) = 5\ cm$

Thickness of steel$(t) = 0.25\ cm$

Outer radius of the bowl$(R) = 5 + 0.25 = 5.25cm$

Outer CSA of the hemispherical bowl $= 2\pi R^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 5.25$

$= 44 \times 0.75 \times 5.25 = 173.25\ cm^2$

The outer curved surface area of the sphere $= 173.25\ cm^2$

9. A right circular cylinder just encloses a sphere of radius r (see Fig. 11.10). Find (i) surface area of the sphere, (ii) curved surface area of the cylinder, (iii) ratio of the areas obtained in (i) and (ii).

Sol: Radius of the cylinder = Radius of the spher $= r$

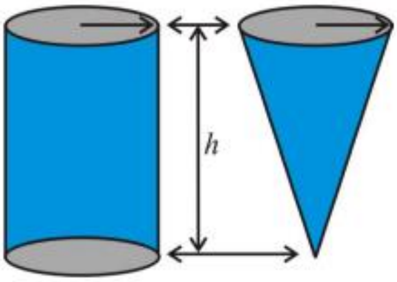
Height of the cylinder $= 2r$

(i)surface area of the sphere $= 4\pi r^2$

(ii)curved surface area of the cylinder $= 2\pi rh = 2\pi r \times 2r = 4\pi r^2$

(iii)ratio of the areas obtained in (i)and (ii) $= \frac{4\pi r^2}{4\pi r^2} = 1:1$

Volume of a Right Circular Cone



Volume of a Cone $= \frac{1}{3}\pi r^2 h$

Example 8 : The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.

Sol: $h = 21\ cm, l = 28\ cm$

$$l^2 = h^2 + r^2 \Rightarrow r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} = \sqrt{(28+21)(28-21)} = \sqrt{49 \times 7} = 7\sqrt{7}\ cm$$

Volume of a Cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times 7\sqrt{7} \times 7\sqrt{7} \times \cancel{21}^7 = 22 \times 49 \times 7 = 7546\ cm^3$

Example 9 : Monica has a piece of canvas whose area is 551 m2 . She uses it to have a conical tent made, with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting, amounts to approximately 1 m2 , find the volume of the tent that can be made with it.

Solution: Radius(r) = 7 m

The area of the canvas = 551 m^2

Area of the canvas lost in wastage = 1 m^2

The area of canvas available for making the tent=551-1=550 m^2

Curved surface area of tent = 550 m^2

$\pi rl = 550$

$\frac{22}{7} \times 7 \times l = 550$

$l = \frac{550 \times 7}{22 \times 7} = 25\ m$

$l^2 = h^2 + r^2 \Rightarrow h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24\ m$

The volume of a Cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times \cancel{24}^8 = 22 \times 7 \times 8 = 1232\ m^3$

EXERCISE 11.3

1. Find the volume of the right circular cone with
(i) radius 6 cm, height 7 cm

Sol: The volume of Cone $=\frac{1}{3}\pi r^2 h=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{6}^2\times 6\times\cancel{7}=22\times 12=264\ cm^3$

(ii) radius 3.5 cm, height 12 cm

Sol: The volume of Cone $=\frac{1}{3}\pi r^2 h=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{3.5}^{0.5}\times 3.5\times\cancel{12}^4=22\times 0.5\times 3.5\times 4=154\ cm^3$

2. Find the capacity in litres of a conical vessel with

(i) Radius 7 cm, slant height 25 cm

$1000\ cm^3 = 1\ l$

Sol: $r=7\ cm, l=25\ cm$

$h=\sqrt{l^2-r^2}=\sqrt{25^2-7^2}=\sqrt{625-49}=\sqrt{576}=24\ cm$

The capacity of the conical vessel $=\frac{1}{3}\pi r^2 h$

$=\frac{1}{\cancel{3}}\times\frac{22}{\cancel{7}}\times\cancel{7}\times 7\times\cancel{24}^8$

$=1232 cm^3$

$=1.232\ litres$

$1000\ m^3 = 1000\ l = 1\ kiololitre$

(ii) Height 12 cm, slant height 13 cm

Sol: $h=12\ cm, l=13\ cm$

$r=\sqrt{l^2-h^2}=\sqrt{13^2-12^2}=\sqrt{169-144}=\sqrt{25}=5\ cm$

The capacity of the conical vessel $=\frac{1}{3}\pi r^2 h$

$=\frac{1}{\cancel{3}}\times\frac{22}{7}\times 5\times 5\times\cancel{12}^4$

$=\frac{2200}{7}cm^3$

$=\frac{2200}{7\times 1000}\ l$

$=\frac{11}{35}\ litres$

3. The height of a cone is 15 cm. If its volume is 1570 cm³ , find the radius of the base. (Use p = 3.14)

Sol: Height$(h)=15\ cm$

Volume of the cone $=1570\ \text{cm}^3$

$\frac{1}{3}\pi r^2 h=1570\text{cm}^3$

$\frac{1}{3}\times\frac{22}{7}\times r^2\times 15=1570$

$r^2=\frac{1570\times 7\times 3}{22\times 15}=100$

$r=10\ cm$

The radius of the base=10 cm

4. If the volume of a right circular cone of height 9 cm is 48πcm³ , find the diameter of its base.

Sol: Height$(h)=9\ cm$

The volume of the cone=48πcm³

$\frac{1}{3}\pi r^2 h = 48\pi$

$\frac{1}{3} \times \pi \times r^2 \times 9 = 48\pi$

$r^2 = \frac{48 \times 3}{9} = 16$

$r = 4\ cm$

The diameter of circular cone base=2×4=8 cm

5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kilolitres?

Sol: Diameter (d) = 3.5 m

Radius$(r) = \frac{3.5}{2} = 1.75\ m$

Depth$(h) = 12\ m$

Volume of the conical $pit = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{1.75}^{0.25} \times 1.75 \times \cancel{12}^{4}$

$= 22 \times 0.25 \times 1.75 \times 4 = 38.5\ m^3 = 38.5$kilolitre.

6. The volume of a right circular cone is 9856 cm3 . If the diameter of the base is 28 cm, find (i) height of the cone (ii) slant height of the cone (iii)curved surface area of the cone

Sol: (i) Diameter of the cone$(d) = 28cm$

Radius$(r) = 14\ cm$

Volume of the cone = 9856 cm^3

$\frac{1}{3}\pi r^2 h = 9856$

$\frac{1}{3} \times \frac{22}{\cancel{7}} \times \cancel{14}^2 \times 14 \times h = 9856$

$h = \frac{9856 \times 3}{22 \times 14} = 48\ cm$

Height of the cone=48 cm

(ii)Slant height of the cone $= l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2} = \sqrt{196 + 2304} = \sqrt{2500} = 50\ cm$

(iii) CSA of the cone $= \pi r l = \frac{22}{\cancel{7}} \times \cancel{14}^2 \times 50 = 2200\ cm^2$

7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol: Radius of the cone$(r) = 5\ cm$

Height of the cone (h)=12 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \pi \times 5 \times 5 \times \cancel{12}^4 = 100\ \pi\ cm^3$

8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Sol: Radius of the cone$(r) = 12\ cm$

Height of the cone (h)=5 cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \pi \times 12 \times \cancel{12}^4 \times 5 = 240\ \pi\ cm^3$

The ratio of the volumes of the two solids obtained $= \frac{100\pi}{240\pi} = \frac{5}{12} = 5:12$

A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Sol: Diameter = 10.5 m

Radius(r) $= \frac{10.5}{2} = 5.25$ m

Height$(h) = 3m$

Slant height of the cone $= l = \sqrt{r^2 + h^2} = \sqrt{5.25^2 + 3^2} = \sqrt{27.5625 + 9} = \sqrt{36.5625} = 6.05\ m$

Volume of the conical heap $= \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 5.25 \times \cancel{3} = 22 \times 0.75 \times 5.25$

$= 86.625\ m^3$

CSA of cone $= \pi r l = \frac{22}{\cancel{7}} \times \cancel{5.25}^{0.75} \times 6.05 = 22 \times 0.75 \times 6.05 = 99.825\ m^2$

The area of the canvas required=99.825 m^2.

Volume of a Sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of hemisphere $= \frac{4}{3}\pi r^3$

Example 10 : Find the volume of a sphere of radius 11.2 cm.

Sol: Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 11.2 \times 11.2 \times 11.2 = 5887.32\ cm^3$

Example 11 : A shot-putt is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g per cm^3 , find the mass of the shot-putt.

Sol: Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{\cancel{7}} \times \cancel{4.9}^{0.7} \times 4.9 \times 4.9\ cm^3 = 493\ cm^3$

Mass of 1 cm^3 of metal= 7.8 g.

Mass of the shot-putt = 7.8 × 493 g=3845.44 g=3.85 kg

Example 12 : A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain?

Sol: Volume of hemispherical bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{\cancel{7}} \times \cancel{3.5}^{0.5} \times 3.5 \times 3.5 = 89.8\ cm^3$

EXERCISE 11.4

1. Find the volume of a sphere whose radius is

(i) 7 cm

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \times 7 = \frac{4312}{3} = 1437\frac{1}{3}\ cm^3$

(ii) 0.63 m

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{0.63}^{\cancel{0.09}^{0.03}} \times 0.63 \times 0.63 = 1.0478\ m^3 = 1.05\ m^3$

2. Find the amount of water displaced by a solid spherical ball of diameter

(i) 28 cm

Sol: Diameter$(d) = 28\ cm$

Radius$(r) = \frac{28\ cm}{2} = 14\ cm$

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{\cancel{7}} \times \cancel{14}^2 \times 14 \times 14$

$= \frac{34496}{3} cm^3 = 11498\frac{2}{3} cm^3$

The amount of water displaced by the solid $= 11498\frac{2}{3} cm^3$

(ii) 0.21 m

Sol: Diameter$(d) = 0.21\ cm$

Radius$(r) = \frac{0.21\ cm}{2} = 0.105\ cm$

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{0.105}^{\cancel{0.015}^{0.005}} \times 0.105 \times 10.105$

$= 0.004851 cm^3$

The amount of water displaced by the solid $== 0.004851 cm^3$

3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm³?

Sol: Diameter$(d) = 4.2\ cm$

Radius$(r) = \frac{4.2\ cm}{2} = 2.1\ cm$

Volume of metallic ball $= \frac{4}{3}\pi r^3 = \frac{4}{\not{3}} \times \frac{22}{\not{7}} \times \not{2.1}^{\not{0.3}^{0.1}} \times 2.1 \times 2.1 = 38.808\ cm^3$

The density of the metal per 1 cm³=8.9 g

The mass of the ball $= 38.808 \times 8.9\ g = 345.39\ g$

4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol: Let radius of the earth $= R$, and radius of moon $= r$

The diameter of the moon = one-fourth of the diameter of the earth.

$2r = \frac{1}{4} \times 2R \Rightarrow \frac{r}{R} = \frac{1}{4}$

$$\frac{\text{Volume of the moon}}{\text{Volume of the earth}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \frac{r^3}{R^3} = \left(\frac{r}{R}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

Required fraction $= \frac{1}{64}$

5. How many litres of milk can a hemispherical bowl of diameter 10.5 cm hold?

Sol: Diameter$(d) = 10.5\ cm$

Radius$(r) = \frac{10.5}{2} = 5.25\ cm$

Volume of hemispherical bowl $= \frac{2}{3}\pi r^3$

$= \frac{2}{\not{3}} \times \frac{22}{\not{7}} \times \not{5.25}^{\not{0.75}^{0.25}} \times 5.25 \times 5.25$

$= 303.1875\ cm^3$

$= \frac{303.1875}{1000}\ l = 0.3031875\ l = 0.303\ l\ (approx)$

0.303 litres of milk can be held in the bowl.

6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol: Inner radius of tank$(r) = 1\ m$

Thickness of iron sheet $= 1\ cm = 0.01\ m$

Outer radius of tank$(R) = 1m + 0.01m = 1.01m$

Volume of the iron $= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7} \times (1.01^3 - 1^3)$

$= \frac{44}{21} \times (1.030301 - 1) = \frac{44}{21} \times 0.030301 = \frac{1.33}{21} = 0.063\ m^3(approx)$

7. Find the volume of a sphere whose surface area is 154 cm^2 .

Sol: Surface area of the sphere $= 154\ cm^2$

$4\pi r^2 = 154$

$4 \times \frac{22}{7} \times r^2 = 154$

$r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$

$r = \frac{7}{2}$

Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{539}{3} = 179\frac{2}{3}\ cm^3$

8. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹ 4989.60. If the cost of white-washing is ₹ 20 per square metre, find the (i) inside surface area of the dome, (ii) volume of the air inside the dome.

Sol: Let inside radius of the dome $= r\ m$

Inside surface area of the dome $= 2\pi r^2 m^2$

The cost of white-washing per 1 m^2= ₹ 20

Total cost of white wash=₹ 20×$2\pi r^2$

$\therefore$ ₹ $20 \times 2\pi r^2 =$ ₹ 4989.60

$20 \times 2 \times \frac{22}{7} \times r^2 = 4989.60$

$r^2 = \frac{4989.60 \times 7}{20 \times 2 \times 22} = 39.69 = (6.3)^2$

$r = 6.3\ m$

(i)Inside surface area of the dome $= 2\pi r^2 = 2 \times \frac{22}{\cancel{7}} \times \cancel{6.3}^{0.9} \times 6.3 = 249.48\ m^2$

(ii)Volume of the air inside the dome $= \frac{2}{3}\pi r^3 = \frac{\cancel{2}}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{6.3}^{\cancel{0.9}\,0.3} \times 6.3 \times 6.3 = 523.9\ m^3$

9. Twenty seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S^1. Find the (i) radius r^1 of the new sphere, (ii) ratio of S and S^1

Sol: Volume of solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of new solid iron sphere $= \frac{4}{3}\pi (r^1)^3$

Volume of new solid iron sphere $= 27 \times$ Volume of solid iron sphere

$\frac{4}{3}\pi (r^1)^3 = 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3r)^3$

$r^1 = 3r$

(i) Radius of the new sphere $r^1=3r$

(ii)Ratio of S and $S^1 = 4\pi r^2 : 4\pi (r^1)^2 = r^2 : (r^1)^2 = r^2 : (3r)^2 = r^2 : 9r^2 = 1:9$

10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much medicine (in mm^3) is needed to fill this capsule?

Sol: Diameter of capsule$(d) = 3.5\ mm$

Radius of capsule$(r) = \frac{3.5}{2} = 1.75\ mm$

Volume of the capsule $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 1.75 \times 1.75 \times 1.75 = 22.46\ mm^3$

$\therefore$ 22.46 mm^3 medicine is needed to fill this capsule.

CHAPTER 12

IX-MATHEMATICS-NCERT(2023-24)

12. STATISTICS(Notes)

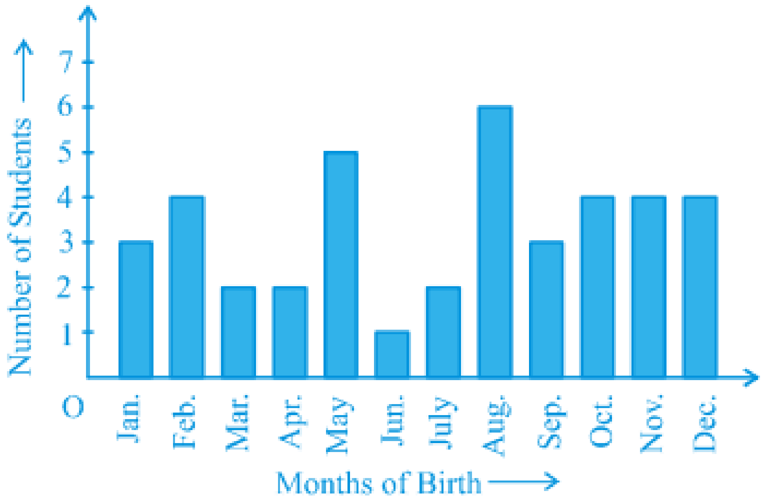
PREPARED BY: BALABHADRA SURESH

Graphical Representation of Data

(A) Bar Graphs:

A bar graph is a pictorial representation of data in which usually bars of uniform width are drawn with equal spacing between them on one axis (say, the x-axis), depicting the variable. The values of the variable are shown on the other axis (say, the y-axis) and the heights of the bars depend on the values of the variable.

Example 1 : In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:



Observe the bar graph given above and answer the following questions:

(i) How many students were born in the month of November?

Sol: 4 .

(ii) In which month was the maximum number of students born?

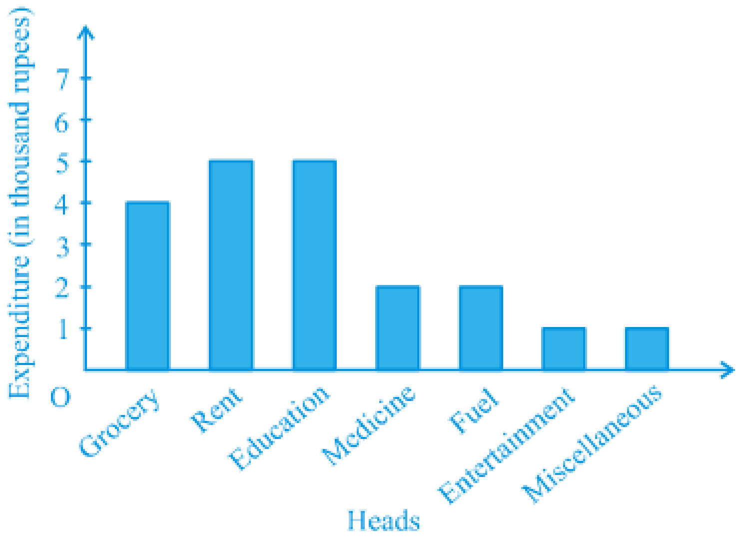
Sol: August.

Example 2 : A family with a monthly income of ` 20,000 had planned the following expenditures per month under various heads:

Heads	Expenditure (in thousand rupees)
Grocery	4
Rent	5
Education of children	5
Medicine	2
Fuel	2
Entertainment	1
Miscellaneous	1

Draw a bar graph for the data above.

Sol:



(B) Histogram

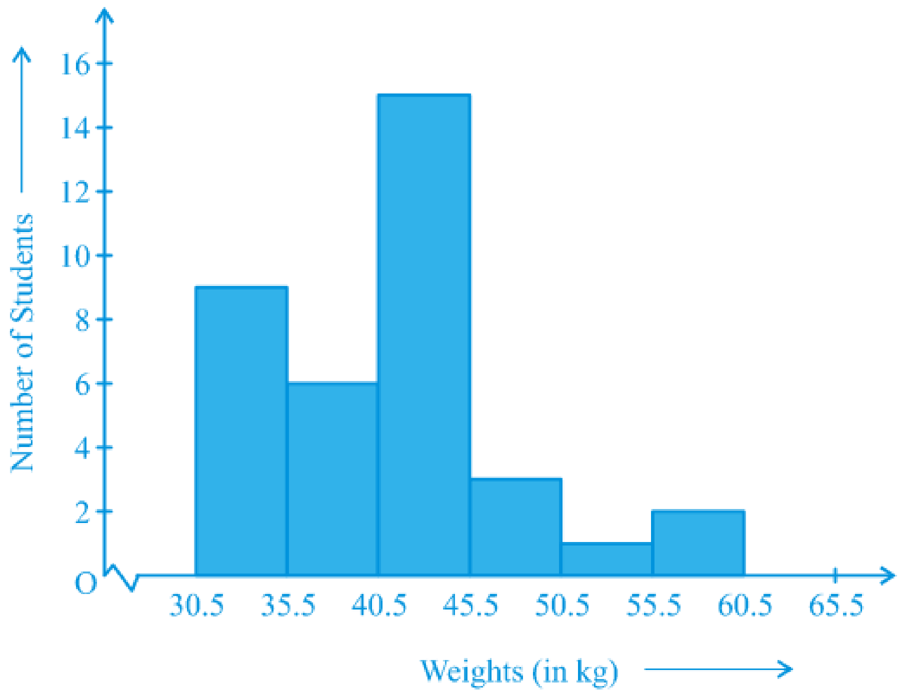
Histogram is a graphical representation of a grouped frequency distribution with continuous classes.

Ex: consider the frequency distribution Table representing the weights of 36 students of a class:

Weights (in kg)	30.5-35.5	35.5-40.5	40.5-45.5	45.5-50.5	50.5-55.5	55.5-60.5	Total
Number of students	9	6	15	3	1	2	36

Draw a histogram for the data above.

Sol:



Example 3 : A teacher wanted to analyse the performance of two sections of students in a mathematics test of 100 marks. Looking at their performances, she found that a few students got under 20 marks and a few got 70 marks or above. So she decided to group them into intervals of varying sizes as follows: 0 - 20, 20 - 30, . . ., 60 - 70, 70 - 100. Then she formed the following table:

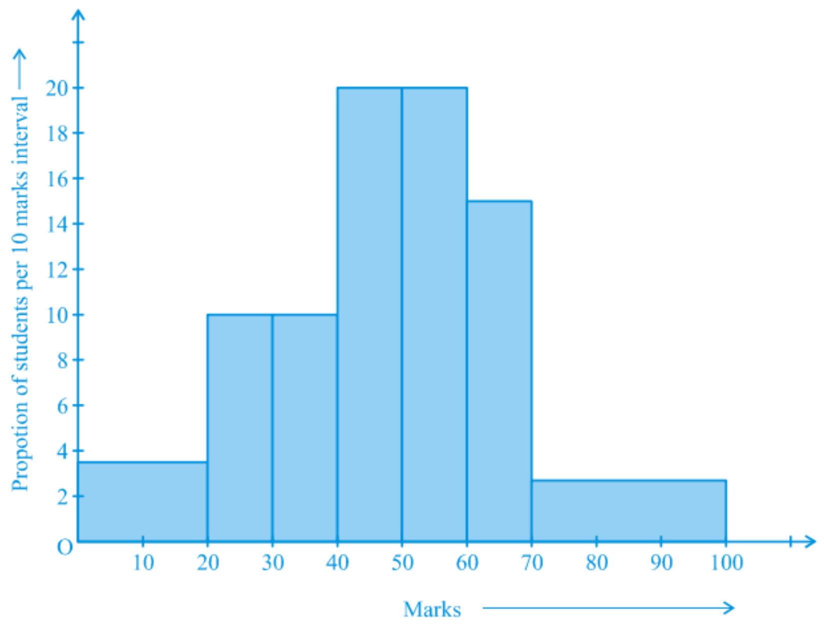
Marks	Number of students
0-20	**7**
20-30	**10**
30-40	**10**
40-50	**20**
50-60	**20**
60-70	**15**
70-above	**8**
Total	**90**

Draw a histogram for this table data above.

Sol: All class intervals are not equal.

So, we need to make certain modifications in the lengths of the rectangles.

Marks	**Number of students**	**Width of the class.**	**Length of the rectangle.**
0-20	7	20	$\frac{7}{20} \times 10 = \frac{7}{2} = 3.5$
20-30	10	10	$\frac{10}{10} \times 10 = 10$
30-40	10	10	$\frac{10}{10} \times 10 = 10$
40-50	20	10	$\frac{20}{10} \times 10 = 20$
50-60	20	10	$\frac{20}{10} \times 10 = 20$
60-70	15	10	$\frac{15}{10} \times 10 = 15$
70-100	8	30	$\frac{8}{30} \times 10 = \frac{8}{3} = 2.67$



(C) Frequency Polygon

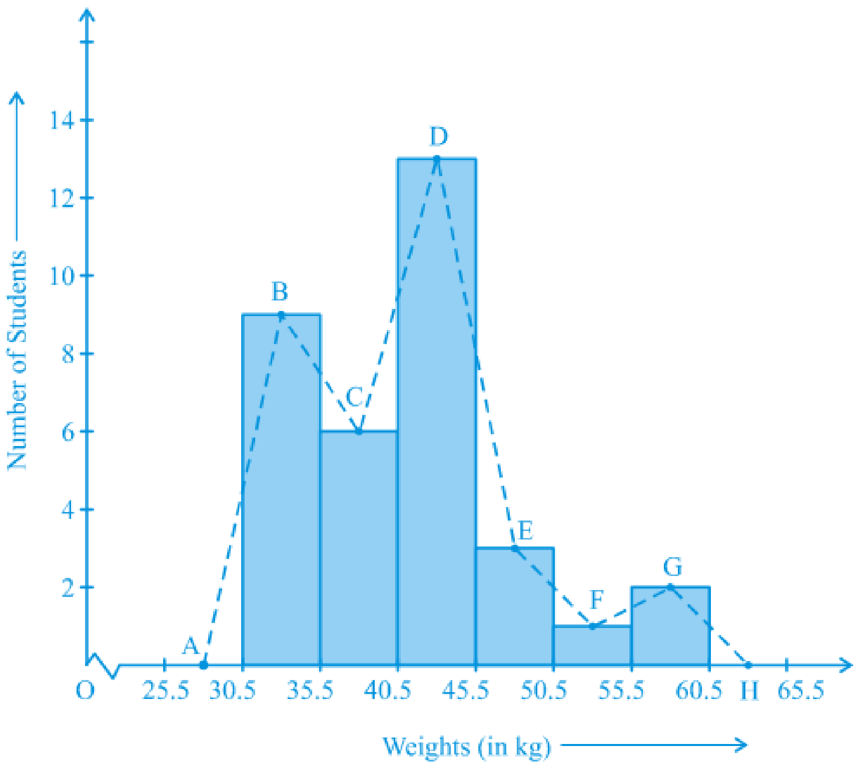
A frequency polygon is a line graph of class frequency plotted against class midpoint. It can be obtained by joining the midpoints of the tops of the rectangles in the histogram.

Example: Draw the frequency of polygon for the given table.

Weights (in kg)	30.5-35.5	35.5-40.5	40.5-45.5	45.5-50.5	50.5-55.5	55.5-60.5	Total
Number of students	9	6	15	3	1	2	36

Sol:

Weights (in kg)	Class mark(midpoint)	Number of students
30.5-35.5	33	9
35.5-40.5	38	6
40.5-45.5	43	15
45.5-50.5	48	3
50.5-55.5	53	1
55.5-60.5	58	2



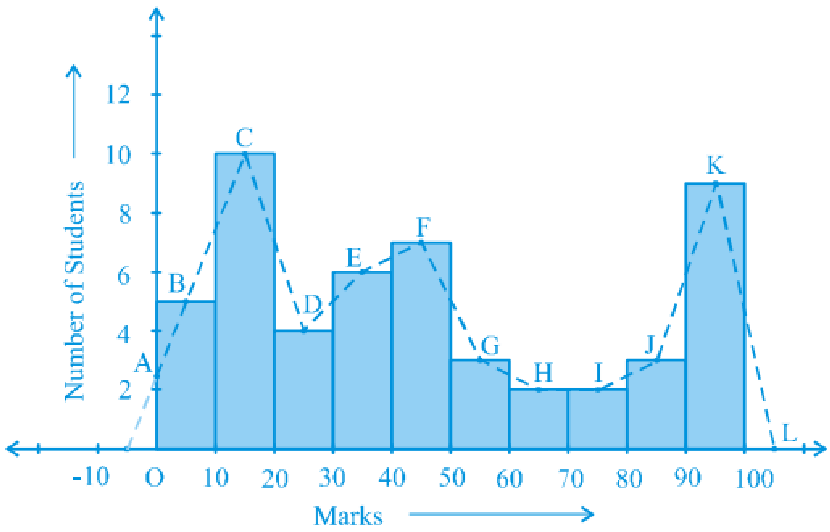
Example 4 : Consider the marks, out of 100, obtained by 51 students of a class in a test, given in Table.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100	Total
Number of students	5	10	4	6	7	3	2	2	3	9	51

Draw a frequency polygon corresponding to this frequency distribution table

Sol:

Marks	Class mark	Number of students
0-10	5	5
10-20	15	10
20-30	25	4
30-40	35	6
40-50	45	7
50-60	55	3
60-70	65	2
70-80	75	2
80-90	85	3
90-100	95	9



Frequency polygons without drawing histograms:

These mid-points of the class-intervals are called class-marks.

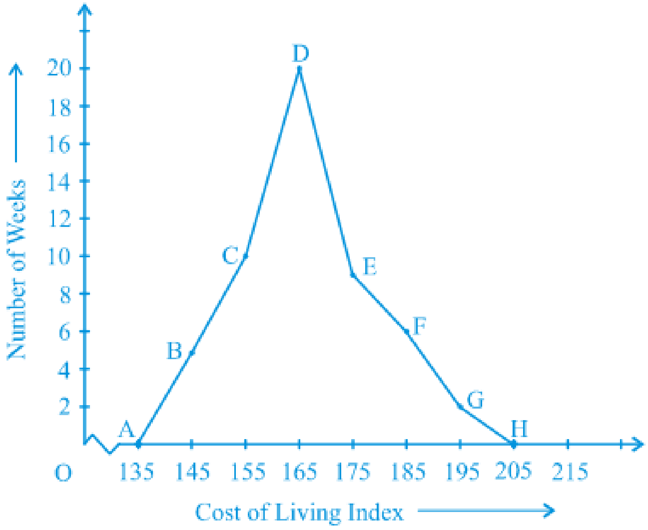
$$\text{Class mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Example 5 : In a city, the weekly observations made in a study on the cost of living index are given in the following table:

Cost of living index	140-150	150-160	160-170	170-180	180-190	190-200	Total
Number of weeks	5	10	20	9	6	2	52

Sol:

Classes	Class-marks	Frequency
140-150	145	5
150-160	155	10
160-170	165	20
170-180	175	9
180-190	185	6
190-200	195	2

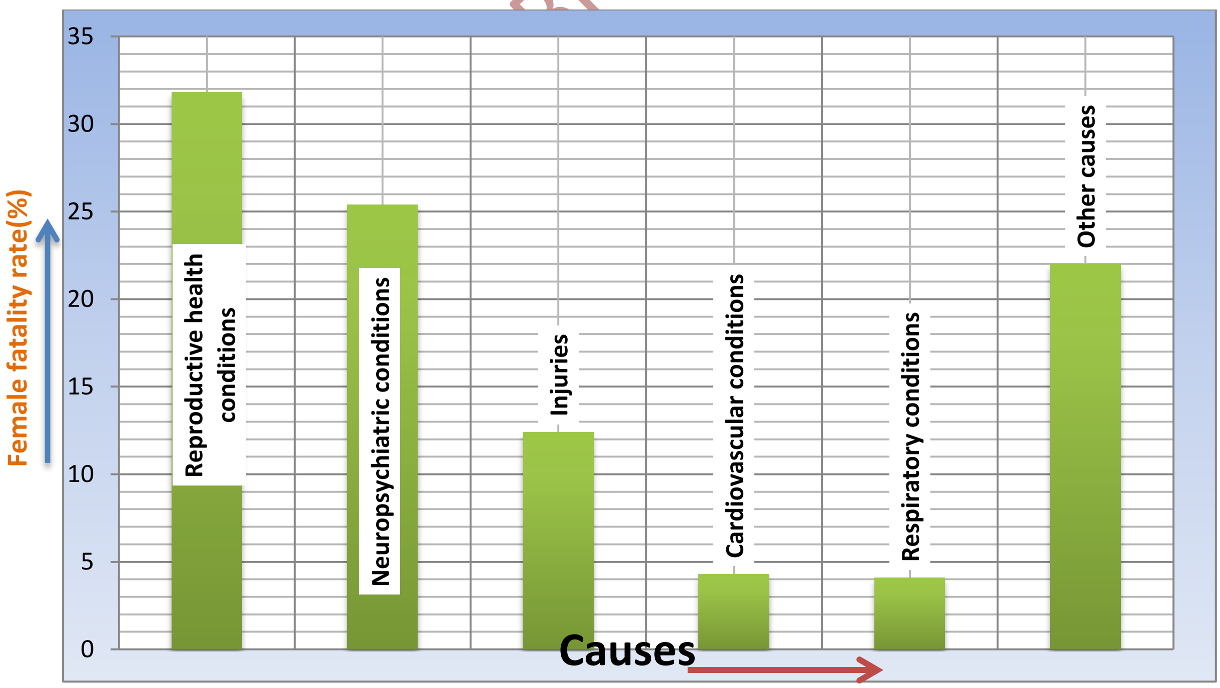


EXERCISE 12.1

1. A survey conducted by an organisation for the cause of illness and death among the women between the ages 15 - 44 (in years) worldwide, found the following figures (in %):

S. No	Causes	Female fatality rate (%)
1.	Reproductive health conditions	31.8
2.	Neuropsychiatric conditions	25.4
3.	Injuries	12.4
4.	Cardiovascular conditions	4.3
5.	Respiratory conditions	4.1
6.	Other causes	22.0

(i) Represent the information given above graphically.



(ii) Which condition is the major cause of women's ill health and death worldwide?

Sol: Reproductive health conditions.

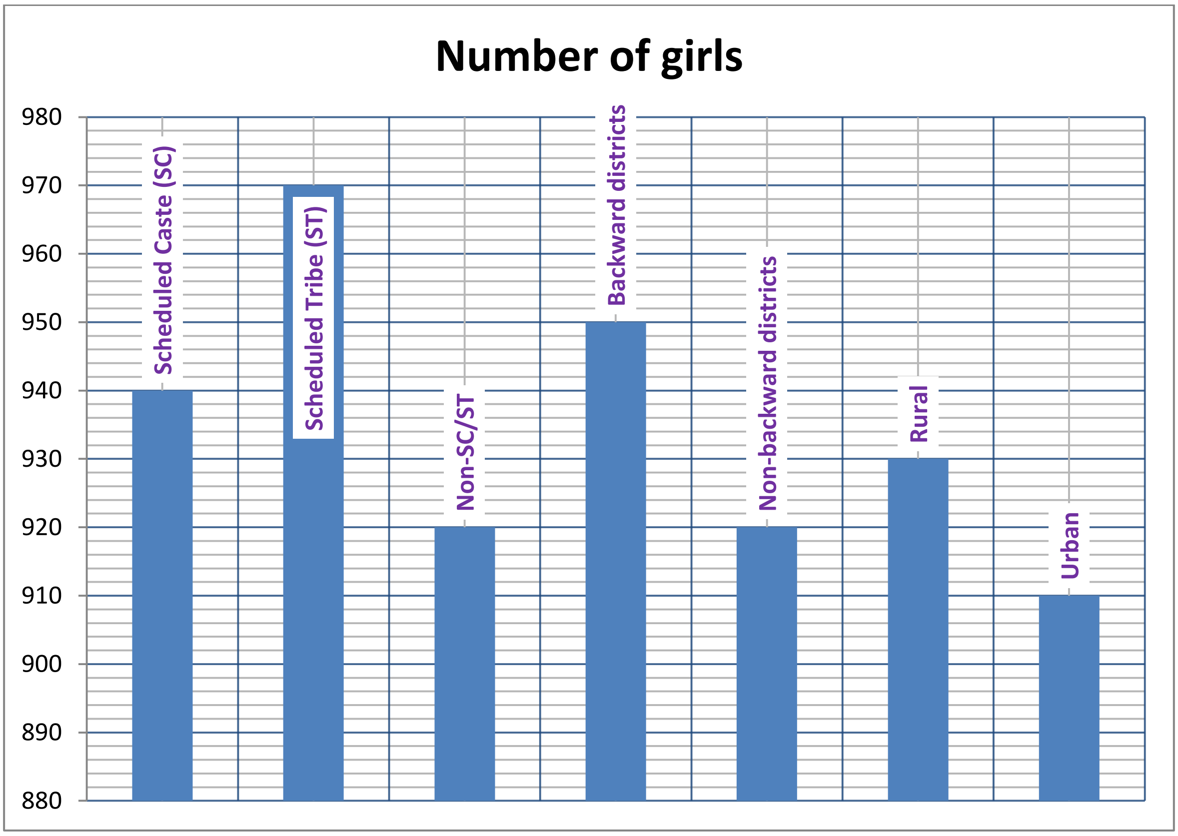
(iii) Try to find out, with the help of your teacher, any two factors which play a major role in the cause in (ii) above being the major cause.

Sol: Lack of awareness, lack of timely medical care, lack of hygiene and diet.

2. The following data on the number of girls (to the nearest ten) per thousand boys in different sections of Indian society is given below.

Section	Number of girls per thousand boys
Scheduled Caste (SC)	940
Scheduled Tribe (ST)	970
Non-SC/ST	920
Backward districts	950
Non-backward districts	920
Rural	930
Urban	910

(i) Represent the information above by a bar graph.



(ii) In the classroom discuss what conclusions can be arrived at from the graph.

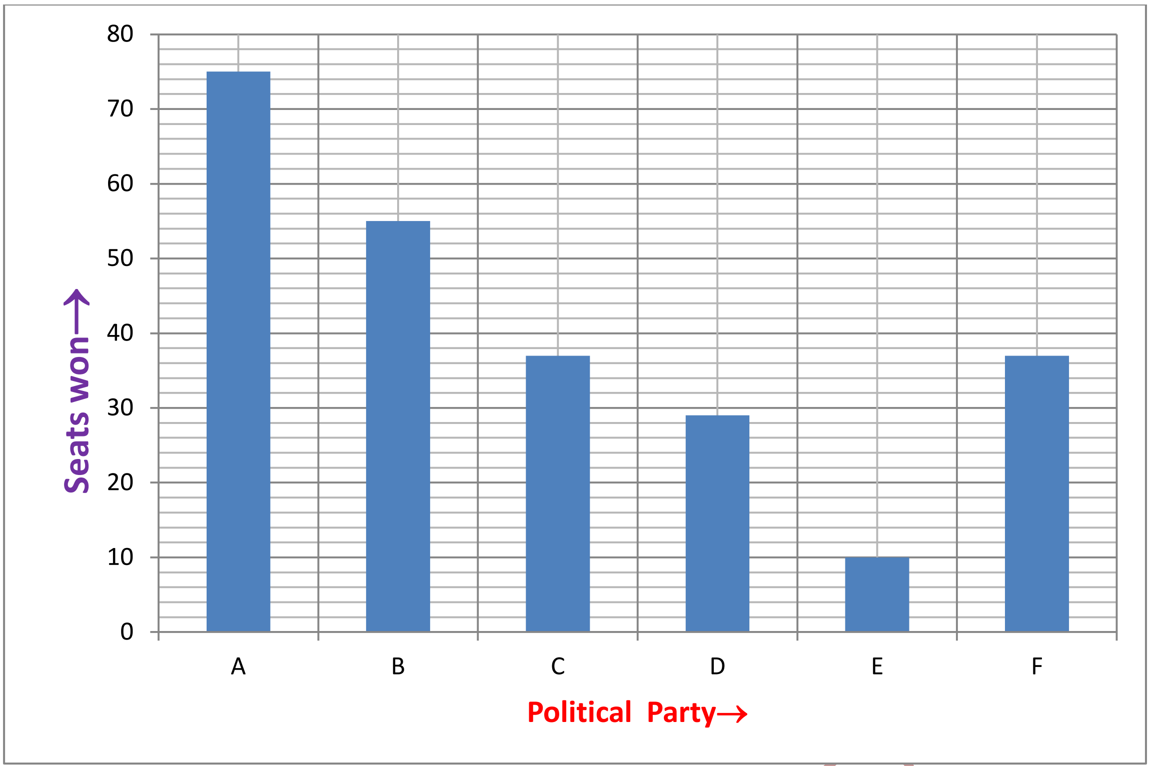
Sol: (a) The number of girls per thousand boys is lowest for urban category.

(b) The number of girls per thousand boys is highest for ST category.

3. Given below are the seats won by different political parties in the polling outcome of a state assembly elections:

Political Party	A	B	C	D	E	F
Seats Won	75	55	37	29	10	37

(i) Draw a bar graph to represent the polling results.



(ii) Which political party won the maximum number of seats?

Sol: Party A

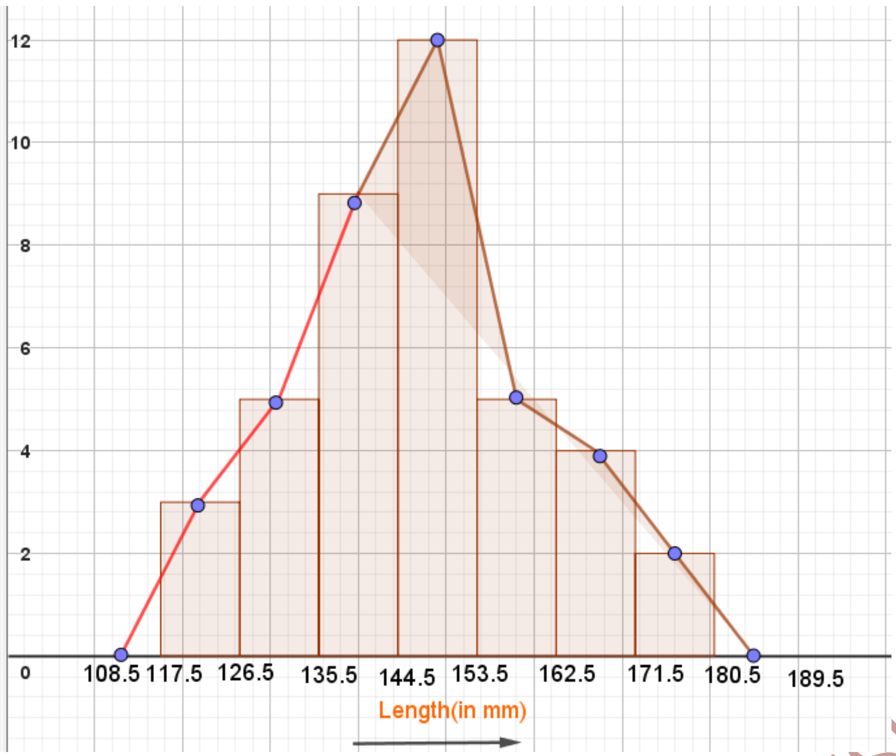
4. The length of 40 leaves of a plant are measured correct to one millimetre, and the obtained data is represented in the following table:

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

(i) Draw a histogram to represent the given data. [Hint: First make the class intervals continuous]

Sol:

Length (in mm)	Continuous class intervals	Number of leaves
118-126	117.5-126.5	3
127-135	126.5-135.5	5
136-144	135.5-144.5	9
145=153	144.5-153.5	12
154-162	153.5-162.5	5
163-171	162.5-171.5	4
172-180	171.5-180.5	2



(ii) Is there any other suitable graphical representation for the same data?

Sol: Frequency polygon.

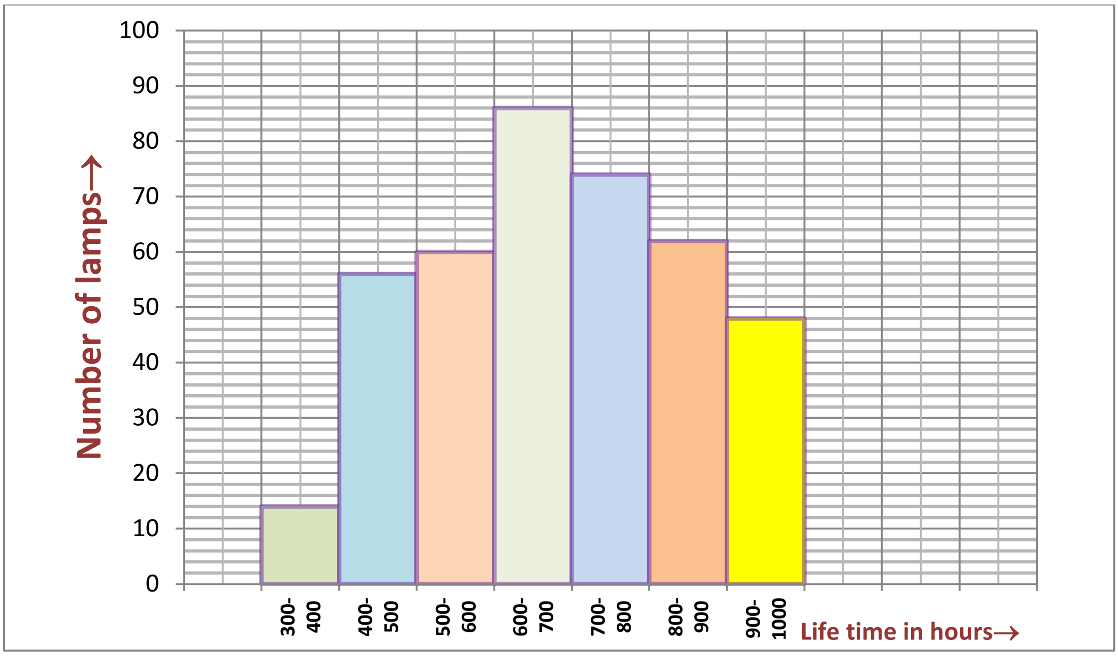
(iii) Is it correct to conclude that the maximum number of leaves are 153 mm long? Why?

Sol: No, The maximum number of leaves have their length lie between 144.5 mm and 153.5 mm.

5. The following table gives the life times of 400 neon lamps:

Life time (in hours)	Number of lamps
300 - 400	14
400 - 500	56
500 - 600	60
600 - 700	86
700 - 800	74
800 - 900	62
900 - 1000	48

(i) Represent the given information with the help of a histogram.



(ii) How many lamps have a life time of more than 700 hours?

Sol: Number of lamps a life time of more than 700 hours=74+62+48=184

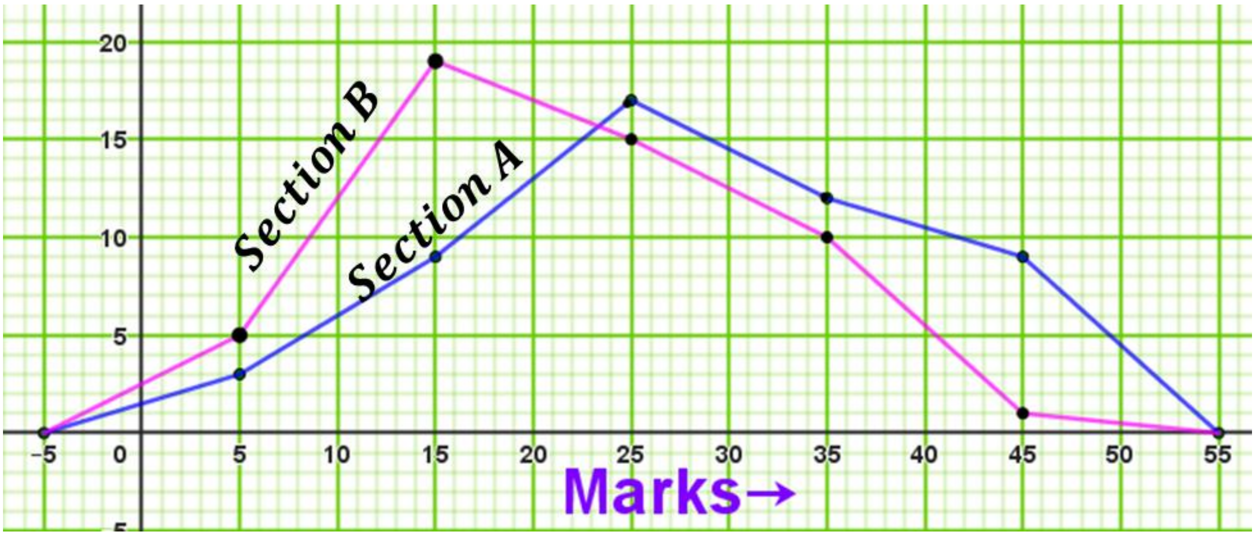
6. The following table gives the distribution of students of two sections according to the marks obtained by them:

Section A		Section B	
Marks	Frequency	Marks	Frequency
0 - 10	3	0 - 10	5
10 - 20	9	10 - 20	19
20 - 30	17	20 - 30	15
30 - 40	12	30 - 40	10
40 - 50	9	40 - 50	1

Represent the marks of the students of both the sections on the same graph by two frequency polygons. From the two polygons compare the performance of the two sections.

Sol:

Section A			Section B		
Marks	Class Mark	Frequency	Marks	Class Mark	Frequency
0-10	5	3	0-10	5	5
10-20	15	9	10-20	15	19
20-30	25	17	20-30	25	15
30-40	35	12	30-40	35	10
40-50	45	9	40-50	45	1



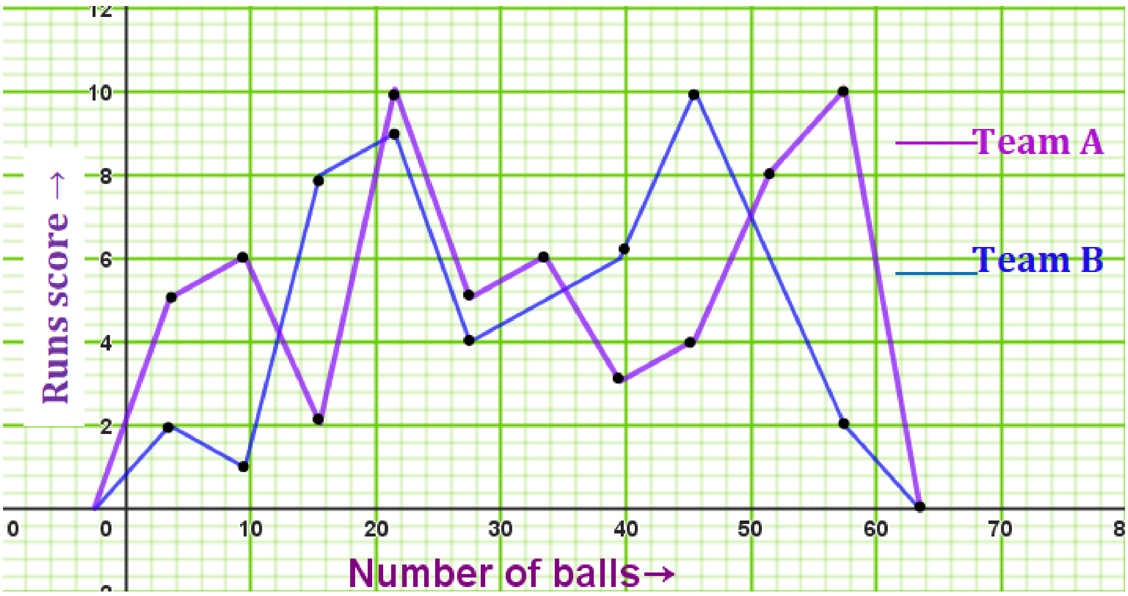
7. The runs scored by two teams A and B on the first 60 balls in a cricket match are given below:

Number of balls	Team A	Team B
1 - 6	2	5
7 - 12	1	6
13 - 18	8	2
19 - 24	9	10
25 - 30	4	5
31 - 36	5	6
37 - 42	6	3
43 - 48	10	4
49 - 54	6	8
55 - 60	2	10

Represent the data of both the teams on the same graph by frequency polygons.
[Hint : First make the class intervals continuous.]

Sol:

Number of balls	Continuous class intervals	Class Mark	Team A	Team B
1-6	0.5-6.5	3.5	2	5
7-12	6.5-12.5	9.5	1	6
13-18	12.5-18.5	15.5	8	2
19-24	18.5-24.5	21.5	9	10
25-30	24.5-30.5	27.5	4	5
31-36	30.5-36.5	33.5	5	6
37-42	36.5-42.5	39.5	6	3
43-48	42.5-48.5	45.5	10	4
49-54	48.5-54.5	51.5	6	8
55-60	54.5-60.5	57.5	2	10



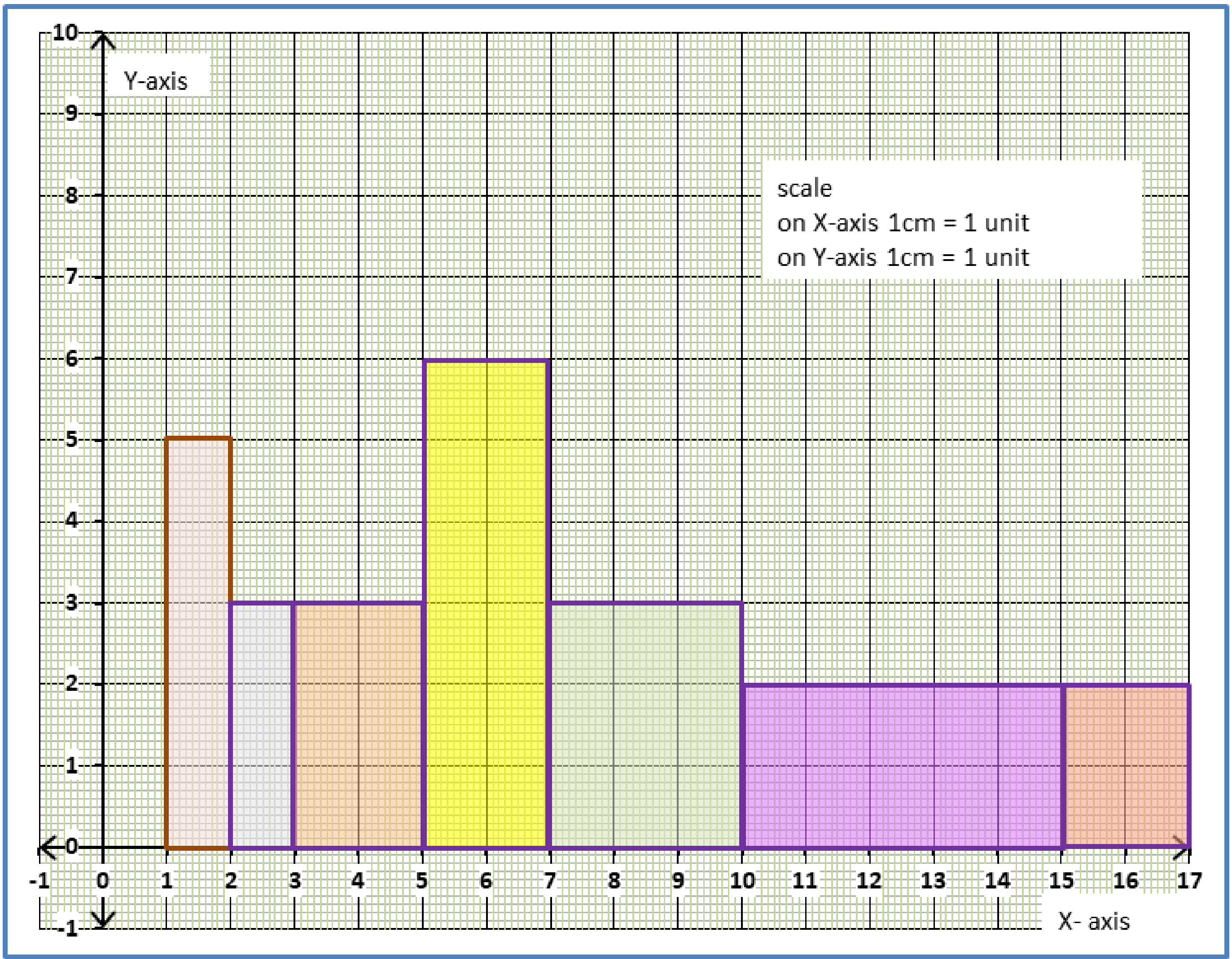
8. **A random survey of the number of children of various age groups playing in a park was found as follows:**

Age (in years)	**1-2**	**2-3**	**3-5**	**5-7**	**7-10**	**10-15**	**15-17**
Number of children	**5**	**3**	**6**	**12**	**9**	**10**	**4**

Draw a histogram to represent the data above.

Sol:

Age (in years)	Number of children	Width of the Class	Length of the rectangle
1-2	5	1	$\frac{5}{1}\times 1 = 5$
2-3	3	1	$\frac{3}{1}\times 1 = 3$
3-5	6	2	$\frac{6}{2}\times 1 = 3$
5-7	12	2	$\frac{12}{2}\times 1 = 6$
7-10	9	3	$\frac{9}{3}\times 1 = 3$
10-15	10	5	$\frac{10}{5}\times 1 = 2$
15-17	4	2	$\frac{4}{2}\times 1 = 2$



9. **100 surnames were randomly picked up from a local telephone directory and a frequency distribution of the number of letters in the English alphabet in the surnames was found as follows:**

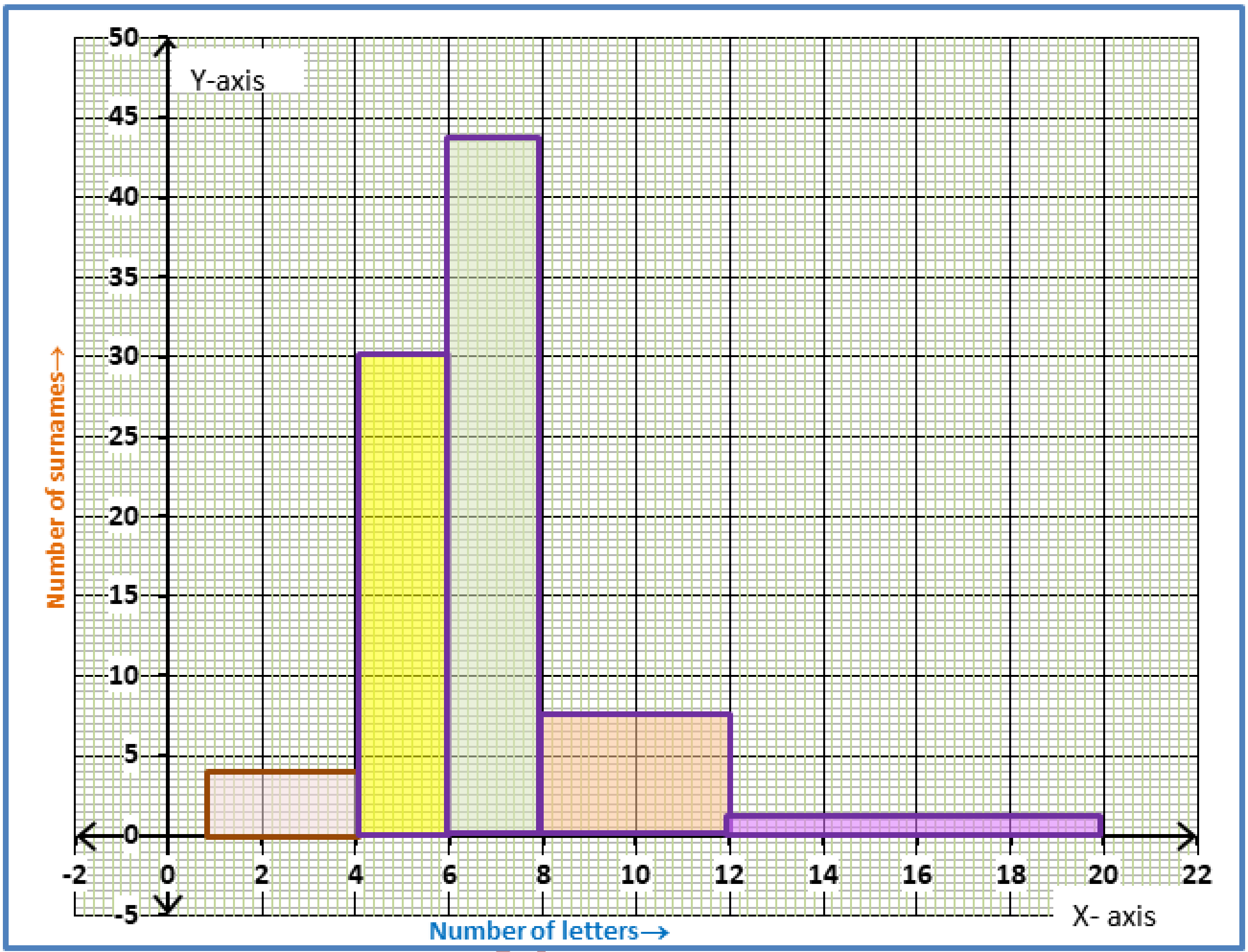
Number of letters	**1-4**	**4-6**	**6-8**	**8-12**	**12-20**
Number of surnames	**6**	**30**	**44**	**16**	**4**

(i) Draw a histogram to depict the given information. (ii) Write the class interval in which the maximum numbers of surnames lie.

Sol:

Number of letters	Number of surnames	Width of the class	Length of rectangle
1-4	6	3	$\frac{6}{3} \times 2 = 2 \times 2 = 4$
4-6	30	2	$\frac{30}{2} \times 2 = 15 \times 2 = 30$
6-8	44	2	$\frac{44}{2} \times 2 = 22 \times 2 = 44$
8-12	16	4	$\frac{16}{4} \times 2 = 4 \times 2 = 8$

12-20	4	8	$\frac{4}{8} \times 2 = \frac{8}{8} = 1$



APPENDIX 2

IX-MATHEMATICS-NCERT-2023-24

INTRODUCTION TO MATHEMATICAL MODELLING

PREPARED BY: BALABHADRA SURESH

1. A mathematical model is a mathematical relation that describes some real-life situation.
2. Mathematical models are used to solve many real-life situations like:
   - launching a satellite.
   - predicting the arrival of the monsoon
   - controlling pollution due to vehicles
   - reducing traffic jams in big cities

The process of mathematical modelling , its Advantages and Limitations

Step 1 : Formulation: Formulation involves the following three steps

(i) Stating the problem.

(ii) Identifying relevant factors.

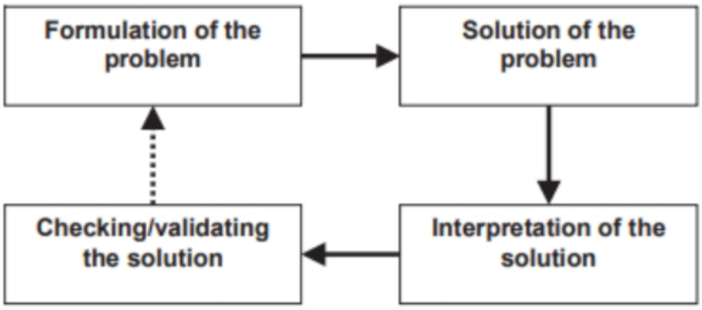
(iii) Mathematical Description,

Step 2 : Finding the solution.

Step 3 : Interpretating the solution.

Step 4 : Validating the solution.

A summary of the order in which the steps in mathematical modelling are carried out.



Example 1 : I travelled 432 kilometres on 48 litres of petrol in my car. I have to go by my car to a place which is 180 km away. How much petrol do I need?

Sol: **Formulation**: The amount of petrol we need varies directly with the distance we travel.

Petrol needed for travelling 432 km = 48 litres

Mathematical Description: Let x = distance I travel y = petrol I need

y varies directly with x

y = kx, where k is a constant.

y = 48, x = 432

$$k = \frac{y}{x} = \frac{48}{432} = \frac{1}{9}$$

$$y = kx \Rightarrow y = \frac{1}{9}x$$

Solution:We want to find the petrol we need to travel 180 kilometres.

$$y = \frac{1}{9} \times 180 = 20$$

Interpretation : Since y = 20,We need 20 litres of petrol to travel 180 kilometres.

Example 2 : Suppose Sudhir has invested ₹ 15,000 at 8% simple interest per year. With the return from the investment, he wants to buy a washing machine that costs ₹ 19,000. For what period should he invest ₹ 15,000 so that he has enough money to buy a washing machine?

Solution:

Step 1 : Formulation of the problem : We have to find the number of years.

$$\text{Mathematical Description : Simple intterest I} = \frac{Pnr}{100}$$

P = Principal,

n = Number of years,

r % = Rate of interest

I = Interest earned

P=15,000; I=19,000-15,000=4,000; r=8%; we want n

$$\frac{15000 \times 8 \times n}{100} = 4000$$

$$n = \frac{4000}{150 \times 8} = \frac{10}{3} = 3\frac{1}{3}$$

$$Solution\ of\ the\ problem: n = 3\frac{1}{3}\ years = 3\ years\ and\ 4\ months$$

Interpretation : Sudhir can buy a washing machine after 3 years and 4 months.

Example 3 : A motorboat goes upstream on a river and covers the distance between two towns on the riverbank in six hours. It covers this distance downstream in five hours. If the speed of the stream is 2 km/h, find the speed of the boat in still water.

Solution :

Step 1 : Formulation : We have to find the speed of the boat in still water.

Mathematical Description : Let us write x for the speed of the boat, t for the time taken and y for the distance travelled. Then y = tx→ (1)

Let d be the distance between the two places.

While going upstream:

The speed of the boat upstream= speed of the boat – speed of the river = (x – 2) km/h,

It takes 6 hours to cover the distance between the towns upstream

d = 6(x – 2) →(2)

The speed of the boat downstream = (x + 2) km/h

The boat takes 5 hours to cover the same distance downstream.

d = 5(x + 2)→ (3)

From (2) and (3), we have 5(x + 2) = 6(x –2) →(4)

Step 2 : Finding the Solution

5(x + 2) = 6(x −2)

$5x + 10 = 6x - 12$

$6x - 5x = 10 + 12$

$x = 22$

Step 3 : Interpretation: Since x = 22, therefore the speed of the motorboat in still water is 22 km/h.

EXERCISE A 2.1

In each of the following problems, clearly state what the relevant and irrelevant factors are while going through Steps 1, 2 and 3 given above.

1. Suppose a company needs a computer for some period of time. The company can either hire a computer for ₹ 2,000 per month or buy one for ₹ 25,000. If the company has to use the computer for a long period, the company will pay such a high rent, that buying a computer will be cheaper. On the other hand, if the company has to use the computer for say, just one month, then hiring a computer will be cheaper. Find the number of months beyond which it will be cheaper to buy a computer.

Sol: The relevant factors are the time period for hiring a computer, and the two costs given to us. We assume that there is no significant change in the cost of purchasing or hiring the computer. So, we treat any such change as irrelevant. We also treat all brands and generations of computers as the same, i.e. these differences are also irrelevant.

The expense of hiring the computer for x months is ₹ 2000x. If this becomes more than the cost of purchasing a computer, we will be better off buying a computer. So, the equation is 2000 x = 25000→ (1)

Step 2 : Solution : Solving (1), x = 25000 2000 = 12.5 Step 3 : Interpretation : Since the cost of hiring a computer becomes more after 12.5 months, it is cheaper to buy a computer, if you have to use it for more than 12 months.

2. Suppose a car starts from a place A and travels at a speed of 40 km/h towards another place B. At the same instance, another car starts from B and travels towards A at a speed of 30 km/h. If the distance between A and B is 100 km, after how much time will the cars meet?

Sol: Step1 : Formulation : We will assume that cars travel at a constant speed. So, any change of speed will be treated as irrelevant. If the cars meet after x hours, the first car would have travelled a distance of 40x km from A and the second car would have travelled 30x km, so that it will be at a distance of (100 – 30x) km from A. So the equation will be 40x = 100 – 30x, i.e., 70x = 100.

Step 2 : Solution : Solving the equation, we get x = $\frac{100}{70}$.

Step 3 : Interpretation : $\frac{100}{70}$ is approximately 1.4 hours. So, the cars will meet after 1.4 hours.

3. The moon is about 3,84,000 km from the earth, and its path around the earth is nearly circular. Find the speed at which it orbits the earth, assuming that it orbits the earth in 24 hours. (Use π = 3.14)

Sol: Step1 : Formulation : The speed at which the moon orbits the earth is $\frac{\text{Length of the orbit}}{\text{Time taken}}$.

Step 2 : Solution : Since the orbit is nearly circular, the length is 2 × p × 384000 km = 2411520 km

The moon takes 24 hours to complete one orbit.

So, speed = $\frac{2411520}{24} =$ = 100480 km/hour.

Step 3 : Interpretation : The speed is 100480 km/h.

4. A family pays ₹ 1000 for electricity on an average in those months in which it does not use a water heater. In the months in which it uses a water heater, the average electricity bill is ₹ 1240. The cost of using the water heater is ₹ 8.00 per hour. Find the average number of hours the water heater is used in a day.

Sol: Formulation : An assumption is that the difference in the bill is only because of using the water heater. Let the average number of hours for which the water heater is used = x Difference per month due to using water heater = ₹ 1240 – ₹ 1000 = ₹ 240 Cost of using water heater for one hour = ₹ 8 So, the cost of using the water heater for 30 days = 8 × 30 × x Also, the cost of using the water heater for 30 days = Difference in bill due to using water heater So, 240x = 240 Solution : From this equation, we get x = 1. Interpretation : Since x = 1, the water heater is used for an average of 1 hour in a day.

A2.3 Some Mathematical Models

Example 4 : Suppose you have a room of length 6 m and breadth 5 m. You want to cover the floor of the room with square mosaic tiles of side 30 cm. How many tiles will you need? Solve this by constructing a mathematical model.

Solution: Formulation : We have to consider the area of the room and the area of a tile for solving the problem. The side of the tile is 0.3 m.

Since the length is 6 m, we can fit in 6 /0.3 = 20 tiles along the length of the room in one row

Since the breadth of the room is 5 metres, we have 5 / 0.3 = 16.67.

So, we can fit in 16 tiles in a column. Since 16 × 0.3 = 4.8, 5 – 4.8 = 0.2 metres along the breadth will not be covered by tiles.

Mathematical Description:

Total number of tiles required = (Number of tiles along the length × Number of tiles along the breadth) + Number of tiles along the uncovered area

(20 × 16) + 20 = 320 + 20 = 340.

Interpretation: We need 340 tiles to cover the floor.

Example 5 : In the year 2000, 191 member countries of the U.N. signed a declaration. In this declaration, the countries agreed to achieve certain development goals by the year 2015. These are called the millennium development goals. One of these goals is to promote gender equality. One indicator for deciding whether this goal has been achieved is the ratio of girls to boys in primary, secondary and tertiary education. India, as a signatory to the declaration, is committed to improve this ratio. The data for the percentage of girls who are enrolled in primary schools is given Table .

Year	Enrolment (in %)
1991-92	41.9
1992-93	42.6
1993-94	42.7
1994-95	42.9
1995-96	43.1
1996-97	43.2
1997-98	43.5
1998-99	43.5
1999-2000	43.6*
2000-01	43.7*
2001-02	44.1*

Solution:

Step 1 : Formulation: we will see how the enrolment grows after 1991 by comparing the number of years that has passed after 1991 and the enrolment. Let us take 1991 as the 0th year, and write 1 for 1992 since 1 year has passed in 1992 after 1991. Similarly, we will write 2 for 1993, 3 for 1994, etc. So, given table will now look like as Table given below. The increase in enrolment is also given in the following table :

Year	Enrolment (in %)	Increase
0	41.9	0
1	42.6	0.7
2	42.7	0.1
3	42.9	0.2
4	43.1	0.2
5	43.2	0.1
6	43.5	0.3
7	43.5	0
8	43.6	0.1
9	43.7	0.1
10	44.1	0.4

$$\text{The mean of the increasing values} = \frac{0.7+0.1+0.2+0.2+0.1+0.3+0+0.1+0.1+0.4}{10} = \frac{2.2}{10}$$

$$= 0.22$$

Let us assume that the enrolment steadily increases at the rate of 0.22 per cent.

Mathematical Description: We have assumed that the enrolment increases steadily at the rate of 0.22% per year.

So, the Enrolment Percentage (EP) in the first year = 41.9 + 0.22

EP in the second year = 41.9 + 2 × 0.22

EP in the third year = 41.9 + 3 × 0.22

So, the enrolment percentage in the n^{th} year = 41.9 + 0.22×n , for ≥ 1.→(1)

Now, we also have to find the number of years by which the enrolment will reach 50%.

So, we have to find the value of in the equation or formula 50=41.9+0.22n→(2)

Step 2: Solution:

$$0.22n = 50 - 41.9 = 8.1$$

$$n = \frac{8.1}{0.22} = 36.8$$

Step 3: Interpretation:

Since the number of years is an integral value, we will take the next higher integer, 37. So, the enrolment percentage will reach 50% in 1991 + 37 = 2028.

Step 4: Validation: Let us check if Formula (2) is in agreement with the reality. Let us find the values for the years we already know, using Formula (2), and compare it with the known values by finding the difference. The values are given in below Table.

Year	Enrolment (in %)	Values given by (2) (in %)	Difference (in %)
0	41.9	41.90	0
1	42.6	42.12	0.48
2	42.7	42.34	0.36
3	42.9	42.56	0.34
4	43.1	42.78	0.32
5	43.2	43.00	0.20
6	43.5	43.22	0.28
7	43.5	43.44	0.06
8	43.6	43.66	–0.06
9	43.7	43.88	–0.18
10	44.1	44.10	0.00

As you can see, some of the values given by Formula (2) are less than the actual values by about 0.3% or even by 0.5%. This can give rise to a difference of about 3 to 5 years since the increase per year is actually 1% to 2%. We may decide that this much of a difference is acceptable and stop here. In this case, (2) is our mathematical model. Suppose we decide that this error is quite large, and we have to improve this model. Then we have to go back to Step 1, the formulation, and change Equation (2). Let us do so. S 1 : R : We still assume that the values increase steadily by 0.22%, but we will now introduce a correction factor to reduce the error. For this, we find the mean of all the errors. This is

$$\frac{0 + 0.48 + 0.36 + 0.34 + 0.2 + 0.28 + 0.06 - 0.06 - 0.18 + 0}{10} = 0.18$$

We take the mean of the errors, and correct our formula by this value.

Revised Mathematical Description : Let us now add the mean of the errors to our formula for enrolment percentage given in (2). So, our corrected formula is:

Enrolment percentage in the n^{th} year = 41.9 + 0.22n + 0.18 = 42.08 + 0.22n , forn≥1→(3)

We will also modify Equation (2) appropriately. The new equation for is: 50=42.08+0.22n →(4)

Step 2 : Altered Solution : Solving Equation (4) for , we get

$$n = \frac{50 - 42.08}{0.22} = \frac{7.92}{0.22} = 36$$

Step 3 : Interpretation : Since = 36, the enrolment of girls in primary schools will reach 50% in the year 1991 + 36 = 2027.

Step 4 : Validation : Once again, let us compare the values got by using Formula (4) with the actual values. Table A2.5 gives the comparison.

Year	Enrolment (in %)	Values given by (2)	Difference between values	Values given by (4)	Difference between values
0	41.9	41.90	0	41.9	0
1	42.6	42.12	0.48	42.3	0.3
2	42.7	42.34	0.36	42.52	0.18
3	42.9	42.56	0.34	42.74	0.16
4	43.1	42.78	0.32	42.96	0.14
5	43.2	43.00	0.2	43.18	0.02
6	43.5	43.22	0.28	43.4	0.1
7	43.5	43.44	0.06	43.62	– 0.12
8	43.6	43.66	– 0.06	43.84	– 0.24
9	43.7	43.88	– 0.18	44.06	– 0.36
10	44.1	44.10	0	44.28	– 0.18

As you can see, many of the values that (4) gives are closer to the actual value than the values that (2) gives. The mean of the errors is 0 in this case.

So, Equation (4) is our mathematical description that gives a mathematical relationship between years and the percentage of enrolment of girls of the total enrolment. We have constructed a mathematical model that describes the growth.

EXERCISE A2.2

1. **We have given the timings of the gold medalists in the 400-metre race from the time the event was included in the Olympics, in the table below. Construct a mathematical model relating the years and timings. Use it to estimate the timing in the next Olympics.**

Year	Timing (in seconds)
1964	52.01
1968	52.03
1972	51.08
1976	49.28
1980	48.88
1984	48.83
1988	48.65
1992	48.83
1996	48.25
2000	49.11
2004	49.41

Sol: Let us first convert the problem into a mathematical problem.

Formulation: Let us take 1964 as 0^{th} year, and write 1^{st} for 1968, 2^{nd} for 1972 and so on. So given table will now look as table

Year	Timing (in seconds)
0	52.01
1	52.03
2	51.08
3	49.28
4	48.88
5	48.83
6	48.65
7	48.83
8	48.25
9	49.11
10	49.11

The reduction in timings of gold medalist in the following table.

Year	Timing (in seconds)	Difference
0	52.01	0
1	52.03	0.02
2	51.08	-0.95
3	49.28	-1.8
4	48.88	-0.4
5	48.83	-0.05
6	48.65	-0.18
7	48.83	0.18
8	48.25	-0.58
9	49.11	0.86
10	49.11	0.3
	Total	-2.6

$$Mean\ of\ differences = \frac{-2.6}{10} = -0.26$$

Let us assume that the timing steadily decreases at the rate of 0.26.

Mathematical Description:

$So, timing\ in\ the\ first\ year\ = 52.01 - 0.26$

$Timing\ in\ the\ second\ year\ = 52.01 - 0.26 \times 2$

$Timing\ in\ the\ n^{th}\ year\ (t) = 52.01 - 0.26 \times n$

Finding the solution:

The estimate timing in the next Olympics (n=11) is

$t = 52.01 - 0.26 \times 11 = 52.01 - 2.86 = 49.15\ sec$

EXERCISE A2.3

1. **How are the solving of word problems that you come across in textbooks different from the process of mathematical modelling?**

Sol: We have already mentioned that the formulation part could be very detailed in real-life situations. Also, we do not validate the answer in word problems. Apart from this word problem have a 'correct answer'. This need not be the case in real-life situations.

2. **Suppose you want to minimise the waiting time of vehicles at a traffic junction of four roads. Which of these factors are important and which are not? (i) Price of petrol. (ii)The rate at which the vehicles arrive in the four different roads. (iii) The proportion of slow-moving vehicles like cycles and rickshaws and fast moving vehicles like cars and motorcycles**

Sol: The important factors are (ii) and (iii). Here (i) is not an important factor although it can have an effect on the number of vehicles sold.